typed up answers for question 3

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April 16, 2025

1 a

$$x_1^{\mathrm{L}} = \frac{\left(1 - \gamma_2^{\mathrm{L}}(x^{\mathrm{L}}) \exp\left[\frac{\Delta_{\mathrm{fus}} H_2(T_{m,2})}{RT}\left(1 - \frac{T}{T_{m,2}}\right)\right]\right)}{\gamma_1^{\mathrm{L}}(x^{\mathrm{L}}) \exp\left[\frac{\Delta_{\mathrm{fus}} H_1(T_{m,1})}{RT}\left(1 - \frac{T}{T_{m,1}}\right)\right] - \gamma_2^{\mathrm{L}}(x^{\mathrm{L}}) \exp\left[\frac{\Delta_{\mathrm{fus}} H_2(T_{m,2})}{RT}\left(1 - \frac{T}{T_{m,2}}\right)\right]}$$

$$x_1^{\mathrm{S}} = \frac{\left(1 - \frac{1}{\gamma_2^{\mathrm{L}}(x^{\mathrm{L}})} \exp\left[-\frac{\Delta_{\mathrm{fus}} H_2(T_{m,2})}{RT}\left(1 - \frac{T}{T_{m,2}}\right)\right]\right)}{\frac{1}{\gamma_1^{\mathrm{L}}(x^{\mathrm{L}})} \exp\left[-\frac{\Delta_{\mathrm{fus}} H_1(T_{m,1})}{RT}\left(1 - \frac{T}{T_{m,1}}\right)\right] - \frac{1}{\gamma_2^{\mathrm{L}}(x^{\mathrm{L}})} \exp\left[-\frac{\Delta_{\mathrm{fus}} H_2(T_{m,2})}{RT}\left(1 - \frac{T}{T_{m,2}}\right)\right]}$$

and now for regular solution theory

$$\ln \gamma_1 = \frac{V_1^L(\delta_1 - \delta_2)^2}{RT}\Phi_2^2 = \frac{\Omega}{RT}x_2^2 \tag{1}$$

just imagine that Ω is whatever Ω needs to be to make that equation true.

$$x_1^{\mathrm{L}} = \frac{\left(1 - \exp\left[\frac{1}{RT}\left(\Omega x_1^2 + \Delta_{\mathrm{fus}} H_2(T_{m,2})\left(1 - \frac{T}{T_{m,2}}\right)\right)\right]\right)}{\exp\left[\frac{1}{RT}\left(\Omega x_2^2 + \Delta_{\mathrm{fus}} H_1(T_{m,1})\left(1 - \frac{T}{T_{m,1}}\right)\right)\right] - \exp\left[\frac{1}{RT}\left(\Omega x_1^2 + \Delta_{\mathrm{fus}} H_2(T_{m,2})\left(1 - \frac{T}{T_{m,2}}\right)\right)\right]}$$

$$x_1^{\mathrm{S}} = \frac{\left(1 - \exp\left[\frac{1}{RT}\left(-\Omega x_1^2 - \Delta_{\mathrm{fus}} H_2(T_{m,2})\left(1 - \frac{T}{T_{m,2}}\right)\right)\right]\right)}{\exp\left[\frac{1}{RT}\left(-\Omega x_2^2 - \Delta_{\mathrm{fus}} H_1(T_{m,1})\left(1 - \frac{T}{T_{m,1}}\right)\right)\right] - \exp\left[\frac{1}{RT}\left(-\Omega x_1^2 - \Delta_{\mathrm{fus}} H_2(T_{m,2})\left(1 - \frac{T}{T_{m,2}}\right)\right)\right]}$$

here are the replacements

$$\gamma_2^{\mathrm{L}}(x^{\mathrm{L}}) = \exp\left(\frac{\Omega}{RT}x_1^2\right) \qquad \gamma_1^{\mathrm{L}}(x^{\mathrm{L}}) = \exp\left(\frac{\Omega}{RT}x_2^2\right)$$

2 b

$$x_1^{\mathrm{L}} = \frac{\left(1 - \frac{1}{\gamma_2^{\mathrm{S}}(x^{\mathrm{S}})} \exp\left[\frac{\Delta_{\mathrm{fus}} H_2(T_{m,2})}{RT}\left(1 - \frac{T}{T_{m,2}}\right)\right]\right)}{\frac{1}{\gamma_1^{\mathrm{S}}(x^{\mathrm{S}})} \exp\left[\frac{\Delta_{\mathrm{fus}} H_1(T_{m,1})}{RT}\left(1 - \frac{T}{T_{m,1}}\right)\right] - \frac{1}{\gamma_2^{\mathrm{S}}(x^{\mathrm{S}})} \exp\left[\frac{\Delta_{\mathrm{fus}} H_2(T_{m,2})}{RT}\left(1 - \frac{T}{T_{m,2}}\right)\right]}$$

