

Assignment 3

Your Name

Baseline Simulation

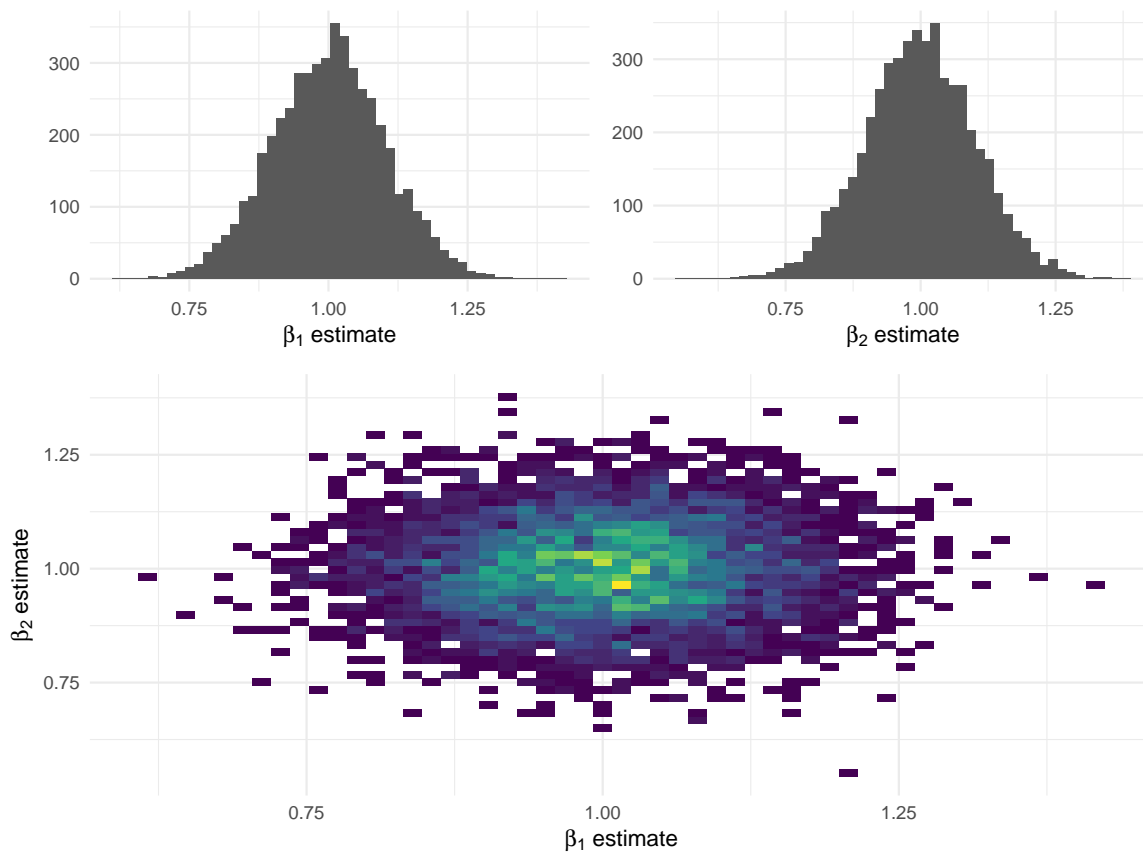
```
# BASELINE CASE (for comparison)
B <- 5000
n <- 100
ests <- matrix(nrow = B, ncol = 3)
for (b in 1:B) {
  # Generate data
  Sigma_X = matrix(c(1, 0, 0, 1), nrow = 2, ncol = 2)
  X <- rmvnorm(n = n, mean = c(1, 1), sigma = Sigma_X)
  X1 <- X[, 1]
  X2 <- X[, 2]

  y <- 10 + X1 * 1 + X2 * 1 + rnorm(n, mean = 0, sd = 1)

  df_b <- data.frame(X1 = X1, X2 = X2, y = y)

  # Estimate OLS
  mod <- lm(y ~ 1 + X1 + X2, data = df_b)
  ests[b, ] <- coef(mod)
}

plot_ests(ests)
```



