Good luck to you!

1. If disposable incomes rise by 5% and demand changes from 100 units to 105, what is the income elasticity of demand? Interpret this number.

   Solution: *(10 pts.)*

   $\%\Delta$ in $Q = \frac{105-100}{100} = 0.05$

   $$\varepsilon_{I,Q} = \frac{\%\Delta \text{ in } Q}{\%\Delta \text{ in } I} = \frac{0.05}{0.05} = 1$$

   To intrepret, a 1% increase in income yields a 1% increase in quantity demanded.

2. The law of demand tells us that when the price of a good goes up, the quantity demanded for that good goes down. Describe why a business might want to know the price elasticity of demand.

   Solution: *(10 pts.)*

   The price elasticity of demand tells you *how much* demand decreases with a price increase. This can be used to help optimally price products. If a good is very inelastic, then you can raise prices without losing many sales, for example.

3. Consider the demand function $x^*(P_x, P_y, I) = \frac{I}{P_x P_y}$

   (a) Does this good satisfy the law of demand?

   (b) Is this good a normal good or a inferior good?

   (c) Are goods x and y substitutes, complements, or neither?

   (d) If the maker of good y raises the price of their good, how does that affect the sales of good x?

   Solution: *(20 pts.)*

   (a) *(5 pts.)* $\frac{\partial x^*}{\partial P_x} < 0$ (denominator grows, so x^* goes down). Therefore, this good satisfies the law of demand ($P_x \uparrow$ implies $x^* \downarrow$)

   (b) *(5 pts.)* $\frac{\partial x^*}{\partial I} = \frac{1}{P_x P_y} > 0$. Therefore $I \uparrow$ implies $x^* \uparrow$. Hence, good x is a normal good.

   (c) *(5 pts)*] $\frac{\partial x^*}{\partial P_y} < 0$. Therefore $P_y \uparrow$ implies $x^* \downarrow$ and hence goods x and ys are complements.

   (d) *(5 pts.)* Since x and y are complements, the demand for good x goes down after an increase in price of y.

4. For the utility function $U(x, y) = 2x + 4y$, Calculate the marginal rate of substitution $MRS_{x,y}$. Interpret this number in words.

5. For the following utility functions, draw three indifference curves

(a) $U(x,y) = 2x + 4y$

(b) $U(x,y) = min(x, 2y)$

Solution: *(15 pts.)*

(a) *(7.5 pts.)* $\bar{U} = 2x + 4y \implies y = \bar{U}/2 - x/2$. Therefore, indifference curves are a bunch of downward-sloping lines

(b) *(7.5 pts.)* "brackets" along the line $x = 2y$.

6. There are two suppliers in the market, Starbucks and Dunkin, who sell coffee. Their supply curves are given by $Q^S_{Starbucks} = -10 + 2P$ and $Q^S_{Dunkin} = -20 + 2P$. Demand for coffee in Boulder is given by $Q^D_{mkt} = 42 - 2P$.

   (a) Solve for the market supply, Q^S_{mkt}.
   Solution:

$$Q^S_{mkt} = \begin{cases} 0 & \text{if } 0 < P < 5 \\ -10 + 2P & \text{if } 5 \leq P \leq 10 \\ -30 + 4P & \text{if } P \geq 10 \end{cases}$$

   (b) What is the equilibrium price and quantity for coffee?
   Solution:

$$P^* = 12 \text{ and } Q^* = -30 + 4 * 12 = 18$$

   (c) Suppose the demand curve, shifts out to $Q^D_{mkt} = 60 - 2P$, will the market price go up or go down? (hint: no need for math)

   Solution: *(15 pts.)*

   (a) *(5 pts.)*

$$Q^S_{mkt} = \begin{cases} 0 & \text{if } 0 < P < 5 \\ -10 + 2P & \text{if } 5 \leq P \leq 10 \\ -30 + 4P & \text{if } P \geq 10 \end{cases}$$

   (b) *(5 pts.)*
   Try $-10 + 2P = 42 - 2P \implies P^* = 13$. This would violate $5 \leq P < 10$.
   Try $-30 + 4P = 42 - 2P \implies P^* = 12$. Plugging into demand gives $Q^* = 42 - 2 * 12 = 18$.

   (c) *(5 pts.)* The demand curve goes up, so the equilibrium price increases.

7. Consider the following consumer optimal consumption problem

$$\max_{x,y} x^{1/5} y^{4/5} \text{ subject to } P_x x + P_y y = I$$

   (a) What is the optimality condition? Interpret in words, why we know that if the consumer is consuming optimally the optimality condition must hold.

(b) Solve for optimal demand $x^*(P_x, P_y, I)$ and $y^*(P_x, P_y, I)$

(c) At price $P_x = 5$, $P_y = 10$, and $I = 100$, what is optimal demand for x and y?

(d) Using the optimal demand, x^*, is good x a normal good or an inferior good? How do you know?

(e) What is the demand curve for good x, $x^*(P_x)$, when $P_y = 10$ and $I = 100$? Draw this curve. From your graph, does this good sastisfy the law of demand?

Solution: *(20 pts.)*

(a) *(10 pts.)*

$$\frac{MU_x}{P_x} = \frac{MU_y}{P_y} \implies y = 4\frac{P_x}{P_y}x$$

(b) *(3 pts.)* Pluging the optimality condition into the budget constraint yields

$$I = P_x * x + P_y * 4\frac{P_x}{P_y}x = 5P_x x \implies x^* = \frac{I}{5P_x}$$

Similarly, $y^* = \frac{4I}{5P_y}$.

(c) *(3 pts.)*

$$x^* = \frac{100}{5 * 5} = 4 \text{ and } y^* = \frac{4 * 100}{5 * 10} = 8$$

(d) *(2 pts.)*

$$\frac{\partial x^*}{\partial I} = \frac{1}{5P_x} > 0 \implies x \text{ is a normal good.}$$

(e) *(2 pts.)*

$$x^*(P_x) = \frac{100}{5P_x} = \frac{20}{P_x}$$

This is a downward sloping curve, which satisfies the law of demand ($P_x \uparrow$ implies $x^* \downarrow$)