Two period model of a non-renewable resource

1. Consider extraction of a non-renewable natural resource. The inverse demand function for the depletable resource is $P = 20 - 2Q$ in both periods 1 and 2 and the marginal cost of supplying it is \$3. The discount rate is 10%. There are 10 units total.

   (a) Explain what the resource constraint is and then write it in mathematical form

   (b) What is the per-unit profit for extracting resources in period 1?

   (c) What is the per-unit profit for extracting resources in period 2? What is the present value of the per-unit profit from the viewpoint of period 1?

   (d) Describe in words why you must equate the marginal unit's profit in each periods (the optimality condition).

   (e) What two pieces of information are needed in order to solve for the optimal extraction Q_1^* and Q_2^*?

   (f) Solve for the optimal extraction Q_1^* and Q_2^*.

   (g) Describe using specific numbers why $Q_1 = 3$ and $Q_2 = 4.5$ is not optimal

   (h) What is the marginal user cost? Interpret this number.

   (i) Now assume $r = 0$. What is the optimal allocation now? Why did optimal allocation change in the direction that it did?

   Solution:

   (a) The resource constraint is that the amount I extract in period 1 plus the amount I extract in period 2 must add up to the supply of 10.

   $$Q_1 + Q_2 = 10$$

   (b) The marginal net benefits curve is given by

   $$MNB_1 = \underbrace{20 - 2Q_1}_{MB=P} - \underbrace{3}_{MC} = 17 - 2Q_1.$$

   (c) The marginal net benefits curve is given by

   $$MNB_2 = \underbrace{20 - 2Q_2}_{MB=P} - \underbrace{3}_{MC} = 17 - 2Q_2.$$

   In present value, we have

   $$PVMNB_2 = \frac{17 - 2Q_2}{1.1} = 15.4545 - 1.8181Q_2.$$

   (d) Suppose that marginal unit's profits in period 1 and period 2 were not equal. Then, I know we are not maximizing total profits since I could extract one unit less from the period with the smaller marginal unit's profits and one more from the period with the higher marginal unit's profits and increase profits while still satisfying the resource constraint.

   (e) The resource constraint $Q_1 + Q_2 = 10$ and $MNB_1 = PVMNB_2$

(f) Plugging $(10 - Q_1) = Q_2$ into the optimality condition yields:

$$17 - 2Q_1 = \frac{17 - 2(10 - Q_1)}{1.1} \implies 18.7 - 2.2Q_1 = 17 - 20 + 2Q_1$$

Solving for Q_1 yields $Q_1^* \approx 5.1667$. Plugging back into the resource constraint yeidls $Q_2^* = 4.8333$.

(g) At $Q_1 = 3$ and $Q_2 = 4.5$, the $MNB_1 = 11$ and the $PVMNB_2 = 7.27$. In this case, I am not extracting optimally because $MNB_1 > PVMNB_2$ and I can increase profits by extracing more in the first period.

(h) $MNB_1(Q_1^*) = 17 - 2 * 5.1667 = 6.666$ which is the marginal user cost. This is the additional profit that could be made if the resource constraint was increased by 1 unit to 11 units.

(i) Plugging $(10 - Q_1) = Q_2$ into the (new) optimality condition yields:

$$17 - 2Q_1 = \frac{17 - 2(10 - Q_1)}{1} \implies 17 - 2Q_1 = 17 - 20 + 2Q_1$$

Solving for Q_1 yields $Q_1^* = 5$. Plugging back into the resource constraint yeidls $Q_2^* = 5$. The reason Q_1^* decreased and Q_2^* increased is because the marginal net benefits increased in the second period when $r = 0$. Therefore, on the margin, resources in period 2 are relatively more attractive to extract.

2. Consider extraction of a non-renewable natural resource. The inverse demand function for the depletable resource is $P = 12 - Q$ in both periods 1 and 2 and the marginal cost of supplying it is \$3. The discount rate is 10%. There are 7.5 units total.