2. Changing sample size

```
B <- 5000
n <- 10
ests <- matrix(nrow = B, ncol = 3)
for (b in 1:B) {
  # Generate data
  Sigma_X = matrix(c(1, 0, 0, 1), nrow = 2, ncol = 2)
  X <- rmvnorm(n = n, mean = c(1, 1), sigma = Sigma_X)
  X1 <- X[, 1]
  X2 <- X[, 2]

  y <- 10 + X1 * 1 + X2 * 1 + rnorm(n, mean = 0, sd = 1)

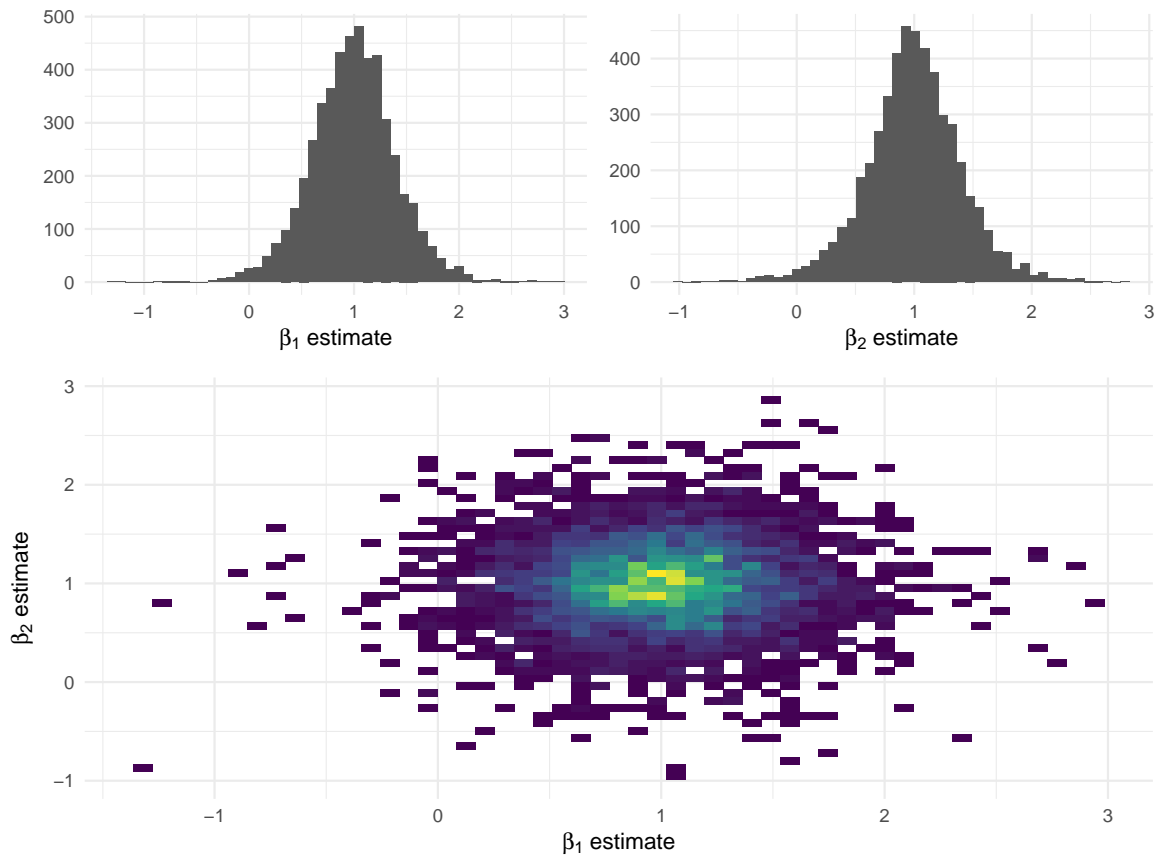
  df_b <- data.frame(X1 = X1, X2 = X2, y = y)
```

```

# Estimate OLS
mod <- lm(y ~ 1 + X1 + X2, data = df_b)
ests[b, ] <- coef(mod)
}

plot_ests(est)

```



```

B <- 5000
n <- 1000
ests <- matrix(nrow = B, ncol = 3)
for (b in 1:B) {
  # Generate data
  Sigma_X = matrix(c(1, 0, 0, 1), nrow = 2, ncol = 2)
  X <- rmvnorm(n = n, mean = c(1, 1), sigma = Sigma_X)
  X1 <- X[, 1]

