Causal Mechanisms in Difference-in-Differences Models

Kyle Butts & Brantly Callaway

June 8, 2022

Motivation

As applied researchers, we often study policies to understand underlying causal mechanisms

Motivation

As applied researchers, we often study policies to understand underlying causal mechanisms

- In some settings, there’s many alternatives and we want to know which is the underlying driver

Motivation

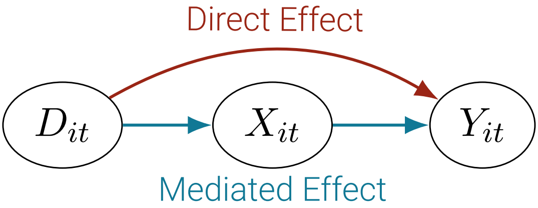
As applied researchers, we often study policies to understand underlying causal mechanisms

- In some settings, there's many alternatives and we want to know which is the underlying driver
- In other settings, we might want to rule out alternative explanations

Motivation

As applied researchers, we often study policies to understand underlying causal mechanisms

- In some settings, there's many alternatives and we want to know which is the underlying driver
- In other settings, we might want to rule out alternative explanations



Examples

Urban Economics

Asquith, Mast, and Reed (2021) find that new appartment construction decreases rental prices in the neighborhood.

Proposed Mechanisms:

1. Improved neighborhood amenities $\implies$ increase
2. Increased supply $\implies$ decrease
3. Congestion and other disamenities $\implies$ decrease

Examples

Health Economics

Goodman-Bacon (2018) find that Medicaid expansion decreased the rate of neonatal fatalities

Proposed Mechanisms:

1. In-hospital births driving the result
2. Rules out increased birth-weight

Examples

Economic Development

Hjort and Poulsen (2019) find that the arrival of fast internet increased employment rates in high-skill sectores.

Proposed Mechanisms:

1. Firm entry
2. Increased firm productivity
3. Firm exporting

Intuition of Mediation Effects

$$y_{it} = \mu_i + \eta_t + \tau_{it} D_{it} + X_{it}\beta + \varepsilon_{it}$$

Consider attending summer school (Math Camp) D_{it} on first-year prelim performance. Take X_{it} to be underlying math skill.

Intuition of Mediation Effects

$$y_{it} = \mu_i + \eta_t + \tau_{it} D_{it} + X_{it}(D_{it})\beta + \varepsilon_{it}$$

Consider attending summer school (Math Camp) D_{it} on first-year prelim performance. Take X_{it} to be ~~underlying~~ math skill.

- Math camp could increase your math skills, i.e. $X_{it}(D_{it})$ should be a function of treatment

Intuition of Mediation Effects

$$y_{it} = \mu_i + \eta_t + \tau_{it} D_{it} + X_{it}(D_{it})\beta(D_{it}) + \varepsilon_{it}$$

Consider attending summer school (Math Camp) D_{it} on first-year prelim performance. Take X_{it} to be underlying math skill.

- Math camp could increase your math skills, i.e. $X_{it}(D_{it})$ should be a function of treatment
- It could also teach you how to apply previous math skills in economics, i.e. $\beta(D_{it})$ should be a function of treatment

Intuition of Mediation Effects

$$y_{it} = \mu_i + \eta_t + \tau_{it} D_{it} + X_{it}(D_{it})\beta(D_{it}) + \varepsilon_{it}$$

Consider attending summer school (Math Camp) D_{it} on first-year prelim performance. Take X_{it} to be underlying math skill.

- Math camp could increase your math skills, i.e. $X_{it}(D_{it})$ should be a function of treatment
- It could also teach you how to apply previous math skills in economics, i.e. $\beta(D_{it})$ should be a function of treatment
- Math camp might improve your prelim scores by other mechanisms, τ_{it} (e.g. scaring you into studying more)

This Paper

This paper does two things:

1. Shows how to semiparametrically decompose the treatment effect into the mediated effect and the direct effect

This Paper

This paper does two things:

1. Shows how to semiparametrically decompose the treatment effect into the mediated effect and the direct effect
2. Shows under a linear setting how to decompose the mediated effect further by Oaxaca-Blinder Decomposition
   - $\rightarrow$ Change to X_{it}, change to β, or combination of both?

