Topic 1: Introduction to Forecasting

ECON 4753 — University of Arkansas

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Fall 2024

Roadmap

Forecasting

Goals of Forecasting

Evaluating Models

Types of Data

Problem of Prediction

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  - → Input: observable characteristics
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- Predict values of a variable in the future, e.g. time-series of stock prices
  - → Input: the time-period
  - → Output: stock price

Prediction model

We have an outcome variable Y and a set of p different predictor variables $X = (X_1, X_2, \ldots, X_p)$.

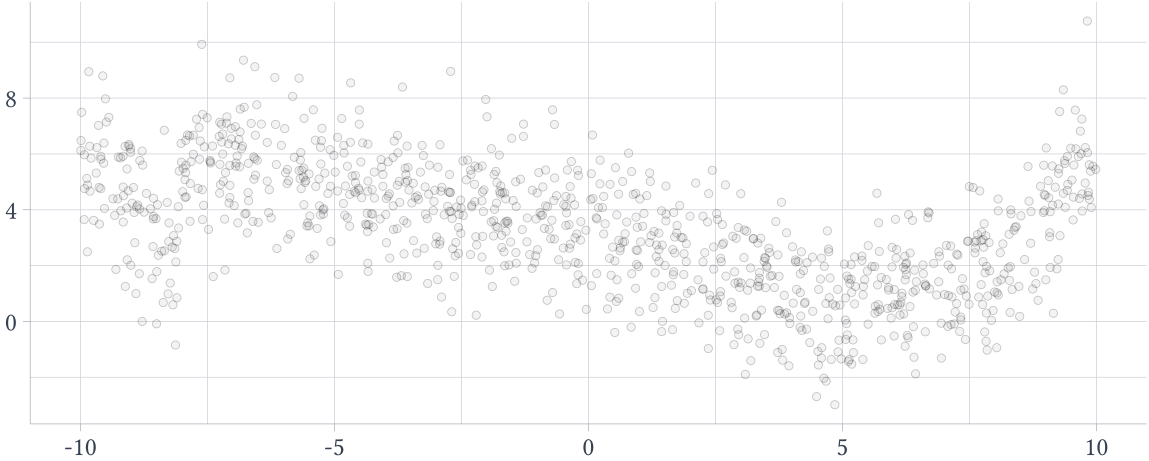
- For some observations we observe both X and Y; this is essential to fit the model

We can write the model in a general form as

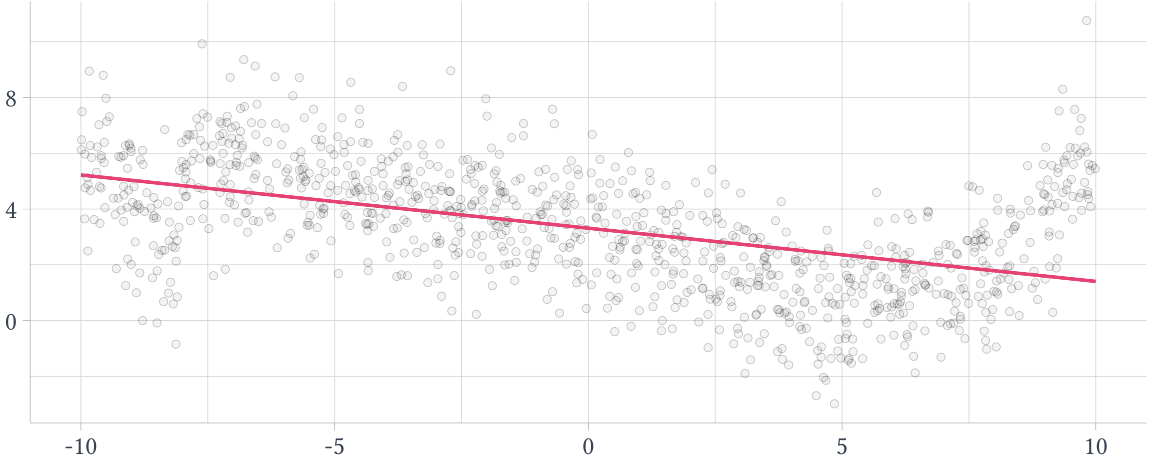
$$Y = f(X) + \varepsilon,$$

where f is some unknown function of X and ε is the error term which we assume is unrelated to X and mean zero.