```

```

X2 <- X[, 2]

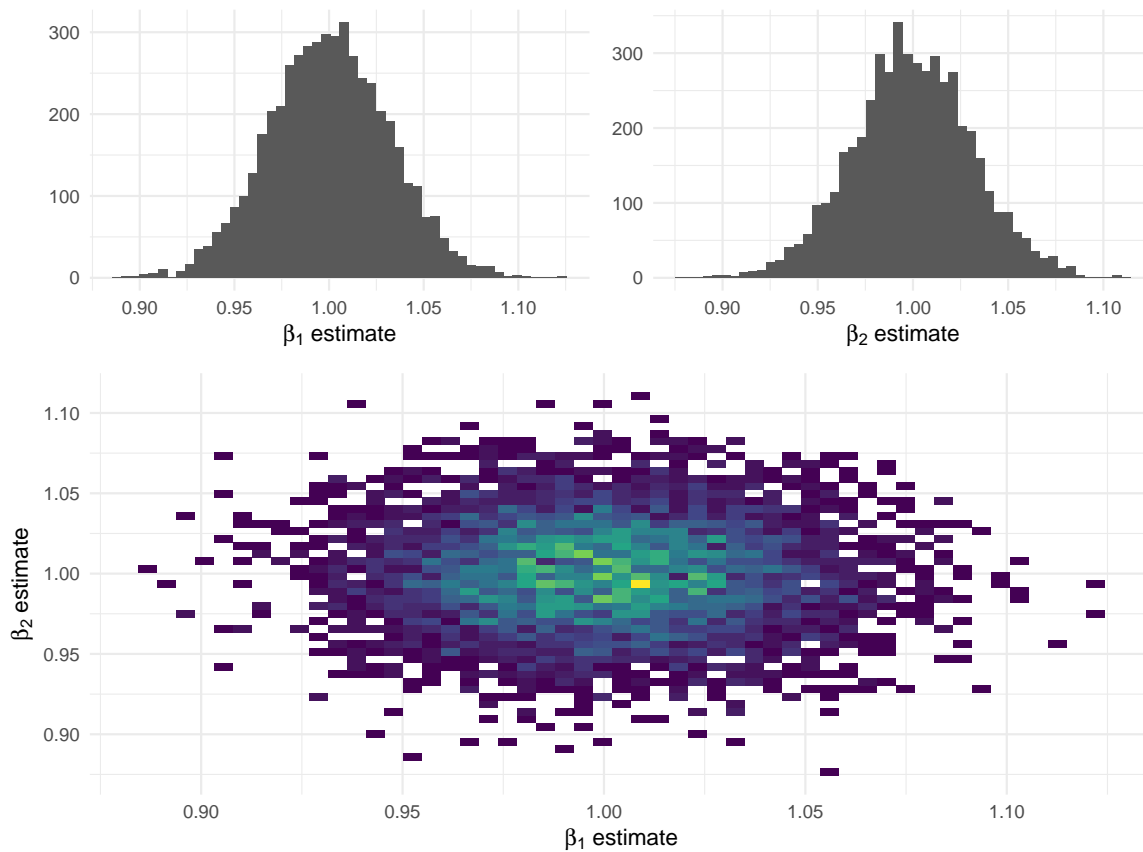
y <- 10 + X1 * 1 + X2 * 1 + rnorm(n, mean = 0, sd = 1)

df_b <- data.frame(X1 = X1, X2 = X2, y = y)

# Estimate OLS
mod <- lm(y ~ 1 + X1 + X2, data = df_b)
ests[b, ] <- coef(mod)
}

plot_ests(est)

```



Answer:

As the sample size grows, the variance of each estimated coefficient's sample distribution

shrinks. The estimates become more precise as the number of observations grow. The shape of the joint distribution remains a circle since the two coefficients are uncorrelated with each other.

