# Lecture 4 - Consumer Choice

ECON 3070 - Intermediate Microeconomic Theory

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# Overview

In the previous lecture, we looked at consumer preferences.

- We considered how consumers ranked various bundles of goods.
- We saw how to graphically and numerically represent these preferences.

In the next lecture, we will consider how consumers choose which bundle to consume.

# The Budget Constraint

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Because a consumer can only spend as much as their income, $I$, their budget constraint can be expressed as:

$$P_x x + P_y y \leq I$$

# Budget Constraint

The budget constraint can be generalized to more than two goods:

$$P_{x_1}x_1 + P_{x_2}x_2 + ... + P_{x_n}x_n \leq I$$

- Because more is better (remember our assumptions), the consumer will always use up their whole budget ($I$).
- In general, however, a consumer can purchase any bundle which costs $I$\$ or less.

# The Budget Line

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The rearranged equation is

$$y = \frac{I}{P_y} - \frac{P_x}{P_y}x$$

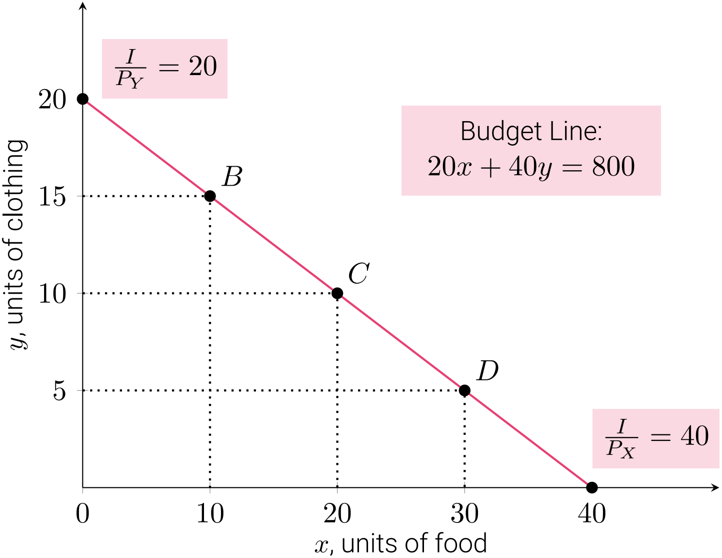
- This equation in slope-intercept form can be used to plot our budget line.

A **budget line** shows all combinations of goods $x$ and $y$ that can be purchased for a given budget $I$, at given prices $P_x$ and $P_y$.

## Try It Yourself

Write the budget constraint with $I = 800$, $P_X = 20$, and $P_Y = 40$. Then, solve for $y$.

*Figure:* The Budget Line



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**Conceptual questions**

- Remember that a change in income shifts the budget line, without changing the slope (why?)
- What would a change in the price of one (or both) of the goods do?

## Try It Yourself

Suppose that Sammy has \$200 in disposable income to spend each month on concert tickets and football tickets. Concert tickets cost \$50 and football tickets cost \$20. The price of concert tickets increases by \$25, and the price of football tickets increases by \$10. Assume that Sammy's disposable income doesn't change.

Draw the budget line before and after.

# Optimal Choice

Now that we know the consumer’s problem (maximize utility), and their constraint (the budget constraint), we can determine the consumer’s optimal choice.

- The **optimal choice** is the amount of each good that should be purchased in order to maximize utility.
- This basket of goods must be located on the budget line (why not below?).

# Optimal Choice

More formally, we can state the consumer's **utility maximization** problem as:

$$\max_{(x,y)} U(x,y) \text{ subject to: } P_x x + P_y y \leq I$$

# Utility Maximization

How can we interpret this graphically?

