Topic #1 Assignment

ECON 5753 — University of Arkansas

Prof. Kyle Butts

These assignments should be completed in groups of 1 or 2 but submitted individually.

Theoretical Questions

1. Let

$$\boldsymbol{A} = \begin{bmatrix} 6 & 7 & 9 \\ 1 & 2 & 3 \\ 8 & 4 & 6 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 10 & 9 & 8 \\ 7 & 5 & 4 \\ 1 & 7 & 6 \end{bmatrix}, \quad \text{and } \boldsymbol{C} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 4 & 0 & 2 \end{bmatrix}.$$

Calculate the following:

i. $3\boldsymbol{B}^T + \boldsymbol{A}$
ii. $\boldsymbol{C}^T - 4\boldsymbol{A}$
iii. $(\boldsymbol{C}\boldsymbol{A})'$

2. Let

$$\boldsymbol{X} = \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix}, \quad \beta = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix}.$$

Verify the following two are equivalent:

$$X\beta \quad \text{and } X_{\cdot,1}\beta_1 + X_{\cdot,2}\beta_2 + X_{\cdot,3}\beta_3$$

3. Match each matrix with its inverse:

$$A = \begin{bmatrix} 1 & 2 & -1 \\ -2 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 4 & 1 \\ -1 & 1 & -1 \\ 1 & 4 & 0 \end{bmatrix}, \quad \text{and } C = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$a = \begin{bmatrix} -4 & -4 & 5 \\ 1 & 1 & -1 \\ 5 & 4 & -6 \end{bmatrix}, \quad b = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 1 & 1 \\ 2 & 3 & 4 \end{bmatrix}, \quad \text{and } c = \text{ Not invertible.}$$

4. Let

$$\varepsilon = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{bmatrix} \right).$$

What is the distribution of $\begin{bmatrix} 1 & 1 \end{bmatrix} \varepsilon$?

5. Let $f(x, y) = x^2 + y^2 + 2xy$.

   i. What is $\frac{\partial}{\partial x} f(x, y)$ and $\frac{\partial}{\partial y} f(x, y)$?

   ii. Use this to create a taylor approximation of this function at (x_0, y_0). Write this in the form of:

$$f(x_0 + dx, y_0 + dy) \approx f(x_0, y_0) + \begin{bmatrix} \frac{\partial}{\partial x} f(x_0, y_0) & \frac{\partial}{\partial x} f(x_0, y_0) \end{bmatrix} \begin{bmatrix} dx \\ dy \end{bmatrix}$$

   iii. Plug in $(x_0, y_0) = (0, 0)$ to create a linear approximation to this function.