(a) Find the equilibrium allocation in each period for resource extraction

(b) Describe using the concept of marignal user cost why $Q_1 = 3$ and $Q_2 = 4.5$ is not optimal

(c) What is the marginal user cost? Interpret this number.

(d) Now assume $r = 0$. What is the optimal allocation now? Why did it change in the direction that it did?

Solution:

(a) Our resource constraint can be written as $Q_2 = 7.5 - Q_1$. The $MNB_1 = 12 - Q_1 - 3 = 9 - Q_1$ and $PVMNB_2 = \frac{12-Q_2-3}{1.1} = \frac{9-Q_2}{1.1}$.
Plugging our resource constraint into the optimality condition $MNB_1 = PVMNB_2$ yields

$$9 - Q_1 = \frac{9 - (7.5 - Q_1)}{1.1} \implies Q_1^* = 4$$

Plugging Q_1^* back into the resouce constraint yields $Q_2^* = 7.5 - 4 = 3.5$.

(b) $MNB_1(3) = 9 - 3 = 6$ and $PVMNB_2(4.5) = \frac{9-4.5}{1.1} = 4.1$. In this case, the value of extracting one more resource in period 1 is greater than the value lost from extracting one fewer unit in period 2. Therefore, total net benefits are not maximized since resource extraction should be moved towards period 1.

(c) In $MUC = MNB_1(Q_1^*) = 9 - 4 = 5$. The marginal user cost implies that adding one additional unit to our resources is worth about \$5 in net benefits.

(d) Following the above work, we have

$$9 - Q_1 = \frac{9 - (7.5 - Q_1)}{1} \implies Q_1^* = 3.75$$

Plugging Q_1^* back into the resouce constraint yields $Q_2^* = 7.5 - 3.75 = 3.75$. Again, since the discount rate decreased, future net benefits are relatively more valuable than when $r = 10\%$. Therefore to maximize benefits, extraction should be moved towards period 2.

3. Consider extraction of a non-renewable natural resource. The inverse demand function for the depletable resource is $P = 12 - Q$ in both periods 1 and 2 and the marginal cost of supplying it is $2 + Q/2$. The discount rate is 6%. There are 15 units total.

(a) Find the equilibrium allocation in each period for resource extraction

(b) What is the marginal user cost? Interpret this number.

Solution: In this example, we have to check if there is unlimited supply or not.

In this case, let's calculate MNB_1 and MNB_2.

$$MNB_1 = MNB_2 = 12 - Q_1 - (2 + Q_1/2) = 10 - 1.5Q_1$$

If there is unlimited supply, then Q_1^* and Q_2^* are found from setting $MNB_1(Q_1) = 0$ and $MNB_2(Q_2) = 0$ respectively. In this case, this yields $Q_1^* = Q_2^* = 6.66$ units. Since $Q_1^* + Q_2^* = 13.33$, there is sufficient supply, so we don't have to solve using the optimality condition. If you did the work and got a negative marginal user cost, that should be your sign that you're in an unlimited supply example (since the marginal net benefit is negative for Q_1^* and Q_2^*).

For additional practice, let's do this problem with $S = 10$ units.

(a) First, our resource constraint is given by $Q_1 + Q_2 = 10$. Next, let's calculate MNB_1 and $PVMNB_2$.

$$MNB_1 = 12 - Q_1 - (2 + Q_1/2) = 10 - 1.5Q_1$$

$$PVMNB_2 = \frac{12 - Q_2 - (2 + Q_2/2)}{1.06} = \frac{10 - 1.5Q_2}{1.06}$$

Plugging $Q_2 = 15 - Q_1$ into our optimality condition yields:

$$10 - 1.5Q_1 = \frac{10 - 1.5(10 - Q_1)}{1.06} \implies Q_1^* \approx 5.04$$

Then, plugging Q_1^* into the resource constraint yields $Q_2^* = 4.96$.

(b) In this example, $MUC = MNB_1(Q_1^*) = 12 - 1.5 * 5.04 = 4.44$. The value of adding an 11th unit to our supply is worth \$4.44 in net benefits.