- Remember that the consumer wants to get to the highest indifference curve possible.
- That means moving as far up and to the right as possible
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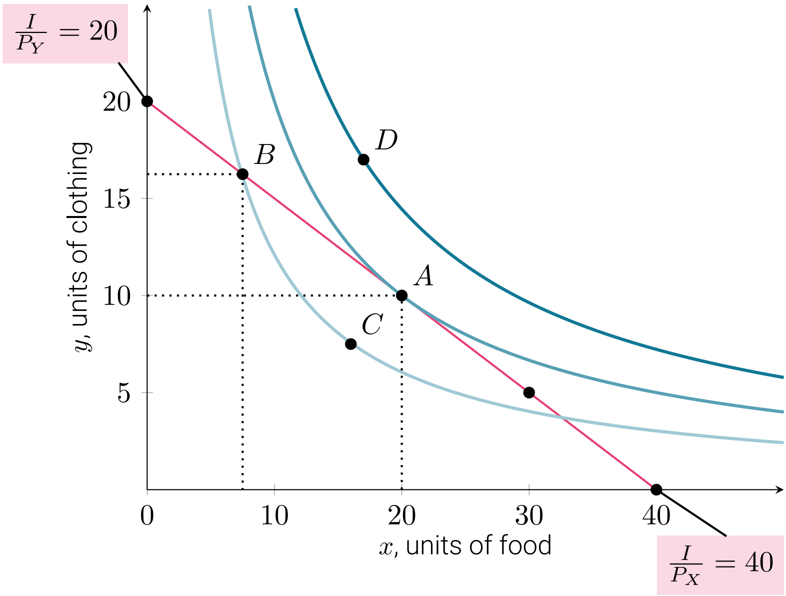
*The optimal bundle ends up being (in most cases) the point where the budget line is tangent with one of the indifference curves.*

*Figure:* Utility Maximization



# Utility Maximization

To find the consumer's optimal level of consumption of $x$ and $y$, we need some calculus.

We will maximize utility in two-ways:

1. The method of lagrange
2. 'plugging-in' the constraint to our problem.

# Method of Lagrange

Define the Lagrangian ($\mathcal{L}$) as

$$\mathcal{L}(x, y, \lambda) = U(x, y) + \lambda(I - P_x x - P_y y)$$

Maximizing the Lagrangian with respect to $x, y$, and $\lambda$ is equivalent to maximizing the original problem.

# Method of Lagrange

Define the Lagrangian ($\mathcal{L}$) as

$$\mathcal{L}(x, y, \lambda) = U(x, y) + \lambda(I - P_x x - P_y y)$$

To maximize the Lagrangian, take derivatives with respect to $x$, $y$, and $\lambda$, and set them equal to zero. The first order conditions for an optimum are:

$$\frac{\delta \mathcal{L}}{\delta x} = 0 \Rightarrow \frac{\delta U(x, y)}{\delta x} = \lambda P_x$$

$$\frac{\delta \mathcal{L}}{\delta y} = 0 \Rightarrow \frac{\delta U(x, y)}{\delta y} = \lambda P_y$$

$$\frac{\delta \mathcal{L}}{\delta \lambda} = 0 \Rightarrow I - P_x x - P_y y = 0$$

# Method of Lagrange

Remember that $\frac{\delta U(x,y)}{\delta x} = MU_x$ and $\frac{\delta U(x,y)}{\delta y} = MU_y$. Then, rearranging terms:

$$\frac{MU_x}{P_x} = \lambda \qquad \text{and} \qquad \frac{MU_y}{P_y} = \lambda$$

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We can combine the first two equations and rearrange terms, and the result is:

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The resulting equality is our optimality condition.

# Unconstrained Optimization

To solve using unconstrained optimization, first solve for $y$ in the budget constraint $P_x x + P_y y = I$.

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We then plug this into our utility function, and our problem becomes:

$$\max_x U(x, \frac{I - P_x x}{P_y})$$

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- Take derivatives, and set them equal to zero.

Taking the total derivative of $U(x, \frac{I-P_x x}{P_y})$ w.r.t $x$ (using the chain-rule!):

$$\begin{aligned} \frac{dU}{dx} &= \frac{\delta U}{\delta x} + \frac{\delta U}{\delta y}\frac{dy}{dx} = 0 \\ &\Rightarrow \frac{\delta U}{\delta x} - \frac{\delta U}{\delta y}\frac{P_x}{P_y} = 0 \\ &\Rightarrow \frac{\delta U}{\delta x}/P_x = \frac{\delta U}{\delta y}/P_y \\ &\rightarrow \frac{MU_x}{P_x} = \frac{MU_y}{P_y} \end{aligned}$$

# The Optimality Condition

We have just found our first optimality condition! Now that we have derived this from first principles, we don't have to do that again!

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- The optimality condition tells us that the consumer maximizes utility when the marginal utility per dollar spent on both goods is equal.
- Think about what would happen if it weren't equal for the two goods.

# The Optimality Condition

We can also rearrange the optimality condition as follows:

$$\frac{MU_x}{MU_y} = \frac{P_x}{P_y}$$

- This tells us that the slope of our indifference curve should equal the slope of our budget line.

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- This tells us that the slope of our indifference curve should equal the slope of our budget line.
- The result is that the optimal basket is located where the budget line is exactly tangent to one of the indifference curves.
- Compare this to our graphical result.

