Homework 1, Problem 3

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I'll start off by bringing in all the data as a dataframe for ease

In [ ]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt

data = {
    'T (C)': [0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 20, 30, 40, 50, 60, 70, 80, 90
    'Volume of 1 gram of H2O (cm3)': [1.0001329, 1.0000733, 1.0000321, 1.00000
    'Volume of 1 gram of Hg (cm3)': [0.0735560, 0.0735694, 0.0735828, 0.073596
}

df = pd.DataFrame(data).set_index('T (C)')
df
```

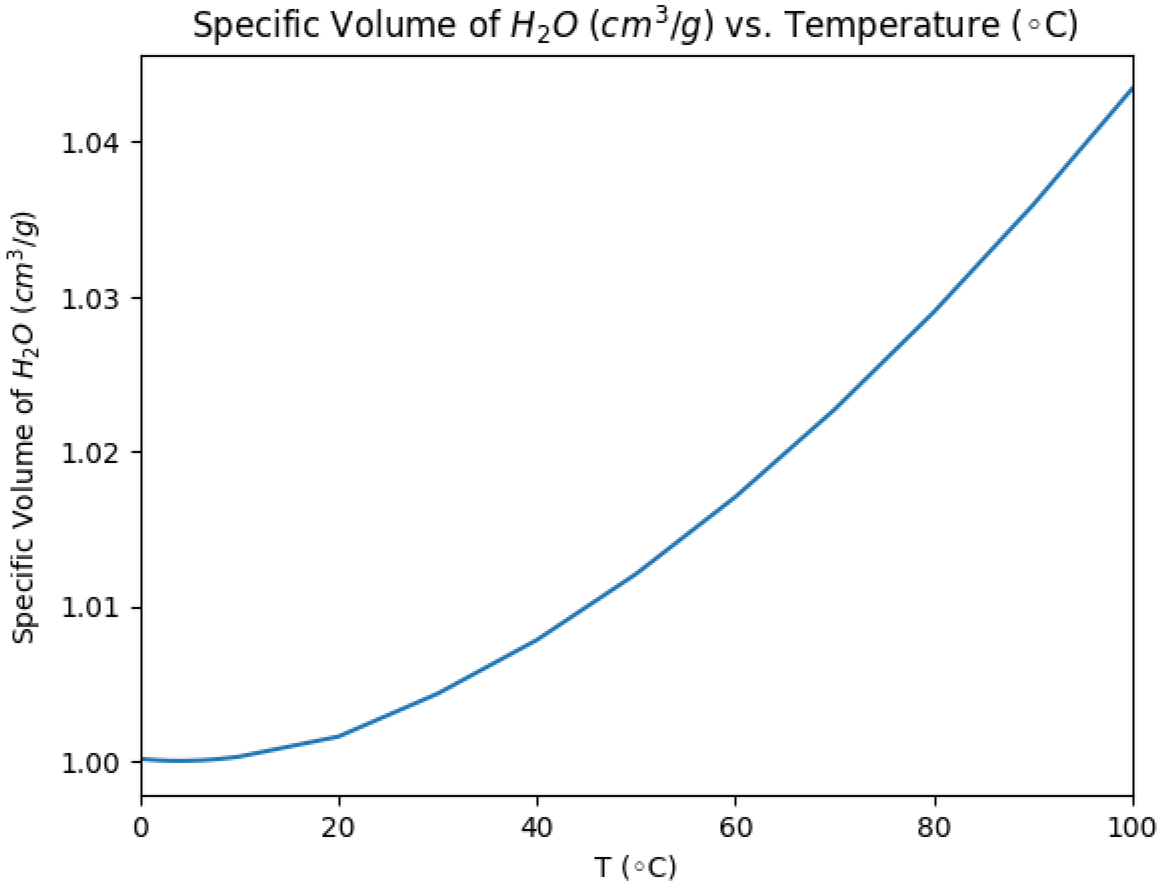
Out[ ]:

T (C)	Volume of 1 gram of H2O (cm3)	Volume of 1 gram of Hg (cm3)
0	1.000133	0.073556
