

### Tradable Permits

1. Two firms can control emissions at the following marginal costs:  $MC_x = 80a_x$  and  $MC_y = 40a_y$  where  $a_x$  and  $a_y$  are, respectively, the amount of emissions reduced by firm  $x$  and firm  $y$ . Assume that with no control at all, each firm would be emitting 50 units of emissions or a total of 100 units for both firms.
  - (a) Which firm is better at abating pollution?
  - (b) If the goal is to reduce total emissions to 60 units. How many units must be abated? Write out the abatement constraint in mathematical terms
  - (c) Consider a uniform standard. How many units must be abated by both firms? How much did each firm have to pay to abate their marginal unit of pollution?
  - (d) Consider a cap-and-trade system that aims for a total 60 units of emissions.
    - i. In words, describe why the marginal abatement costs for each firm must be equal to each other in order to be at equilibrium (the optimality condition).
    - ii. Using the optimality condition and the abatement constraint, solve for the equilibrium allocation of permits to each firm?
    - iii. At what price would these permits sell for at an auction?
  - (e) Assume that the control authority wanted to reach its objective by using an emissions charge system instead.
    - i. What tax amount should they impose to reach this equilibrium?
    - ii. How much revenue would the government collect?
  - (f) Why is cap-and-trade more cost-effective than a uniform standard where each firm reduces pollution by the same amount?
2. Two firms can control emissions at the following marginal costs:  $MC_x = 200a_x$  and  $MC_y = 100a_y$  where  $a_x$  and  $a_y$  are, respectively, the amount of emissions reduced by firm  $x$  and firm  $y$ . Assume that with no control at all, each firm would be emitting 20 units of emissions or a total of 40 units for both firms.
  - (a) Consider a cap-and-trade system that aims for a total reduction of 21 units of emissions is necessary.
    - i. What is the equilibrium allocation of permits to each firm?
    - ii. At what price would these permits sell for at an auction
  - (b) Assume that the control authority wanted to reach its objective by using an emissions charge system instead.
    - i. What tax amount should they impose to reach this equilibrium?
    - ii. How much revenue would the government collect?
  - (c) Why is cap-and-trade more cost-effective than a uniform standard where each firm reduces pollution by 10.5 units?
3. Two firms can control emissions at the following marginal costs:  $MC_x = 5 + 10a_x$  and  $MC_y = 11a_y$  where  $a_x$  and  $a_y$  are, respectively, the amount of emissions reduced by firm  $x$  and firm  $y$ . Assume that with no control at all, each firm would be emitting 10 units of emissions or a total of 20 units for both firms.

- (a) Consider a cap-and-trade system that aims for a total reduction of 10 units of emissions.
  - i. What is the equilibrium allocation of permits to each firm?
  - ii. At what price would these permits sell for at an auction
- (b) Assume that the control authority wanted to reach its objective by using an emissions charge system instead.
  - i. What tax amount should they impose to reach this equilibrium?
  - ii. How much revenue would the government collect?