3. Changing error term distribution

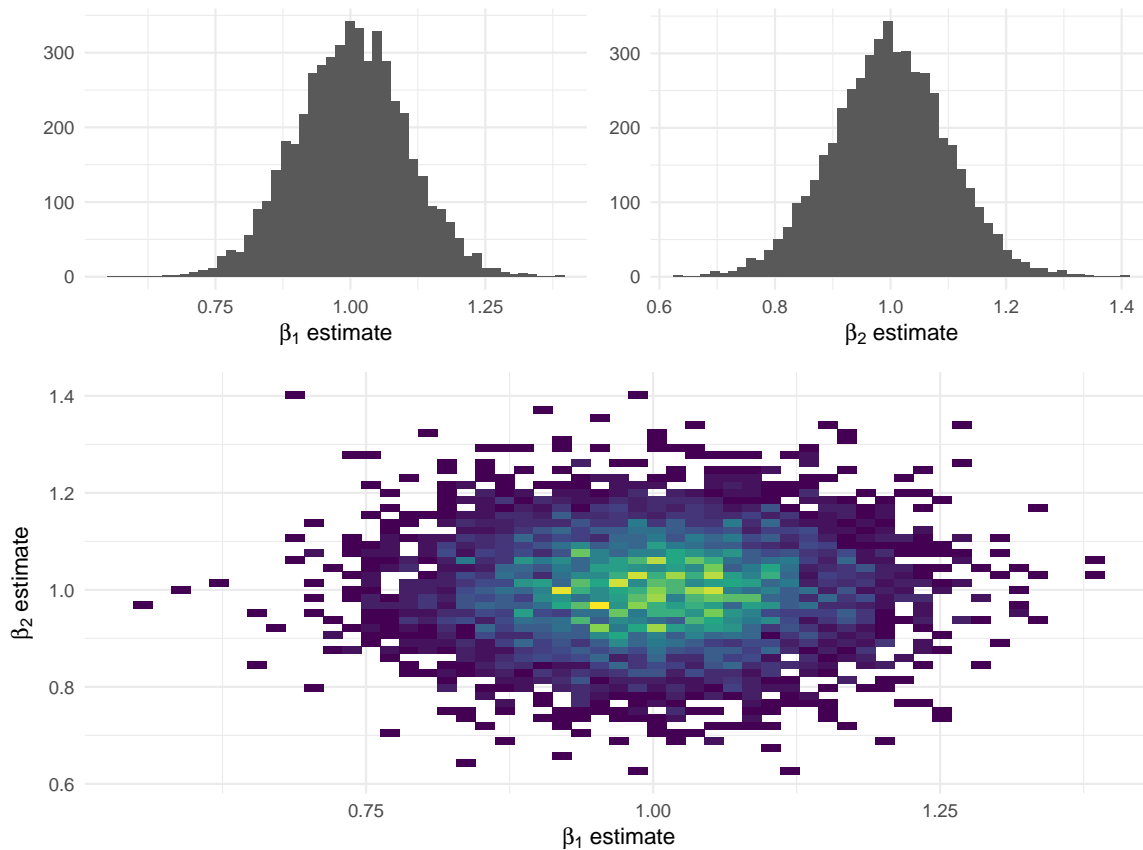
```
B <- 5000
n <- 100
ests <- matrix(nrow = B, ncol = 3)
for (b in 1:B) {
  # Generate data
  Sigma_X = matrix(c(1, 0, 0, 1), nrow = 2, ncol = 2)
  X <- rmvnorm(n = n, mean = c(1, 1), sigma = Sigma_X)
  X1 <- X[, 1]
  X2 <- X[, 2]

  y <- 10 + X1 * 1 + X2 * 1 + rnorm(n, mean = 0, sd = 1)

  df_b <- data.frame(X1 = X1, X2 = X2, y = y)

  # Estimate OLS
  mod <- lm(y ~ 1 + X1 + X2, data = df_b)
