Two period model of a non-renewable resource

Consider extraction of a non-renewable natural resource. The inverse demand function for the depletable resource is $P = 20 - 2Q$ in both periods 1 and 2 and the marginal cost of supplying it is \$3. The discount rate is 10%. There are 10 units total.

1. Explain what the resource constraint is and then write it in mathematical form
2. What is the per-unit profit for extracting resources in period 1?
3. What is the per-unit profit for extracting resources in period 2? What is the present value of the per-unit profit from the viewpoint of period 1?
4. Describe in words why you must equate the marginal unit's profit in each periods (the optimality condition).
5. What two pieces of information are needed in order to solve for the optimal extraction Q_1^* and Q_2^*?
6. Solve for the optimal extraction Q_1^* and Q_2^*.
7. Describe using specific numbers why $Q_1 = 3$ and $Q_2 = 4.5$ is not optimal
8. What is the marginal user cost? Interpret this number.
9. Now assume $r = 0$. What is the optimal allocation now? Why did optimal allocation change in the direction that it did?

Tradable Permits

Two firms can control emissions at the following marginal costs: $MC_1 = 80a_x$ and $MC_2 = 40a_y$ where a_x and a_y are, respectively, the amount of emissions reduced by the first and second firms. Assume that with no control at all, each firm would be emitting 50 units of emissions or a total of 100 units for both firms.

1. Which firm is better at abating pollution?
2. If the goal is to reduce total emissions to 60 units. How many units must be abated? Write out the abatement constraint in mathematical terms
3. Consider a uniform standard. How many units must be abated by both firms? How much did each firm have to pay to abate their marginal unit of pollution?
4. Consider a cap-and-trade system that aims for a total 60 units of emissions.
   (a) In words, describe why the marginal abatement costs for each firm must be equal to eachother in order to be at equilibrium (the optimality condition).
   (b) Using the optimality condition and the abatement constraint, solve for the equilibrium allocation of permits to each firm?
   (c) At what price would these permits sell for at an auction?
5. Assume that the control authority wanted to reach its objective by using an emissions charge system instead.
   (a) What tax amount should them impose to reach this equilibrium?
   (b) How much revenue would the government collect?
6. Why is cap-and-trade more cost-effective than a uniform standard where each firm reduces pollution by the same amount?