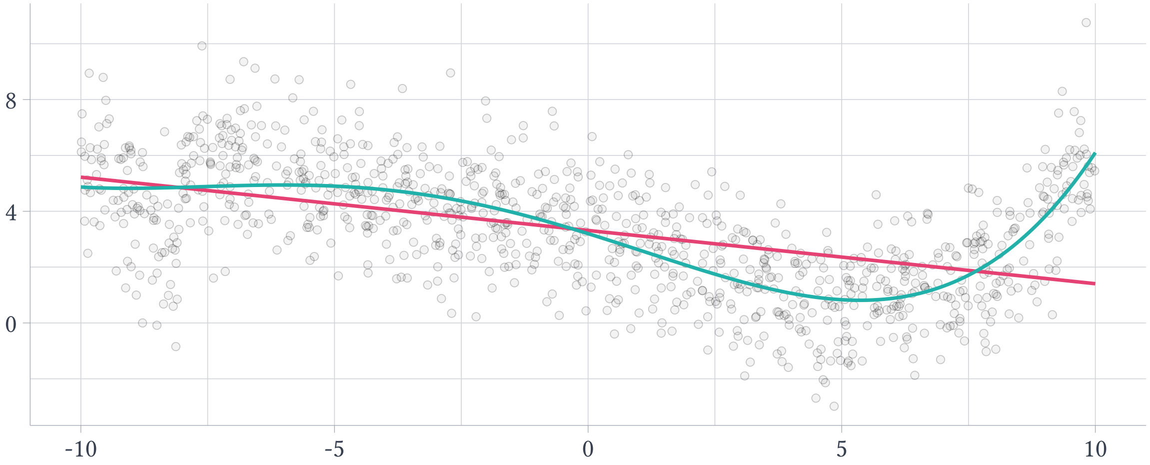
Examples of f:



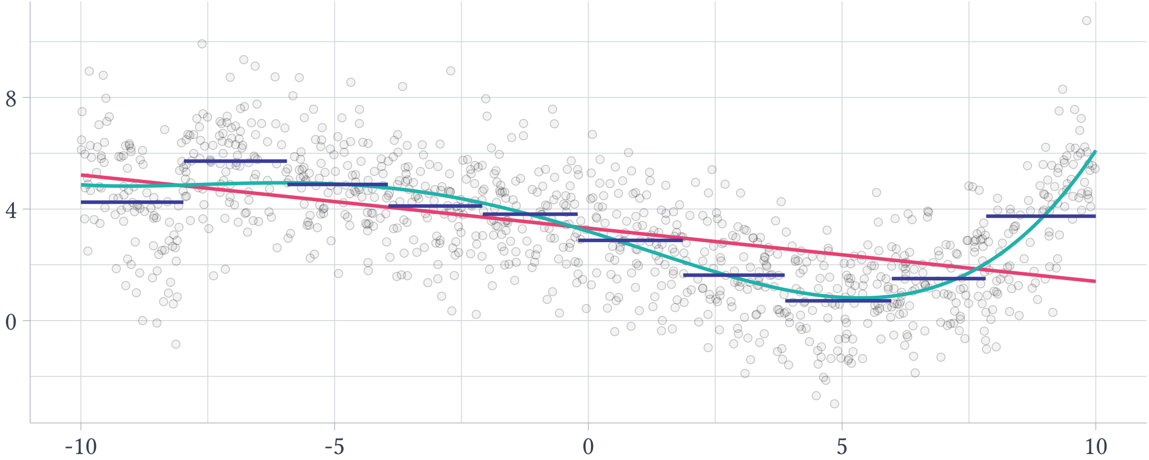
Examples of f: Line



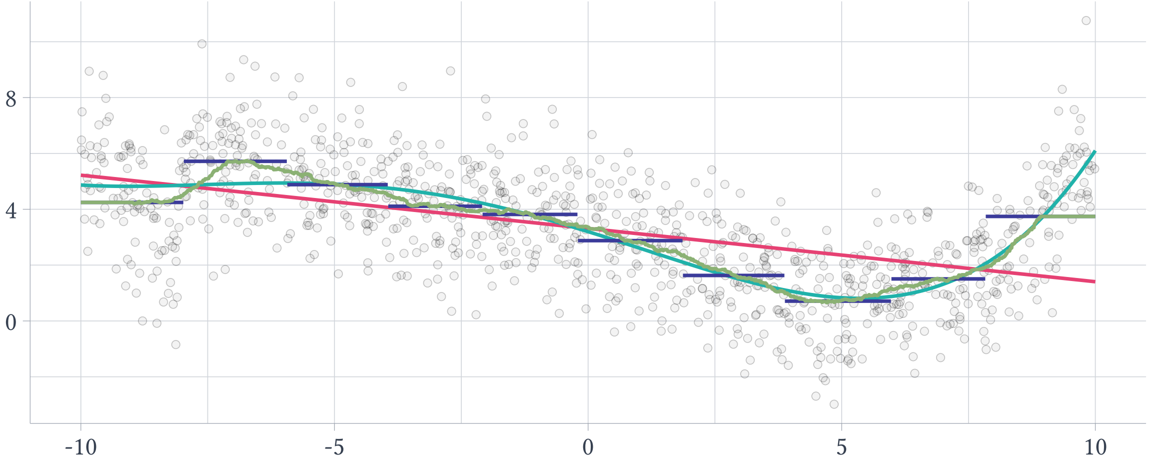
Examples of f: Line, Polynomial (x^4)



Examples of f: Line, Polynomial $\left(x^4\right)$, Bins of x



Examples of f: Line, Polynomial $\left(x^4\right)$, Bins of x, KNN of x

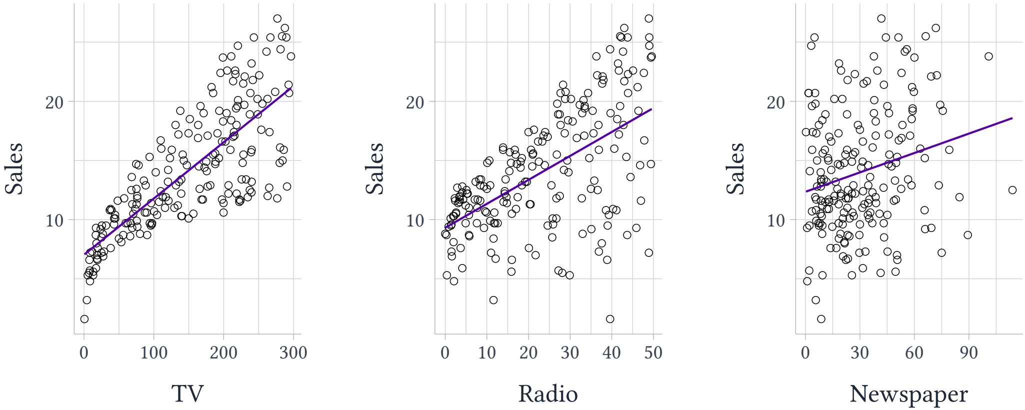


Advertising Example

Let's given an example. Say you're a business and you want to use advertising to boost sales. You have a bunch of different markets (e.g. cities) and you have data on how you've spent your advertising budget in those markets and the sales in that market

Advertising Example

Single-variable predictors



Advertising Example

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- Do places with more TV ads also have more radio ads? Then how can we tell if it is TV ads that are helping or if it is really radio ads