  ests[b, ] <- coef(mod)
}

plot_ests(ests)
```



```

B <- 5000
n <- 100
ests <- matrix(nrow = B, ncol = 3)
for (b in 1:B) {
  # Generate data
  Sigma_X = matrix(c(1, 0, 0, 1), nrow = 2, ncol = 2)
  X <- rmvnorm(n = n, mean = c(1, 1), sigma = Sigma_X)
  X1 <- X[, 1]
  X2 <- X[, 2]

  y <- 10 + X1 * 1 + X2 * 1 + rnorm(n, mean = 0, sd = 3)

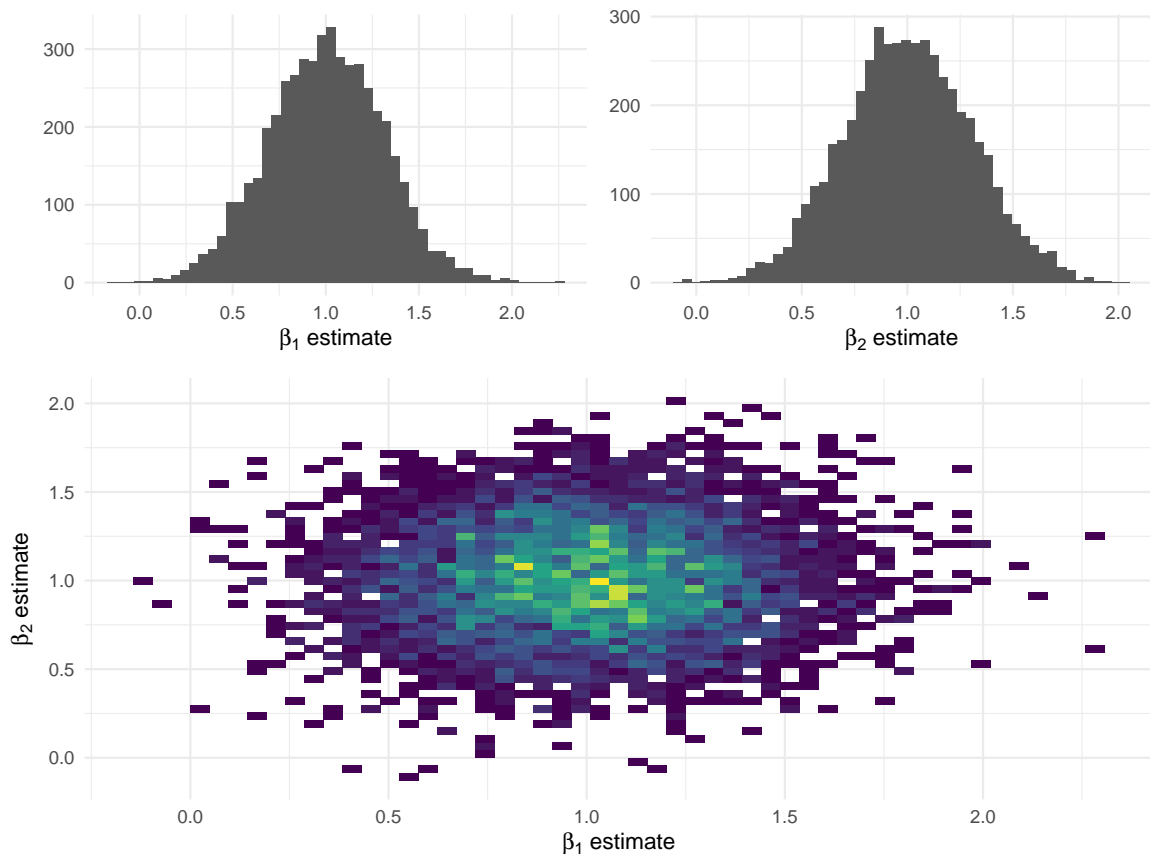
  df_b <- data.frame(X1 = X1, X2 = X2, y = y)

  # Estimate OLS
  mod <- lm(y ~ 1 + X1 + X2, data = df_b)