1	1.000073	0.073569
2	1.000032	0.073583
3	1.000008	0.073596
4	1.000000	0.073609
5	1.000008	0.073623
6	1.000032	0.073636
7	1.000070	0.073650
8	1.000124	0.073663
9	1.000191	0.073676
10	1.000272	0.073689
20	1.001568	0.073823
30	1.004341	0.073957
40	1.007811	0.074091
50	1.012074	0.074225
60	1.017046	0.074359
70	1.022694	0.074494
80	1.028987	0.074628
90	1.035904	0.074763
100	1.043427	0.074898

Graphing the specific volume of H_2O as a function of temperature

In [ ]:
```
plt.plot(df['Volume of 1 gram of H2O (cm3)'], label='H2O')
plt.xlim(0, 100)
plt.xlabel('T (◦C)')
plt.ylabel('Specific Volume of $H_2O$ $(cm^3/g)$')
plt.title('Specific Volume of $H_2O$ ($cm^3 / g$) vs. Temperature (◦C)')
```

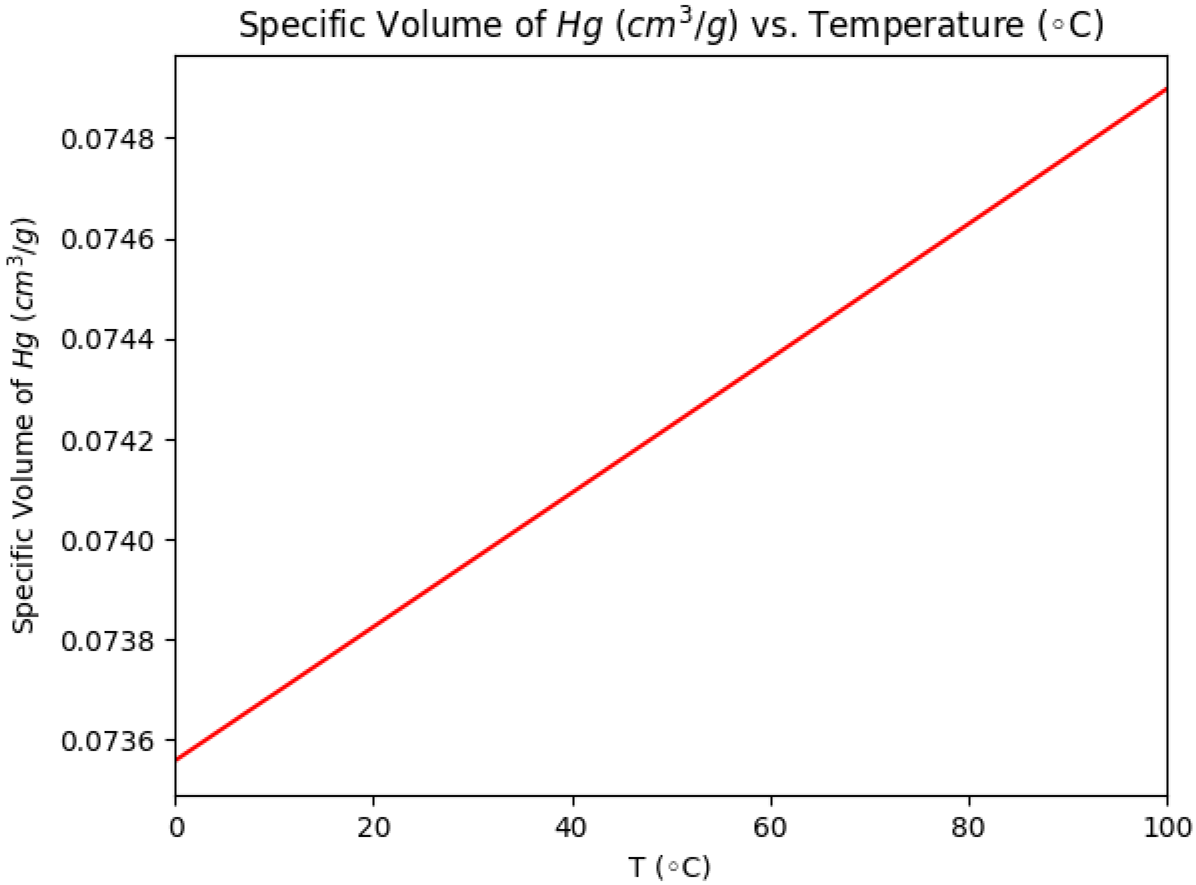
Out[ ]:
```
Text(0.5, 1.0, 'Specific Volume of $H_2O$ ($cm^3 / g$) vs. Temperature (◦C)')
```



doing the same for mercury

In [ ]:
```
plt.plot(df['Volume of 1 gram of Hg (cm3)'], label='Hg', color='red')
plt.xlim(0, 100)
plt.xlabel('T (◦C)')
plt.ylabel('Specific Volume of $Hg$ $(cm^3/g)$')
plt.title('Specific Volume of $Hg$ ($cm^3 / g$) vs. Temperature (◦C)')
```

Out[ ]:
```
Text(0.5, 1.0, 'Specific Volume of $Hg$ ($cm^3 / g$) vs. Temperature (◦C)')
```



a

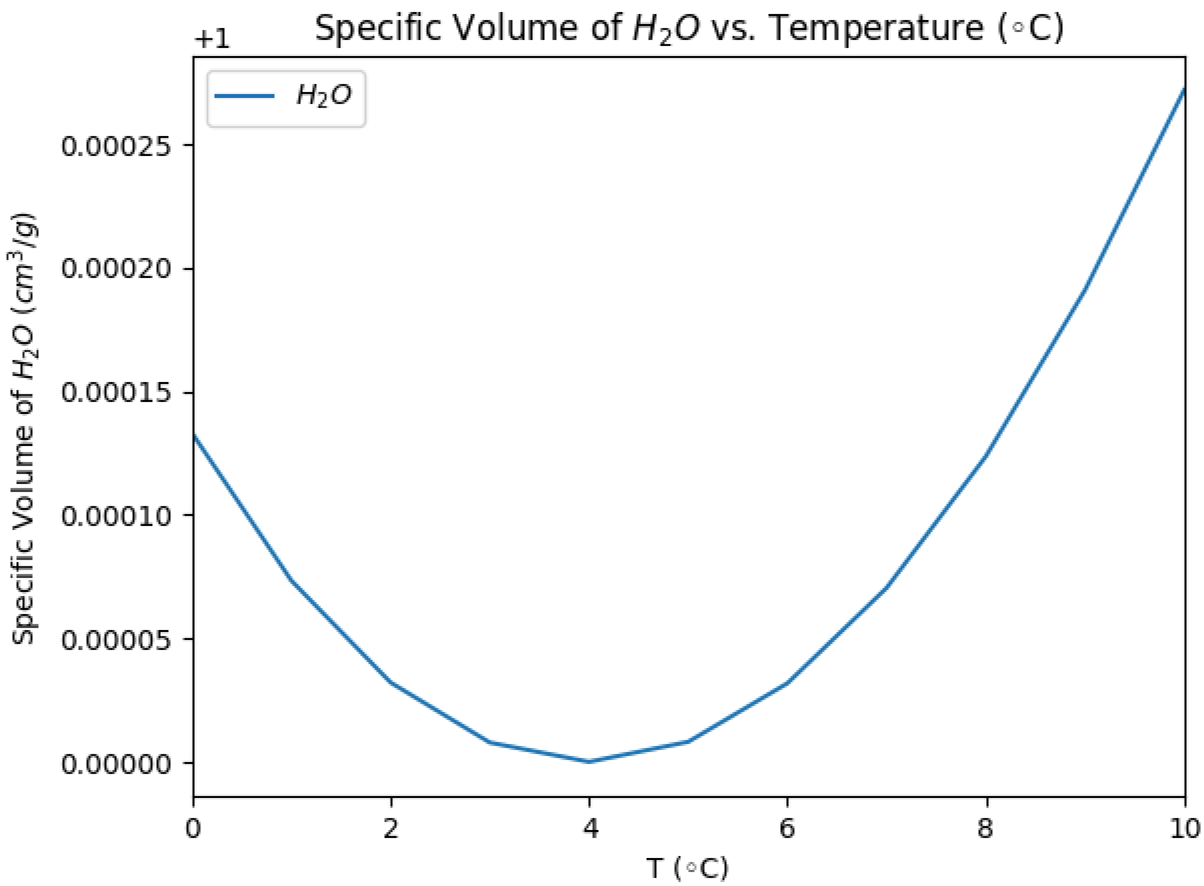
visualizing the specific volume of water from T=0^oC to T=10^oC

In [ ]:

```
plt.plot(df['Volume of 1 gram of H20 (cm3)'][:11], label='$H_20$')
plt.xlim(0, 10)
plt.xlabel('T (∘C)')
plt.ylabel('Specific Volume of $H_20$ $(cm^3/g)$')
plt.title('Specific Volume of $H_20$ vs. Temperature (∘C)')
plt.legend()
```

Out[ ]:

```
<matplotlib.legend.Legend at 0x13abd8a10>
```



water wouldnt be very good on this range of temperatures since its hard to tell 0-2 apart from 6-8 and also water as a whole (from the earlier visualization) is quite nonlinear. since it's not strictly increasing/decreasing the temperature could be rising but the thermometer level could be changing directions. this is not what you want a thermometer to do!

b

first simulate one that's only calibrated at 0^oC and 100^oC

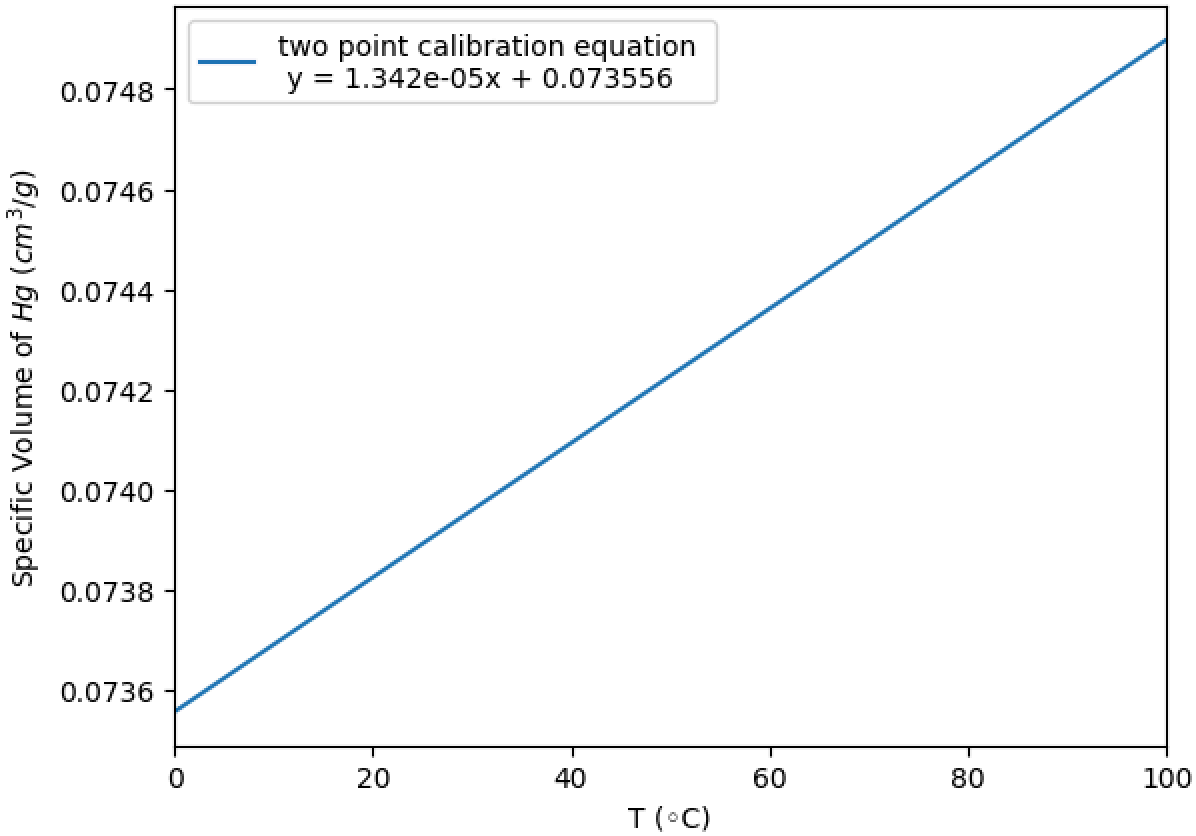
In [ ]:
```
volumes = [df['Volume of 1 gram of Hg (cm3)'][0], df['Volume of 1 gram of Hg (
temperatures = [0, 100]

m, b = np.polyfit(temperatures, volumes, 1)
print(f'equation: y = {m:.3e}x + {b:.6f}')

plt.plot(temperatures, m*np.array(temperatures) + b, label=f'two point calibra
plt.xlim(0, 100)
plt.xlabel('T (◦C)')
plt.ylabel('Specific Volume of $Hg$ $(cm^3/g)$')
plt.legend()
```