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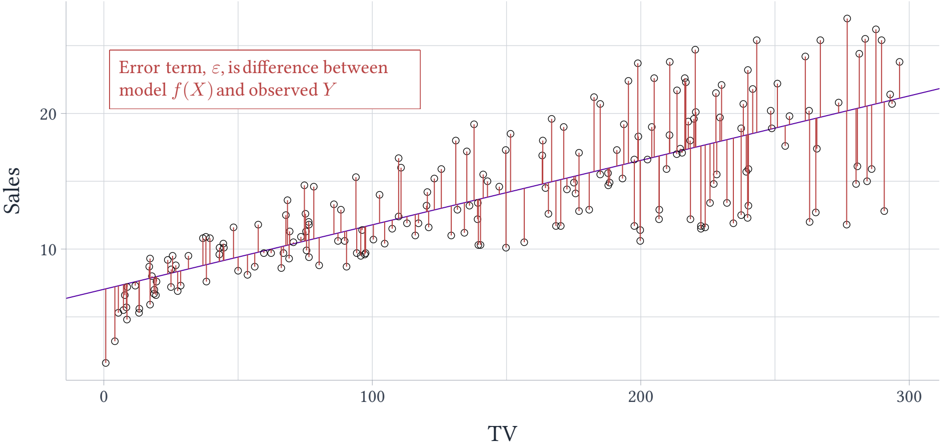
Key takeaway: Forecasting models get better the more carefully you think about the context you are in

Error term



Error term, ε, is difference between model $f(X)$ and observed Y
20
10
Sales
0
100
200
300
TV

Error term

In the previous figure, we were able to determine ε because we assumed we *knew* the model f and therefore could observe $f(X)$ for each market.

In reality, we do not know f and can never observe ε. But, we can try and estimate it ...

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Types of Data

Why estimate f?

There are two related reasons to try and predict f:

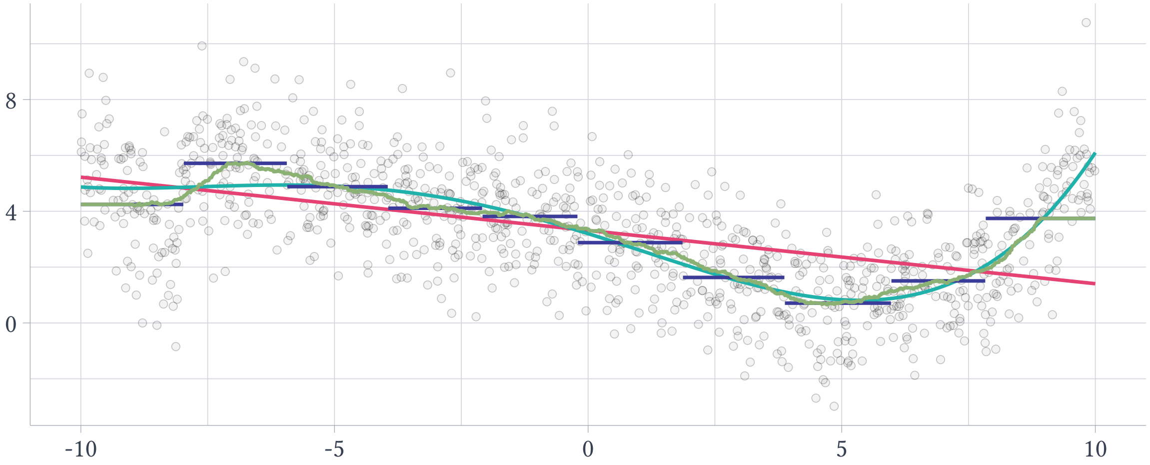
1. Predict y as good as possible (prediction)
   - $\rightarrow$ Think of prediction as a 'black box' where the goal is to do as good of a job at predicting y as possible
2. Understand the relationship between x and y (inference)
   - $\rightarrow$ If our goal is being able to describe the relationship between x and y.

