Tradable Permits

1. Two firms can control emissions at the following marginal costs: $MC_x = 80a_x$ and $MC_y = 40a_y$ where a_x and a_y are, respectively, the amount of emissions reduced by firm x and firm y. Assume that with no control at all, each firm would be emitting 50 units of emissions or a total of 100 units for both firms.

   (a) Which firm is better at abating pollution?

   (b) If the goal is to reduce total emissions to 60 units. How many units must be abated? Write out the abatement constraint in mathematical terms

   (c) Consider a uniform standard. How many units must be abated by both firms? How much did each firm have to pay to abate their marginal unit of pollution?

   (d) Consider a cap-and-trade system that aims for a total 60 units of emissions.

      i. In words, describe why the marginal abatement costs for each firm must be equal to eachother in order to be at equilibrium (the optimality condition).

      ii. Using the optimality condition and the abatement constraint, solve for the equilibrium allocation of permits to each firm?

      iii. At what price would these permits sell for at an auction?

   (e) Assume that the control authority wanted to reach its objective by using an emissions charge system instead.

      i. What tax amount should them impose to reach this equilibrium?

      ii. How much revenue would the government collect?

   (f) Why is cap-and-trade more cost-effective than a uniform standard where each firm reduces pollution by the same amount?

   Solution:

   (a) Firm y has a lower cost of abatement, so it is the 'better' firm at abating.

   (b) Going from 100 units to 60 units of pollution implies an abatement goal of 40 units. In mathematical notation, we have

   $$a_x + a_y = 40.$$

   (c) Each firm needs to abate half, so $a_x = a_y = 20$. For the 20th unit, we have $MAC_x = 80 * 20 = 1600$ and $MAC_y = 40 * 20 = 800$. In words, the 20th unit cost firm x \$1600 to abate and firm y \$800 to abate.

   (d) Consider a cap-and-trade system that aims for a total 60 units of emissions.

      i. The marginal abatement costs must be equal to each other in order to ensure that the total cost of abatement is as cheap as possible. Consider the case where (a_x, a_y) is such that $MAC_x(a_x) > MAC_y(a_y)$ and $a_x + a_y = 40$. We can lower cost by having firm y abate one more unit, spending $MAC_y(a_y)$ dollars and having firm x abate one fewer unit, saving $MAC_x(a_x)$. The abatement goal will still be hit, but costs will be cheaper since savings $MAC_x(a_x)$ are bigger than additional costs $MAC_y(a_y)$. The same will hold with $MAC_y(a_y) > MAC_x(a_x)$. Therefore, we need equality to hold in order for the abatement goal to be achieved at as low of a cost as possible.

  ii. Our optimality condition is $80a_x = 40a_y$ which implies $2a_x = a_y$. Plugging into our constraint yields $a_x + 2a_x = 40 \implies a_x^* = 40/3 \approx 13.33$. This implies $a_y^* = 40 - 40/3 = 80/3 \approx 26.67$.
  iii. The permit price is equal to $MAC_x(a_x^*) = MAC_y(a_y^*) = 80 * 40/3 = 3200/30 \approx \$106.67$.

(e) Assume that the control authority wanted to reach its objective by using an emissions charge system instead.
  i. The tax amount should be equal to the permit price solved for above, so $T = \$106.67$.
  ii. After the policy, the firms will polute 60 units, so the revenue equals $106.67 * 60 \approx \$6400$.

(f) In the uniform standard, the firm with higher abatement costs is abating more than is optimal. The cap-and-trade system balances this trade-off by setting $MAC_x = MAC_y$ as required for efficiency as argued above.

2. Two firms can control emissions at the following marginal costs: $MC_x = 200a_x$ and $MC_y = 100a_y$ where a_x and a_y are, respectively, the amount of emissions reduced by firm x and firm y. Assume that with no control at all, each firm would be emitting 20 units of emissions or a total of 40 units for both firms.

(a) Consider a cap-and-trade system that aims for a total reduction of 21 units of emissions is necessary.
  i. What is the equilibrium allocation of permits to each firm?
  ii. At what price would these permits sell for at an auction

(b) Assume that the control authority wanted to reach its objective by using an emissions charge system instead.
  i. What tax amount should them impose to reach this equilibrium?
  ii. How much revenue would the government collect?

(c) Why is cap-and-trade more cost-effective than a uniform standard where each firm reduces pollution by 10.5 units?

Solution:

(a) Our constaint is $a_x + a_y = 21$ and our optimality condition is $200a_x = 100a_y \implies 2a_x = a_y$.
  i. Plugging the optimality condition into the constraint yields $a_x + 2a_x = 21 \implies a_x^* = 7$. Plugging a_x^* back into the budget constraint yields $a_y^* = 14$.
  ii. Permits would sell for $MC_x(a_x^*) = MC_y(a_y^*) = \$700$.

(b) Assume that the control authority wanted to reach its objective by using an emissions charge system instead.
  i. The tax price should equal \$700 to reach the same outcome.
  ii. The firms would pollute 19 units generating total government revenue of $19 * 700 = \$13,300$.

(c) When $a_x = a_y = 10.5$, we have $MAC_x(10.5) = 1050 < MAC_y(10.5) = 2100$. In this case, total costs are not minimized since firm y has a higher marginal abatement cost than firm x.

3. Two firms can control emissions at the following marginal costs: $MC_x = 5 + 10a_x$ and $MC_y = 11a_y$ where a_x and a_y are, respectively, the amount of emissions reduced by firm x and firm y. Assume that with no control at all, each firm would be emitting 10 units of emissions or a total of 20 units for both firms.

   (a) Consider a cap-and-trade system that aims for a total reduction of 10 units of emissions.

      i. What is the equilibrium allocation of permits to each firm?

      ii. At what price would these permits sell for at an auction

   (b) Assume that the control authority wanted to reach its objective by using an emissions charge system instead.

      i. What tax amount should them impose to reach this equilibrium?

      ii. How much revenue would the government collect?

   Solution:

   (a) Our constaint is $a_x + a_y = 10$ and our optimality condition is $5 + 10a_x = 11a_y$.

      i. Rewriting the constraint as $a_x = 10 - a_y$ and plugging it into the optimality condition yields

$$5 + 10 * (10 - a_y) = 11a_y \implies 105 = 21a_y \implies a_y^* = 5.$$

      Plugging a_y^* into the budget constraint yields $a_x^* = 5$.

      ii. Permits would sell for $MC_x(a_x^*) = MC_y(a_y^*) = \$55$.

   (b) Assume that the control authority wanted to reach its objective by using an emissions charge system instead.

      i. The tax price should equal \$55 to reach the same outcome.

      ii. The firms would pollute 10 units generating total government revenue of $10 * 55 = \$550$.