# The Optimality Condition

So far, we only considered the case where baskets contain two goods. However, the optimality condition holds for any number of goods.

$$\frac{MU_{x_1}}{P_{x_1}} = \frac{MU_{x_2}}{P_{x_2}} = ... = \frac{MU_{x_n}}{P_{x_n}}$$

- As before, the utility per dollar spent on each good (or "bang for your buck"), must be equal.

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This is a *key insight*! This optimality condition is how we think individuals balance trade-offs on what to buy

## Try It Yourself

Consider a market with only two goods, bread and cheese. Suppose that at Francis' current level of consumption, his marginal utility of bread is 4, and is marginal utility of cheese is 6. Suppose that a loaf of bread costs $3, and a pound of cheese costs $6. What should Francis do?

A) Consume more bread and less cheese.
B) Consume more cheese and less bread.
C) Leave his consumption of bread and cheese unchanged.
D) It is impossible to tell without knowing how much bread and cheese he is consuming.

# Finding the Optimal Bundle

Now that we know something about the optimal bundle (from the optimality condition), how do we actually find it?

- Well we also know that the consumer has only $\$I$ to spend.
- This gives us a system of two equations (the optimality condition and the budget constraint), and two unknowns (quantity consumed of goods $x$ and $y$).

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All that's left is some (sometimes messy) algebra.

# Finding the Optimal Bundle

Suppose $U(x,y) = \sqrt{x}\sqrt{y}$. Let $P_x = 2, P_y = 3$ and $I = 24$.

Calculate $MU_x$ and $MU_y$ and the optimality condition

# Finding the Optimal Bundle

Suppose $U(x,y) = \sqrt{x}\sqrt{y}$. Let $P_x = 2, P_y = 3$ and $I = 24$.

$MU_x = \frac{\sqrt{y}}{2\sqrt{x}}$ and $MU_y = \frac{\sqrt{x}}{2\sqrt{y}}$

Then our optimality condition is $\frac{\sqrt{y}}{2\sqrt{x}} / \frac{\sqrt{x}}{2\sqrt{y}} = \frac{2}{3}$

$$\implies y = \frac{2}{3}x$$

Plugging the optimality condition into our budget constraint

$$2x + 3y = 24 \implies 2x + 3(\frac{2}{3}x) = 24 \implies x^* = 6$$

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Plugging $x^*$ back into the optimality condition, $y^* = \frac{2}{3}6 = 4$

# Finding the Optimal Bundle

We could also leave out prices and income, and simply solve for $x$ and $y$ in terms of $P_x$, $P_y$, and $I$.

Take the rearranged optimality condition $y = x\frac{P_x}{P_y}$. Plug into budget constraint

$$P_x x + P_y x(\frac{P_x}{P_y}) = I \implies x^* = \frac{I}{2P_x}$$

Plug $x$ back into the optimality condition:

$$y^* = \frac{I}{2P_x}\frac{P_x}{P_y} = \frac{I}{2P_y}$$

# Demand Functions

The resulting **demand functions** are:

$$x^*(P_x, P_y, I) = \frac{I}{2P_x} \quad \text{and} \quad y^*(P_x, P_y, I) = \frac{I}{2P_y}$$

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The **demand function** is useful since we do the work once and then can predict what that person will consumer under any $(P_x, P_y, I)$ combination for comparitive statics

- I'll likely do this on the test

# Three Utility Functions

How do you find demand for the three utility functions?

1. **Cobb-Douglas**: $U(x, y) = x^{\alpha} y^{\beta}$

2. **Perfect Subsitutes**: $U(x, y) = ax + by$

3. **Perfect Complements**: $U(x, y) = \min\big(ax, by\big)$

# Cobb-Douglas

$U(x, y) = x^\alpha y^\beta$. Our optimality condition is

$$\frac{MU_x}{P_x} = \frac{MU_a}{P_y} \implies \frac{\alpha x^{\alpha-1} y^\beta}{P_x} = \frac{\beta x^\alpha y^{\beta-1}}{P_y} \implies x = \frac{\alpha P_y}{\beta P_x} y$$

Our budget constraint is

$$I = P_x x + P_y y$$

Solve these two equations for $x^*$ and $y^*$.

# Perfect Substitutes

$U(x, y) = ax + by$. Our optimality condition is

$$\frac{MU_x}{P_x} = \frac{MU_a}{P_y} \implies \frac{a}{P_x} = \frac{b}{P_y}$$

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**Wait!** This isn't always true!!