Model Flexibility

There is a limit to how flexible we can make our model

1. If our goal is prediciton, we only have a finite amount of data to use to fit the model, so there's a limit on how much we can learn
   $\rightarrow$ Face the risk of overfitting the data (chasing after the random noise ε)
2. If our goal is inference, then added flexibility is harder to summarize to stakeholders.

Examples of f: Line, Polynomial $\left(x^4\right)$, Bins of x, KNN of x



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Prediction Error

Given our model, we will want to be able to evaluate how good our model does at predicting observations y

Define the prediction error as

$$\hat{\varepsilon} = \underbrace{y}_{\text{true value}} - \underbrace{\hat{y}}_{\text{predicted value}}$$

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Large $\hat{\varepsilon}$ means you did a poor job of predicting that observation. That could be because

1. The linear model is bad at predicting y
2. Or, the true noise ε is making y far away from the systematic component $f(X)$ for this observation

Mean-square prediction error

To provide a summary measure of fit, we want to *average* prediction error over many observations. This will find a ‘average’ prediction error

- If we took the simple mean of prediction error, positive and negative prediction errors would cancel out

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The mean-square (prediction) error (MSE) is calculated as:

$$\text{MSE} \equiv \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2 = \frac{1}{n}\sum_{i=1}^{n}\hat{\varepsilon}_i^2 \tag{1}$$

- Average of squared prediction error

Mean-square prediction error

y_i	$\hat{y}_i$	$\hat{\varepsilon}_i$
3.7	4.20	
4.1	4.18	
5.6	5.48	
2.9	3.29	
8.8	8.81	

Calculate mean-square prediction error:

Mean-square prediction error

y_i	$\hat{y}_i$	$\hat{\varepsilon}_i$
3.7	4.20	0.5
4.1	4.18	0.08
5.6	5.48	-0.12
2.9	3.29	0.39
8.8	8.81	0.01

Calculate mean-square prediction error:

$$\text{MSPE} = \frac{1}{5}\left(0.5^2 + 0.08^2 + -0.12^2 + 0.39^2 + 0.01^2\right)$$
$$= 0.0846$$

In-sample vs. Out-of-sample prediction error

As a forecaster, you will fit a model using a set of observations $\{(x_1, y_1), \ldots, (x_n, y_n)\}$. This is called the training data.

We can calculate the in-sample MSE by formula (1) averaging over all observations in the training data.

- This tells us how good we do at predicting the data *we trained the model on*.

In-sample vs. Out-of-sample prediction error

If our goal is prediction, we really want to know how the model would predict on *new* observations that we *have not seen before*

- It is common to hold out a set of test data that is NOT used for training the model, but just for evaluating it's performance

Why use ‘test data’?

It is common to try and ‘pick’ from a set of models based on how they do at in-sample prediction:

- That is, select the model with the smallest *in-sample MSE*.