```
equation: y = 1.342e-05x + 0.073556
```

Out[ ]:
```
<matplotlib.legend.Legend at 0x13ad3ec90>
```



The two point calibration gives us

$$\hat{V} = 1.342 \cdot 10^{-5} \cdot T + 0.073556$$

Which means

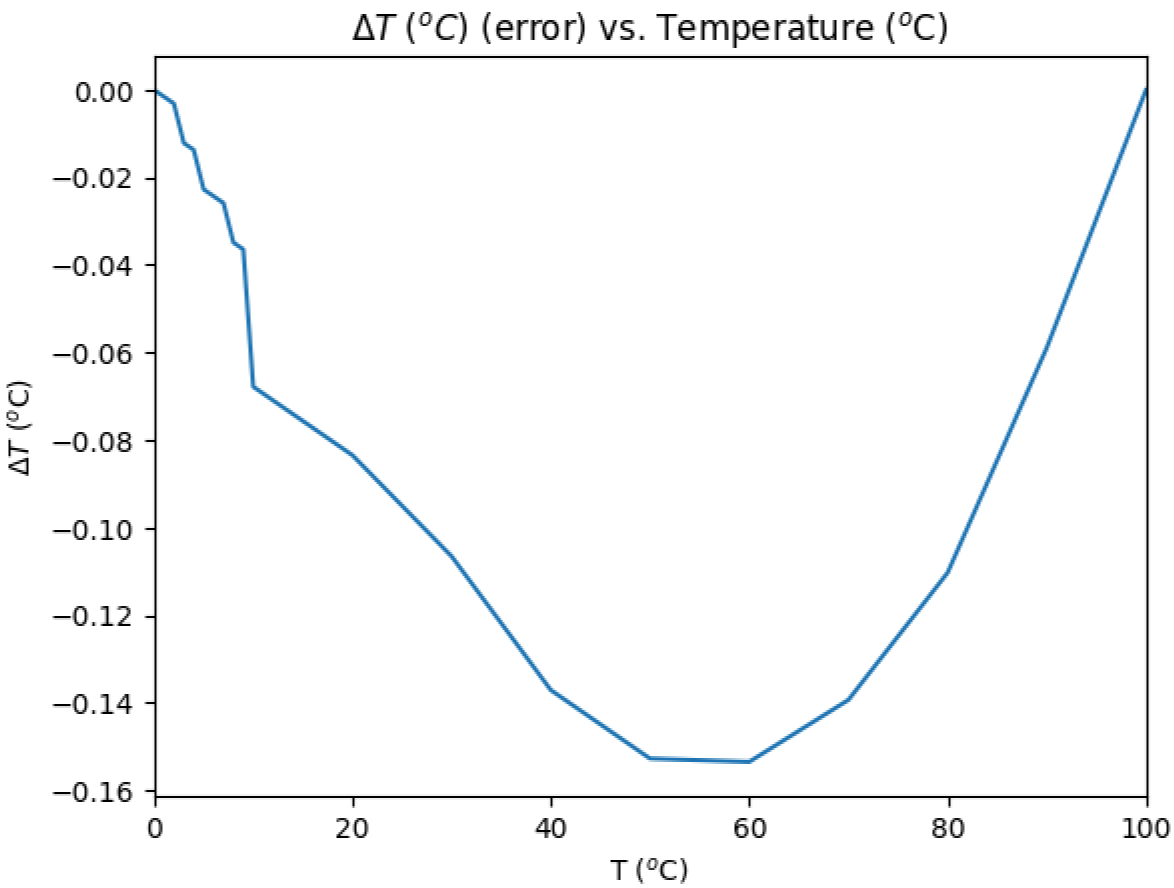
$$T_{meas} = \frac{\hat{V} - 0.073556}{1.3402 \cdot 10^{-5}}$$

so we go through each $\bar{V}$ in the table and calculate what temp that correlates to on the two point calibration, subtract the actual temperature from it, and then put together a nice little graph

In [ ]:

```
volumes = df['Volume of 1 gram of Hg (cm3)']
temperatures = df.index
measured_temps = (volumes - b) / m
delta_T = measured_temps - temperatures

plt.plot(temperatures, delta_T, label='Measured Temperatures')
plt.xlim(0, 100)
plt.xlabel('T ($^o$C)')
plt.ylabel('$\Delta T$ ($^o$C)')
plt.title('$\Delta T \ (^oC)$ (error) vs. Temperature ($^o$C)');
```



C

The change in height of the mercury is proportional to how much the volume increases and inversely proportional to the cross sectional area of the tube/capilarry. To put this in math:

$$\Delta h_{mercury} = \frac{\Delta V}{A}$$

So if you want to maximize the thermometers response to changes in temperatures you'd minimize the A, which is the purpose of using the capilarry tube, and maximize the ΔV, which is the purpose of the large bulb since having more mass of Hg increases the ΔV