Related Work

Topic 1: Difference-in-Differences with Covariates

[Caetano et al. (2022), Callaway and Sant'Anna (2020)]

- This paper shows how to use covariates to decompose treatment effects

Topic 2: Mediation Analysis

[Sloczyński (2015), Rahimi and Hashemi Nazari (2021), Mora (2008), Kline (2011)]

- This paper extends this literature in the panel-setting

Theory

Theory

Model

Consider a panel of units $i = 1, \ldots, N$ and periods $t = 1, \ldots, T$ with treatment turning on at a particular period for all units (not needed, but easier). Researchers observe K covariates $X_{k,it}$ that are time-varying.

$$y_{it}(D_{it}) = \mu_i + \eta_t + \tau_{it} D_{it} + \sum_{k=1}^{K} g_k(X_{k,it}(D_{it}), D_{it}) + \varepsilon_{it}$$

- $X_{k,it}(D_{it})$ can depend on treatment
- g_k is a non-parametric function whose form can depend on treatment

Theory

Role of Covariates

$$y_{it}(D_{it}) = \mu_i + \eta_t + \tau_{it} D_{it} + \sum_{k=1}^{K} g_k(X_{k,it}(D_{it}), D_{it}) + \varepsilon_{it}$$

- Parallel Trends might only be plausible after controlling for covariates $X_{k,it}(0)$
- However, controlling for post-treatment covariates, $X_{k,it}(1)$, can absorb some of the treatment effect
  - $\rightarrow$ Controlling for improved math ability absorbs a treatment effect of math camp

So should you include covariates? Caetano et al. (2022)

Theory

Treatment Effects

For an individual, their treatment effect is given by

$$y_{it}(1) - y_{it}(0) = \tau_{it} + \sum_{k=1}^{K} \left[g_k(X_{k,it}(1), 1) - g_k(X_{k,it}(0), 0)\right]$$

What do we need to estimate?

1. $X_{k,it}(0)$
2. $g_k(x, 0)$
3. $g_k(x, 1)$

Theory

Identification Problem

$$\tau_{it} + \sum_{k=1}^{K} \left[g_k(X_{k,it}(1), 1) - g_k(X_{k,it}(0), 0) \right]$$

Note that you can't seperately identify a constant from g_k's. Take $\tau_{it} + \kappa$ and then take $g_k(x, 1) - \kappa$ for some k, and you will have the same fit.

Theory

Identification Problem

$$\tau_{it} + \sum_{k=1}^{K} [g_k(X_{k,it}(1), 1) - g_k(X_{k,it}(0), 0)]$$

Note that you can't seperately identify a constant from g_k's. Take $\tau_{it} + \kappa$ and then take $g_k(x, 1) - \kappa$ for some k, and you will have the same fit.

For this reason, we assume $g_k(0, d) = 0$ for all k and $d \in \{0, 1\}$. With our series estimator, this is equivalent to writting

$$g_k(X, d) \approx X_{k,it}\beta_{k,1}(d) + X_{k,it}^2\beta_{k,2}(d) + \cdots + X_{k,it}^L\beta_{k,L}(d)$$

Theory

Identification of $X_{k,it}(0)$

First, we need a way to impute $X_{k,it}(0)$. For this, we assume:

$$X_{k,it}(0) = \tilde{\mu}_i + \tilde{\eta}_t + \tilde{\epsilon}_{it},$$

and the parallel trends assumption $\mathbb{E}\left[\tilde{\epsilon}_{it+1} - \tilde{\epsilon}_{it} \mid D_{it}\right] = 0$

Then, identification follows an imputation estimator:

- Use untreated/not-yet-treated observations ($D_{it} = 0$) to estimate the above model.
- Then predict $X_{k,it}(0)$ for the treated observations ($D_{it} = 1$).

