cheg304 hw6 q1

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(a)

for $n = 4$, we get (10 ± 0.075). since n is pretty small, we need to **assume the underlying distribution is normal** to continue this analysis. once we do that, we know that the sampling distribution of a normally distributed random variable is as follows

$$\mu_{\bar{x}} = \mu \qquad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

now we can find our z value for the range and calculate the probability from there. also if just calculate one side then we can use symmetry

$$z_{\text{left}} = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{-0.075}{0.10/\sqrt{4}} = -1.5$$

now we use table or like any software to find (where $f(x)$ is the z distribution)

$$\int_{-\infty}^{-1.5} f(x)\,dx$$

▶ Code

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p: 0.134
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(b)

this is the same setup as before, but we need to calculate z_{left} and z_{right} and add their probabilities

$$z_{\text{right}} = \frac{\bar{x}_{\text{right}} - \mu}{\sigma/\sqrt{n}} = \frac{10.075 - 10.10}{0.10/\sqrt{4}} = -0.5$$

$$z_{\text{left}} = \frac{\bar{x}_{\text{left}} - \mu}{\sigma/\sqrt{n}} = \frac{9.925 - 10.10}{0.10/\sqrt{4}} = -3.5$$

the probability can then be calculated easily noting that

$$\int_{-\infty}^{-0.5} f(x)\,dx = \int_{0.5}^{\infty} f(x)\,dx$$

which makes the left

$$\int_{-\infty}^{-3.5} f(x)\,dx$$

and the right

$$\int_{0.5}^{\infty} f(x)\,dx = 1 - \int_{-\infty}^{-0.5} f(x)\,dx$$

▶ Code

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probability: 0.692
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now just making a quick visualization to compare (a) and (b)

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