Chapter 6

1. For each of the production functions below, answer the following questions:

   (i) What is the marginal product of each of the inputs?

   (ii) Does the marginal product of L diminish, remain constant, or increase as the level of L increases?

   (iii) What is the marginal rate of substitution ($MRTS$) of L and K?

   (iv) Is the $MRTS_{L,K}$ diminishing, constant, or increasing as the firm substitutes more L for K, holding the level of output constant?

   (v) Does the production function exhibit increasing, constant, or decreasing returns to scale? Show your work.

   (a) $Q(K,L) = 6L^{\frac{1}{2}}K^{\frac{1}{2}}$

   (b) $Q(K,L) = 24L - 1/2L^2 + 30K - K^2$

(c) $Q(K, L) = \sqrt{L} + K$

(d) $Q(K, L) = K + 2L$

2. Suppose that a firm originally has the production function $Q(K, L) = 10L + 10K$. Over time as the company learns, the production function changes to $Q(K, L) = 40L + 20K$.

   (a) Show that the innovation has resulted in technological progress in the sense defined in the notes.

   (b) Is the technological progress neutral, labor-saving, or capital-saving? How can you tell?

3. Suppose that a firm originally has the production function $Q(K, L) = \sqrt{KL}$. Over time as the company learns, the production function changes to $Q(K, L) = \sqrt{KL}$.

   (a) Show that the innovation has resulted in technological progress in the sense defined in the notes.

   (b) Is the technological progress neutral, labor-saving, or capital-saving? How can you tell?

4. Suppose that a firm originally has the production function $Q(K, L) = KL$. Over time as the company learns, the production function changes to $Q(K, L) = K^2 L^2$.

   (a) Show that the innovation has resulted in technological progress in the sense defined in the notes.

   (b) Is the technological progress neutral, labor-saving, or capital-saving? How can you tell?

5. Let T represent car tires and F represent car frames. The production of a car requires 4 tires and 1 frame.

   (a) Draw at least two isoquants for car production, with tires on the horizontal axis and frames on the vertical axis. Be sure to label the axes and the isoquants.

   (b) Write the production function in mathematical notation, $Q(T, F)$, for the car producer's production function.