Theory

Identification of $g_k(x, 0)$

Similarly, we can estimate $g_k(x, 0)$ using untreated/not-yet-treated observations:

$$y_{it} = \mu_i + \eta_t + \sum_{k=1}^{K} X_{k,it}\beta_{k,1}(0) + X_{k,it}^2\beta_{k,2}(0) + \cdots + X_{k,it}^L\beta_{k,L}(0) + \varepsilon_{it}$$

Under parallel trends $\mathbb{E}\left[\varepsilon_{it+1} - \varepsilon_{it} \mid D_{it}, X_{it}(0)\right] = 0$, we can identify all these terms using a series estimator.

While we're at it, impute $\hat{y}_{it}(0)$ for treated observations.

Theory

Identification of $g_k(x, 1)$

Take our imputed $\hat{y}_{it}(0)$ and subtrace that from our observed y_{it}. Then we can estimate $g_k(x, 1)$ with a series estimator using treated observations ($D_{it} = 1$):

$$y_{it} - \hat{y}_{it}(0) = \tau + \sum_{k=1}^{K} X_{k,it}\beta_{k,1}(1) + X_{k,it}^2\beta_{k,2}(1) + \cdots + X_{k,it}^L\beta_{k,L}(1) + u_{it}$$

Identification of Direct and Mediation Effects

From our previous regerssion, $\hat{\tau}$ will serve as our estimate of the direct effect.

For each k, we can estimate the mediated effect by:

$$\hat{\gamma_k} \equiv \hat{\mathbb{E}}\left[\hat{g}_k(X_{k,it}, 1) - \hat{g}_k(\hat{X}_{k,it}(0), 0) \mid D_{it} = 1\right]$$

This is a true decomposition:

$$\widehat{\text{Overall ATT}} = \hat{\tau} + \sum_{k=1}^{K} \hat{\gamma_k}$$

Oaxaca-Blinder Decomposition

Linear in Covariates

Now assume $g_k(X_{k,it}, D_{it}) = X_{k,it}(D_{it})\beta(D_{it})$.

The three steps above can be summarized as:

1. Estimate $X_{k,it}(0)$
2. Estimate $\beta(0)$
3. Estimate $\beta(1)$

Linear in Covariates

Now assume $g_k(X_{k,it}, D_{it}) = X_{k,it}(D_{it})\beta(D_{it})$.

The three steps above can be summarized as:

1. Estimate $X_{k,it}(0)$
2. Estimate $\beta(0)$
3. Estimate $\beta(1)$

With observed data, we have two 'groups':

1. Treated group: $\{y_{it}(1), X_{it}(1), \beta(1)\}$
2. Untreated group: $\{y_{it}(0), X_{it}(0), \beta(0)\}$

Oaxaca-Blinder Decomposition

This falls directly into the Oaxaca-Blinder Decomposition. Treatment effects can be decomposed into:

$$\underbrace{\color{darkred}{\tau_{it}}}_{(1)} + \sum_{k=1}^{K} (\underbrace{\beta_k(0)\left[X_{k,it}(1) - X_{k,it}(0)\right]}_{(2)} + \underbrace{X_{k,it}(0)\left[\beta_k(1) - \beta_k(0)\right]}_{(3)}$$

$$+ \underbrace{\left[\beta_k(1) - \beta_k(0)\right]\left[X_{k,it}(1) - X_{k,it}(0)\right]}_{(4)})$$

(1) - Direct Effect operating outside of X_ks

Oaxaca-Blinder Decomposition

This falls directly into the Oaxaca-Blinder Decomposition. Treatment effects can be decomposed into:

$$\underbrace{\tau_{it}}_{(1)} + \sum_{k=1}^{K} \Big(\underbrace{\beta_k(0)\left[X_{k,it}(1) - X_{k,it}(0)\right]}_{(2)} + \underbrace{X_{k,it}(0)\left[\beta_k(1) - \beta_k(0)\right]}_{(3)}$$

$$+ \underbrace{\left[\beta_k(1) - \beta_k(0)\right]\left[X_{k,it}(1) - X_{k,it}(0)\right]}_{(4)} \Big)$$