```

```
ests[b, ] <- coef(mod)
}

plot_ests(ests)
```



Answer:

As the variance of the error term grows, the variance of each estimated coefficient's sample distribution grows. The shape of the joint distribution remains a circle since the two coefficients are uncorrelated with each other.

4. Changing covariance matrix of X

```
B <- 5000
n <- 100
```

```

ests <- matrix(nrow = B, ncol = 3)
for (b in 1:B) {
  # Generate data
  Sigma_X = matrix(c(1, 0.25, 0.25, 1), nrow = 2, ncol = 2)
  X <- rmvnorm(n = n, mean = c(1, 1), sigma = Sigma_X)
  X1 <- X[, 1]
  X2 <- X[, 2]

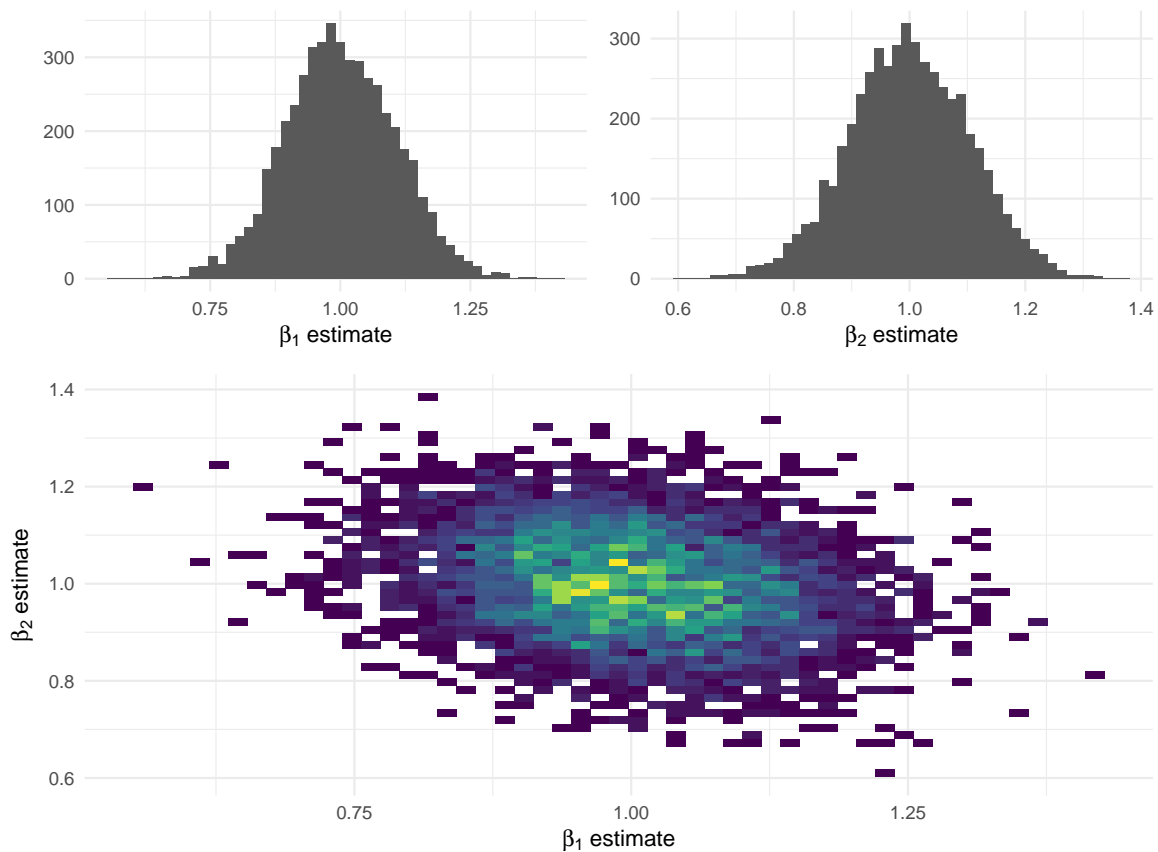
  y <- 10 + X1 * 1 + X2 * 1 + rnorm(n, mean = 0, sd = 1)

  df_b <- data.frame(X1 = X1, X2 = X2, y = y)

  # Estimate OLS
  mod <- lm(y ~ 1 + X1 + X2, data = df_b)
  ests[b, ] <- coef(mod)
}

plot_ests(ests)

