

Midterm 2 Prep

Production Functions

1. Consider the production function $Q(K, L) = 5K + L$.
 - (a) What is the marginal product of capital and labor? Interpret these in words.
 - (b) What is marginal rate of technical substitution $MRTS_{K,L}$?
 - (c) What does this production function tell you about the substitutability of labor and capital?
 - (d) Does this production function exhibit constant, increasing or decreasing returns to scale?
 - (e) Draw 3 isoquant curves for this production function.
 - (f) Holding fixed capital at $K = 10$, what happens to the marginal product of labor as the firm increases L ?
2. Consider the production function $Q(K, L) = 5KL$.
 - (a) What is the marginal product of capital and labor? Interpret these in words.
 - (b) What is marginal rate of technical substitution $MRTS_{K,L}$?
 - (c) Does this production function exhibit constant, increasing or decreasing returns to scale?
 - (d) Draw 3 isoquant curves for this production function.
 - (e) Holding fixed capital at $K = 10$, what happens to the marginal product of labor as the firm increases L ?
 - (f) If the production function becomes $Q(K, L) = 5K^{3/2}L$, is this capital-biased, labor-biased, or neutral technological change?
3. Let T represent car tires and F represent car frames. The production of a car requires 4 tires and 1 frame.
 - (a) Draw 3 isoquant curves for this production function.
 - (b) Write this as a mathematical production function.

Cost Minimization

1. For the following scenarios, find the cost-minimizing input bundles and the total costs of producing the given quantity
 - (a) $Q(K, L) = 2L + 4K$, $r = 8$, $w = 2$, and $\bar{Q} = 40$
 - (b) $Q(K, L) = 2L + 4K$, $r = 8$, $w = 6$, and $\bar{Q} = 40$
 - (c) $Q(K, L) = K^{1/2}L^{1/2}$, $r = 8$, $w = 8$, and $\bar{Q} = 40$
 - (d) $Q(K, L) = \min(K, 2L)$, $r = 8$, $w = 2$, and $\bar{Q} = 40$
2. Describe in words, why a firm producing with cobb-douglas technology needs to have $MP_K/r = MP_L/w$.

3. A firm is producing the required amount of output, \bar{Q} units with $MP_k/r = 2$ and $MP_L/w = 4$. Is this firm producing at the lowest-possible cost? If not, explain how the firm could shift between inputs and lower costs.
4. Consider the production function $Q(K, L) = KL^{1/2}$.
 - (a) Solve for the conditional input demand functions.
 - (b) Are labor and capital normal or inferior inputs?
 - (c) In the short-run, the firm's capital is fixed at $\bar{K} = 10$. Find the short-run conditional labor demand function.
 - (d) What is the labor demanded to produce $\bar{Q} = 20$ units in the short-run when $\bar{K} = 10$.

Cost Curves

1. Consider the production function $Q(K, L) = K^{1/2}L^{1/2}$.
 - (a) Let $w = 2$ and $r = 4$. Solve for the total cost function of producing \bar{Q} units.
 - (b) Now, solve for the long-run total cost curve as a function of \bar{Q} , w , and r .
 - (c) In the short-run, the firm's capital is fixed at $\bar{K} = 16$. Find the short-run conditional labor demand function.
 - (d) Find the short-run total cost curve when $\bar{K} = 16$. Is it true that $TC(\bar{Q}, w, r) \leq SRTC(\bar{Q}, w, r)$?
 - (e) In words, explain why the short-run total cost curve has to be larger than the long-run total cost curve.
2. Consider the total cost curve $TC(Q) = \frac{Q}{25}\sqrt{wr}$.
 - (a) What is the marginal cost of producing the 11th unit of output when $w = 4, r = 4$?
 - (b) What is the firm's average total cost of producing 40 units when $w = 2$ and $r = 8$? Interpret this in words
 - (c) Does this firm experience economies of scale?
3. Consider the total cost curve $TC(Q) = Q^2 - 2Q + 10$.
 - (a) What is the average total cost curve, $ATC(Q)$ and the marginal cost curve, $MC(Q)$?
 - (b) What is the quantity where the firm is producing at the minimum efficient scale?
 - (c) When is the firm experiencing economies of scale and diseconomies of scale? (*Hint*: use the previous question)
 - (d) In the short run, the firm can not change the amount of labor they use because of worker's contracts. What is the relationship between the short-run marginal cost and the long-run marginal cost?