(2) - Effect of changing X_k holding fixed marginal returns $\beta_k(0)$

Oaxaca-Blinder Decomposition

This falls directly into the Oaxaca-Blinder Decomposition. Treatment effects can be decomposed into:

$$\underbrace{\tau_{it}}_{(1)} + \sum_{k=1}^{K} (\underbrace{\beta_k(0)\left[X_{k,it}(1) - X_{k,it}(0)\right]}_{(2)} + \underbrace{X_{k,it}(0)\left[\beta_k(1) - \beta_k(0)\right]}_{(3)}$$

$$+ \underbrace{\left[\beta_k(1) - \beta_k(0)\right]\left[X_{k,it}(1) - X_{k,it}(0)\right]}_{(4)})$$

(3) - Effect of changing marginal returns β_k holding fixed $X_k(0)$

Oaxaca-Blinder Decomposition

This falls directly into the Oaxaca-Blinder Decomposition. Treatment effects can be decomposed into:

$$\underbrace{\tau_{it}}_{(1)} + \sum_{k=1}^{K} \Big(\underbrace{\beta_k(0)\left[X_{k,it}(1) - X_{k,it}(0)\right]}_{(2)} + \underbrace{X_{k,it}(0)\left[\beta_k(1) - \beta_k(0)\right]}_{(3)}$$

$$+ \underbrace{\left[\beta_k(1) - \beta_k(0)\right]\left[X_{k,it}(1) - X_{k,it}(0)\right]}_{(4)} \Big)$$

(4) - The remaining mediation effect from switching both β_k and X_k

Oaxaca-Blinder Decomposition

With the steps in the previous section, we could plug in all the values and take sample averages of (1), (2), (3), (4)

Conclusion

This paper unlocks a set of tools to understand why a treatment affects an outcome:

1. Decompose the overall average treatment effect into a mediated effect operating through a covariate and the remaining direct effect operating through other channels
2. Under a linear-in-covariates assumption, further decompose the mediation effect

Thank you!

References I

Asquith, Brian J., Evan Mast, and Davin Reed (May 2021). "Local Effects of Large New Apartment Buildings in Low-Income Areas". en. *The Review of Economics and Statistics*, pp. 1–46. ISSN: 0034-6535, 1530-9142. DOI: 10.1162/rest_a_01055.

Caetano, Carolina et al. (Feb. 2022). *Difference in Differences with Time-Varying Covariates*. en. Tech. rep. arXiv: 2202.02903. URL: http://arxiv.org/abs/2202.02903.

Callaway, Brantly and Pedro H.C. Sant'Anna (Dec. 2020). "Difference-in-Differences with Multiple Time Periods". en. *Journal of Econometrics*, S0304407620303948. ISSN: 03044076. DOI: 10.1016/j.jeconom.2020.12.001.

References II

Goodman-Bacon, Andrew (2018). "Public insurance and mortality: evidence from Medicaid implementation". *Journal of Political Economy* 126.1, pp. 216–262.

Hjort, Jonas and Jonas Poulsen (2019). "The arrival of fast internet and employment in Africa". *American Economic Review* 109.3, pp. 1032–79.

Kline, Patrick (May 2011). "Oaxaca-Blinder as a Reweighting Estimator". en. *American Economic Review* 101.3, pp. 532–537. ISSN: 0002-8282. DOI: 10.1257/aer.101.3.532.

Mora, Ricardo (2008). "A nonparametric decomposition of the Mexican American average wage gap". *Journal of Applied Econometrics* 23.4, pp. 463–485.

References III

Rahimi, Ebrahim and Seyed Saeed Hashemi Nazari (Dec. 2021). “A detailed explanation and graphical representation of the Blinder-Oaxaca decomposition method with its application in health inequalities”. en. *Emerging Themes in Epidemiology* 18.1, p. 12. ISSN: 1742-7622. DOI: 10.1186/s12982-021-00100-9.

Słoczyński, Tymon (Aug. 2015). “The Oaxaca-Blinder Unexplained Component as a Treatment Effects Estimator”. en. *Oxford Bulletin of Economics and Statistics* 77.4, pp. 588–604. ISSN: 03059049. DOI: 10.1111/obes.12075.