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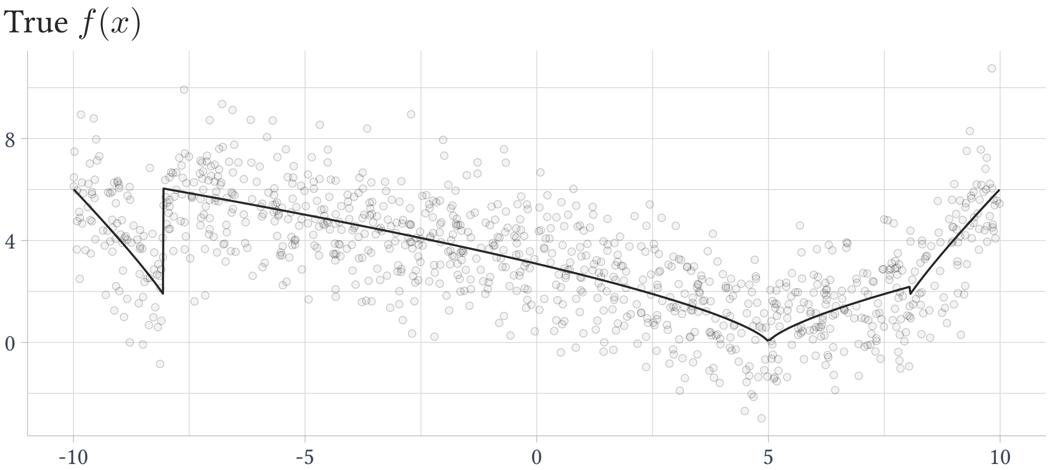
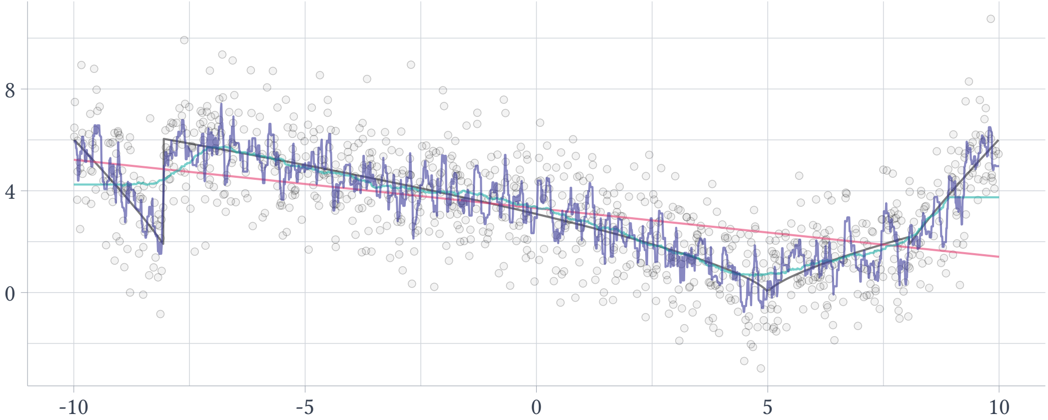
This is *a bad thing to do*; by focusing on fitting the current sample very well, you are risking overfitting the data

Flexibility vs. Overfitting



Flexibility vs. Overfitting

True $f(x)$, Line, Somewhat flexible, Highly flexible



Flexibility vs. Overfitting

By making the model more and more *flexible*, you risk overfitting more and more

- A solution is to evaluate your model fit using outside ‘testing data’

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This technique is not as common when you care more about the associations between variables (interpreting the model)

- Not really a good reason other than "that is more complicated"

Bias-variance trade-off

This discussing of increasing flexiblity leading to increasing the noise of the model fit is a well-known problem. It is called the Bias-Variance Tradeoff:

1. Bias: When the model we fit, $\hat{f}(x)$, does a poor job fitting the true model $f(x)$
2. Variance: When the model we fit, $\hat{f}(x)$, is very variable across samples

    $\rightarrow$ In repeated sampling, the model we estimate varies from estimate to estimate

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This is a 'trade-off'. To lower bias by adding flexibility, you're adding variance (noisiness) to the estimate

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Cross-sectional Data

Cross-sectional data consists of many different *units* viewed at a point in time:

school_id	avg_sat_math	pct_white	pct_b
01M539	657	28.6%	13.3%
02M294	395	11.7%	38.5%
02M308	418	3.1%	28.2%
02M545	613	1.7%	3.1%
01M292	410	3.9%	24.4%
01M696	634	45.3%	17.2%

Time-series Data

Time-series data consists of a single observational unit viewed over multiple points in time:

month	day	hour	bikers	temp
1	1	0	16	0.24
1	1	1	40	0.22
1	1	2	32	0.22
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
12	31	21	52	0.40
12	31	22	38	0.38
12	31	23	31	0.36

Panel Data

Panel data is like time-series data, but for many different observational units:

hedge_fund_manager	month	return
1	1	-3.34%
1	2	3.76%
1	3	12.97%
⋮	⋮	⋮
2000	48	-3.76%
2000	49	2.25%
2000	50	6.68%