cheg304 hw6 q2

AUTHOR
author

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(2)

since our population variance is unknown, we should use the t-distribution. specifically, we want the t value where $\nu = 63$ for both 95% and 99%. then we can go back to our definition of t

$$t = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

then if we rearrange slightly

$$\bar{x} - \mu = t \frac{\sigma}{\sqrt{n}}$$

now we can answer the question (in code of course!!!!)

- note that scipy inverse cdfs default to being from the left and single tail.
- this is easy since t-distribution is symmetric

```
from scipy.stats import t
import numpy as np

mu = 178.0
sigma = 12.0
n = 64
nu = n-1

alpha = 0.05
x_mu = t.ppf(1 - alpha/2, nu) * sigma / np.sqrt(n)
ci_95 = (mu - x_mu, mu + x_mu)
print(f'95% CI: {np.round(ci_95, 1)}')

alpha = 0.01
x_mu = t.ppf(1 - alpha/2, nu) * sigma / np.sqrt(n)
ci_99 = (mu - x_mu, mu + x_mu)
print(f'99% CI: {np.round(ci_99, 1)}')
```

```
95% CI: [175. 181.]
99% CI: [174. 182.]
```

(b)

for this part we very quickly see that $\bar{x} - \mu = 2$, so

$$2 = t \frac{12}{\sqrt{64}}$$

$$t(\nu = 63) = \frac{4}{3}$$

now we just take our integral of the t-distribution pdf from $-\frac{4}{3}$ to $\frac{4}{3}$ and that will be our answer

```
integral = t.cdf(4/3, 63) - t.cdf(-4/3, 63)
print(f'level of confidence: {integral:.1%}')
```

```
level of confidence: 81.3%
```

we can also quickly check this by generating the confidence interval for this level of confidence

```
alpha = 1 - integral
x_mu = t.ppf(1 - alpha/2, nu) * sigma / np.sqrt(n)
ci_99 = (mu - x_mu, mu + x_mu)
print(f'{integral:.1%} CI: {np.round(ci_99, 1)}')
```

```
81.3% CI: [176. 180.]
```

```
# filler text
```