

Problem Set - Matrix Algebra

CU Denver Math Camp

1. Let

$$A = \begin{bmatrix} 6 & 7 & 9 \\ 1 & 2 & 3 \\ 8 & 4 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} 10 & 9 & 8 \\ 7 & 5 & 4 \\ 1 & 7 & 6 \end{bmatrix}, \quad \text{and } C = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 4 & 0 & 2 \end{bmatrix}$$

calculate the following:

i. $3B^T + A$

ii. $C^T - 4A$

iii. $(CA)^T$

2. What is the column rank of each of the following matrices (show your work):

i.

$$\begin{bmatrix} 6 & 7 & 9 \\ 1 & 2 & 3 \\ 8 & 4 & 6 \end{bmatrix}$$

ii.

$$\begin{bmatrix} 2 & 4 & 8 \\ 1 & 2 & 3 \\ 2 & 4 & 7 \end{bmatrix}$$

iii.

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & -1 \\ -2 & -4 & 4 \end{bmatrix}$$

3. Match each matrix with its inverse:

$$A = \begin{bmatrix} 1 & 2 & -1 \\ -2 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 4 & 1 \\ -1 & 1 & -1 \\ 1 & 4 & 0 \end{bmatrix}, \quad \text{and } C = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$a = \begin{bmatrix} -4 & -4 & 5 \\ 1 & 1 & -1 \\ 5 & 4 & -6 \end{bmatrix}, \quad b = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 1 & 1 \\ 2 & 3 & 4 \end{bmatrix}, \quad \text{and } c = \text{Not invertible.}$$

4. Convert the following systems of equations to matrices and solve:

i.

$$x_1 + 6x_2 = 4$$

$$3x_1 - 2x_2 = 2$$

ii.

$$2x - 5y = 15$$

$$3x + y = 31$$