```

```

B <- 5000
n <- 100
ests <- matrix(nrow = B, ncol = 3)
for (b in 1:B) {
  # Generate data
  Sigma_X = matrix(c(1, 0.99, 0.99, 1), nrow = 2, ncol = 2)
  X <- rmvnorm(n = n, mean = c(1, 1), sigma = Sigma_X)
  X1 <- X[, 1]
  X2 <- X[, 2]

  y <- 10 + X1 * 1 + X2 * 1 + rnorm(n, mean = 0, sd = 1)

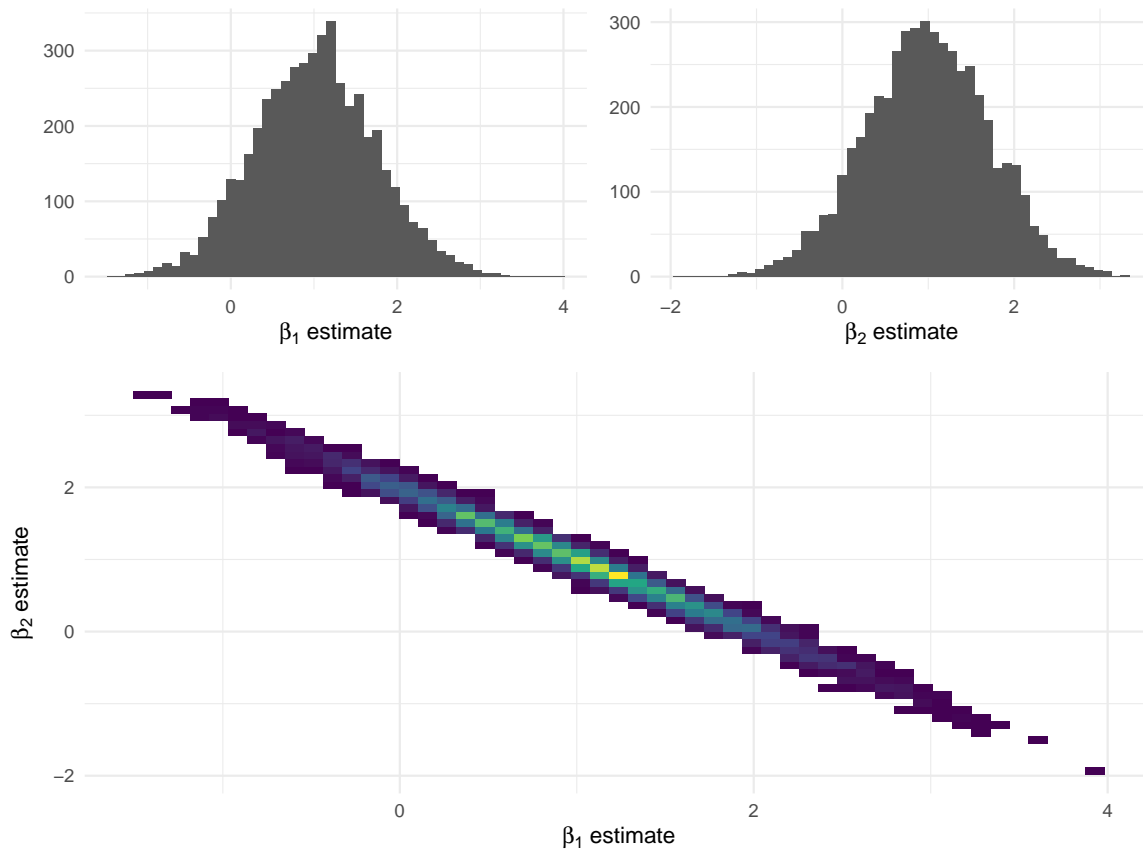
  df_b <- data.frame(X1 = X1, X2 = X2, y = y)

  # Estimate OLS
  mod <- lm(y ~ 1 + X1 + X2, data = df_b)

```

```
ests[b, ] <- coef(mod)
}

plot_ests(ests)
```



Answer:

As the covariance between the two variables get stronger, the marginal distributions get wider. This is because when the two are correlated, there is less “independent” variation of each variable. This makes it harder for OLS to pin down each parameter.

The joint distribution of the coefficients show that the estimates are correlated with each other.

5. Intuition on extreme collinearity

Note that in this extreme case, the OLS coefficients seem to fall on the line $\hat{\beta}_1 + \hat{\beta}_2 = 2$. In the case of the $Cov(X_1, X_2) = 0.99$, the two are nearly collinear. Therefore, OLS is not sure if to give “credit” to X_2 or X_1 . But, it is able to tell that a one unit increase in X_1 (and therefore \approx a 1 unit increase in X_2) yields a 2 unit increase in y . That’s why $\hat{\beta}_1 + \hat{\beta}_2 = 2$.