cheg304 hw7 q1

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a

part 1

here our hypotheses are the following

- null: the average height is 1.0 meters
- alternative the average height is not 1.0 meters

in math, where μ is the true average height

$$H_0: \ \mu = 1.0\,\mathrm{m}$$

$$H_a: \ \mu \neq 1.0\,\mathrm{m}$$

part 2

we need to compute the sample mean and standard deviation

```
import numpy as np
from scipy.stats import t, norm

data = "0.937 0.886 1.149 0.957 0.915 0.910 1.026 0.987 0.782 1.047"
data = data.split(' ')
data = np.array(data).astype(float)

print(f'sample mean: {data.mean():.3f}')
print(f'sample std: {data.std(ddof=1):.3f}')
```

```
sample mean: 0.960
sample std: 0.100
```

now since enszer directly said that the underlying distribution is normally distributed but we don't know the population standard deviation, we can use the t distribution

$$t = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{\bar{x} - \mu}{s/\sqrt{N}}$$

```
t_ = (data.mean() - 1.0) / (data.std(ddof=1) / np.sqrt(len(data)))
print(f't statistic: {t_:.3f}')
```

```
t statistic: -1.273
```

now we can also calculate the p value associated with this z statistic

```
p = t.cdf(t_, len(data) - 1)
print(f'p: {p:.3f}')
```

```
p: 0.117
```

part 3

when we compare p and $\alpha/2$ we see that $p > \alpha/2$. this means we fail to reject the null.

part 4

we fail to reject the null and conclude that the true mean height in the tank could be 1.0 meters

b

part 1

here our hypotheses are the following

- null: the average height is 1.0 meters
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in math, where μ is the true average height

$$H_0: \ \mu = 1.0\,\mathrm{m}$$

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these are the same as (a)

part 2

we need to compute the sample mean, but don't care about the sample standard deviation

```
print(f'sample mean: {data.mean():.3f}')
```

```
sample mean: 0.960
```

since we somehow know the population standard deviation, we know the sample means follow the normal distribution

$$z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{\bar{x} - \mu}{\sigma/\sqrt{N}}$$

```
z = (data.mean() - 1.0) / (0.1 / np.sqrt(len(data)))
print(f'z statistic: {z:.3f}')
```

```
z statistic: -1.278
```

we can again calculate p

```
p = norm.cdf(z)
print(f'p: {p:.3f}')
```

```
p: 0.101
```

part 3

when we compare p and $\alpha/2$ we see that $p > \alpha/2$. this means we fail to reject the null.

part 4

we fail to reject the null and conclude that the true mean height in the tank could be 1.0 meters

```
# filler
```