Tradable Permits

1. Two firms can control emissions at the following marginal costs: $MC_x = 80a_x$ and $MC_y = 40a_y$ where a_x and a_y are, respectively, the amount of emissions reduced by firm x and firm y. Assume that with no control at all, each firm would be emitting 50 units of emissions or a total of 100 units for both firms.

   (a) Which firm is better at abating pollution?

   (b) If the goal is to reduce total emissions to 60 units. How many units must be abated? Write out the abatement constraint in mathematical terms

   (c) Consider a uniform standard. How many units must be abated by both firms? How much did each firm have to pay to abate their marginal unit of pollution?

   (d) Consider a cap-and-trade system that aims for a total 60 units of emissions.

      i. In words, describe why the marginal abatement costs for each firm must be equal to eachother in order to be at equilibrium (the optimality condition).

      ii. Using the optimality condition and the abatement constraint, solve for the equilibrium allocation of permits to each firm?

      iii. At what price would these permits sell for at an auction?

   (e) Assume that the control authority wanted to reach its objective by using an emissions charge system instead.

      i. What tax amount should them impose to reach this equilibrium?

      ii. How much revenue would the government collect?

   (f) Why is cap-and-trade more cost-effective than a uniform standard where each firm reduces pollution by the same amount?

2. Two firms can control emissions at the following marginal costs: $MC_x = 200a_x$ and $MC_y = 100a_y$ where a_x and a_y are, respectively, the amount of emissions reduced by firm x and firm y. Assume that with no control at all, each firm would be emitting 20 units of emissions or a total of 40 units for both firms.

   (a) Consider a cap-and-trade system that aims for a total reduction of 21 units of emissions is necessary.

      i. What is the equilibrium allocation of permits to each firm?

      ii. At what price would these permits sell for at an auction

   (b) Assume that the control authority wanted to reach its objective by using an emissions charge system instead.

      i. What tax amount should them impose to reach this equilibrium?

      ii. How much revenue would the government collect?

   (c) Why is cap-and-trade more cost-effective than a uniform standard where each firm reduces pollution by 10.5 units?

3. Two firms can control emissions at the following marginal costs: $MC_x = 5 + 10a_x$ and $MC_y = 11a_y$ where a_x and a_y are, respectively, the amount of emissions reduced by firm x and firm y. Assume that with no control at all, each firm would be emitting 10 units of emissions or a total of 20 units for both firms.

(a) Consider a cap-and-trade system that aims for a total reduction of 10 units of emissions.

   i. What is the equilibrium allocation of permits to each firm?

   ii. At what price would these permits sell for at an auction

(b) Assume that the control authority wanted to reach its objective by using an emissions charge system instead.

   i. What tax amount should them impose to reach this equilibrium?

   ii. How much revenue would the government collect?