- e.g. what if $U(x, y) = x + y$, $P_x = 1$, and $P_y = 2$. This says $1 = 1/2$ which isn't true

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Since both goods are perfectly substitutable, you should consumer **ONLY** the good that provides marginal utility per dollar (bang for your buck)

# Perfect Substitutes

$U(x, y) = ax + by$. For perfect substitutes our demand rule is simple:

1. If $MU_x/P_x > MU_y/P_y$:

$$x^* = I/P_x \quad \text{and} \quad y^* = 0$$

2. If $MU_x/P_x < MU_y/P_y$:

$$x^* = 0 \quad \text{and} \quad y^* = I/P_y$$

3. If $MU_x/P_x = MU_y/P_y$:

   Pick any bundle on your budget line

# Perfect Complements

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Instead, recognize that perfect complements means you have to consume always in the proportion $ax = by$.

- If $ax > by$, you are wasting money on $x$ and vice-versa if $by > ax$

$ax = by$ is therefore our optimality condition! Our budget constraint is

$$I = P_x x + P_y y$$

Solve these two equations for $x^*$ and $y^*$.

# Clubs and Two-Part Tariffs

Suppose that you have \$300 to spend on CDs (what are those?), and that each CD costs \$20 (call this budget line 1)

- You can purchase 15 CDs at most, or spend some of that money on other things.

But what if you could pay \$100, and each CD only cost \$10 (call this budget line 2). This is called a **two-part tariff**. Part 1: pay to enter the club. Part 2: pay for quantity you want.

*Figure:* Consumer signs up for club

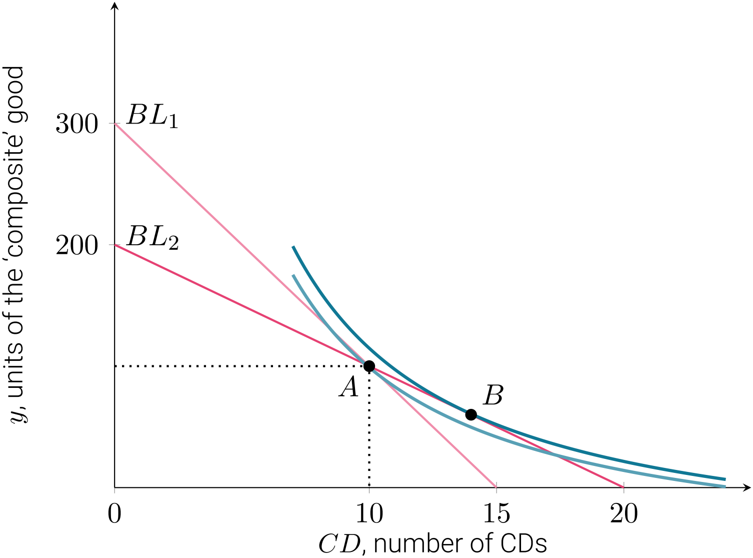
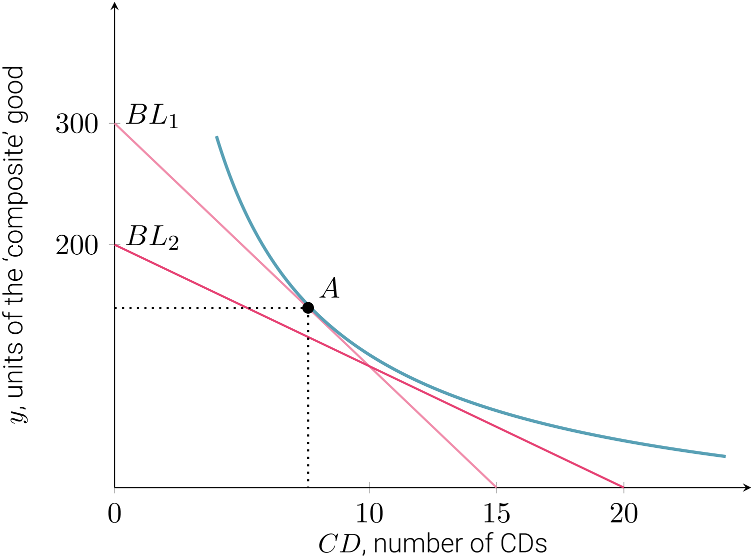


*Figure:* Consumer does not sign up for club



# Clubs and Two-Part Tariffs

Note that with the two-part tariff:

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What are some other examples of products where you can pay a flat rate in exchange for a lower per-unit price?