ECON 3535 Math Practice 1

Two period model of a non-renewable resource

1. Consider extraction of a non-renewable natural resource. The inverse demand function for the depletable resource is P = 20 - 2Q in both periods 1 and 2 and the marginal cost of supplying it is \$3. The discount rate is 10%. There are 10 units total.

- (a) Explain what the resource constraint is and then write it in mathematical form
- (b) What is the per-unit profit for extracting resources in period 1?
- (c) What is the per-unit profit for extracting resources in period 2? What is the present value of the per-unit profit from the viewpoint of period 1?
- (d) Describe in words why you must equate the marginal unit's profit in each periods (the optimality condition).
- (e) What two pieces of information are needed in order to solve for the optimal extraction Q_1^* and Q_2^* ?
- (f) Solve for the optimal extraction Q_1^* and Q_2^* .
- (g) Describe using specific numbers why $Q_1 = 3$ and $Q_2 = 4.5$ is not optimal
- (h) What is the marginal user cost? Interpret this number.
- (i) Now assume r = 0. What is the optimal allocation now? Why did optimal allocation change in the direction that it did?
- 2. Consider extraction of a non-renewable natural resource. The inverse demand function for the depletable resource is P = 12 Q in both periods 1 and 2 and the marginal cost of supplying it is \$3. The discount rate is 10%. There are 7.5 units total.
 - (a) Find the equilibrium allocation in each period for resource extraction
 - (b) Describe using the concept of marignal user cost why $Q_1 = 3$ and $Q_2 = 4.5$ is not optimal
 - (c) What is the marginal user cost? Interpret this number.
 - (d) Now assume r = 0. What is the optimal allocation now? Why did it change in the direction that it did?
- 3. Consider extraction of a non-renewable natural resource. The inverse demand function for the depletable resource is P = 12 Q in both periods 1 and 2 and the marginal cost of supplying it is 2 + Q/2. The discount rate is 6%. There are 15\$ units total.
 - (a) Find the equilibrium allocation in each period for resource extraction
 - (b) What is the marginal user cost? Interpret this number.