$$x_1^{\mathrm{S}} = \frac{\left(1 - \gamma_2^{\mathrm{S}}(x^{\mathrm{S}}) \exp\left[-\frac{\Delta_{\mathrm{fus}} H_2(T_{m,2})}{RT}\left(1 - \frac{T}{T_{m,2}}\right)\right]\right)}{\gamma_1^{\mathrm{S}}(x^{\mathrm{S}}) \exp\left[-\frac{\Delta_{\mathrm{fus}} H_1(T_{m,1})}{RT}\left(1 - \frac{T}{T_{m,1}}\right)\right] - \gamma_2^{\mathrm{S}}(x^{\mathrm{S}}) \exp\left[-\frac{\Delta_{\mathrm{fus}} H_2(T_{m,2})}{RT}\left(1 - \frac{T}{T_{m,2}}\right)\right]}$$

and now subbing in

$$x_1^{\mathrm{L}} = \frac{\left(1 - \exp\left[\frac{1}{RT}\left(-\Omega x_1^2 + \Delta_{\mathrm{fus}} H_2(T_{m,2})\left(1 - \frac{T}{T_{m,2}}\right)\right)\right]\right)}{\exp\left[\frac{1}{RT}\left(-\Omega x_2^2 + \Delta_{\mathrm{fus}} H_1(T_{m,1})\left(1 - \frac{T}{T_{m,1}}\right)\right)\right] - \exp\left[\frac{1}{RT}\left(-\Omega x_1^2 + \Delta_{\mathrm{fus}} H_2(T_{m,2})\left(1 - \frac{T}{T_{m,2}}\right)\right)\right]}$$

$$x_1^{\mathrm{S}} = \frac{\left(1 - \exp\left[\frac{1}{RT}\left(\Omega x_1^2 - \Delta_{\mathrm{fus}} H_2(T_{m,2})\left(1 - \frac{T}{T_{m,2}}\right)\right)\right]\right)}{\exp\left[\frac{1}{RT}\left(\Omega x_2^2 - \Delta_{\mathrm{fus}} H_1(T_{m,1})\left(1 - \frac{T}{T_{m,1}}\right)\right)\right] - \exp\left[\frac{1}{RT}\left(\Omega x_1^2 - \Delta_{\mathrm{fus}} H_2(T_{m,2})\left(1 - \frac{T}{T_{m,2}}\right)\right)\right]}$$

and for readability here is what was replaced

$$\gamma_1^{\mathrm{S}}(x^{\mathrm{S}}) = \exp\left(\frac{\Omega}{RT} x_2^2\right) \qquad \gamma_2^{\mathrm{S}}(x^{\mathrm{S}}) = \exp\left(\frac{\Omega}{RT} x_1^2\right)$$

3 c

$$x_1^{\mathrm{L}} = \frac{\left(1 - \frac{\gamma_2^{\mathrm{L}}(x^{\mathrm{L}})}{\gamma_2^{\mathrm{S}}(x^{\mathrm{S}})} \exp\left[\frac{\Delta_{\mathrm{fus}} H_2(T_{m,2})}{RT}\left(1 - \frac{T}{T_{m,2}}\right)\right]\right)}{\frac{\gamma_1^{\mathrm{L}}(x^{\mathrm{L}})}{\gamma_1^{\mathrm{S}}(x^{\mathrm{S}})} \exp\left[\frac{\Delta_{\mathrm{fus}} H_1(T_{m,1})}{RT}\left(1 - \frac{T}{T_{m,1}}\right)\right] - \frac{\gamma_2^{\mathrm{L}}(x^{\mathrm{L}})}{\gamma_2^{\mathrm{S}}(x^{\mathrm{S}})} \exp\left[\frac{\Delta_{\mathrm{fus}} H_2(T_{m,2})}{RT}\left(1 - \frac{T}{T_{m,2}}\right)\right]}$$

$$x_1^{\mathrm{S}} = \frac{\left(1 - \frac{\gamma_2^{\mathrm{S}}(x^{\mathrm{S}})}{\gamma_2^{\mathrm{L}}(x^{\mathrm{L}})} \exp\left[-\frac{\Delta_{\mathrm{fus}} H_2(T_{m,2})}{RT}\left(1 - \frac{T}{T_{m,2}}\right)\right]\right)}{\frac{\gamma_1^{\mathrm{S}}(x^{\mathrm{S}})}{\gamma_1^{\mathrm{L}}(x^{\mathrm{L}})} \exp\left[-\frac{\Delta_{\mathrm{fus}} H_1(T_{m,1})}{RT}\left(1 - \frac{T}{T_{m,1}}\right)\right] - \frac{\gamma_2^{\mathrm{S}}(x^{\mathrm{S}})}{\gamma_2^{\mathrm{L}}(x^{\mathrm{L}})} \exp\left[-\frac{\Delta_{\mathrm{fus}} H_2(T_{m,2})}{RT}\left(1 - \frac{T}{T_{m,2}}\right)\right]}$$

suprise suprise we‘re making the same substitutions

$$\gamma_2^{\mathrm{L}}(x^{\mathrm{L}}) = \exp\left(\frac{\Omega_L}{RT}x_1^2\right) \qquad \gamma_1^{\mathrm{L}}(x^{\mathrm{L}}) = \exp\left(\frac{\Omega_L}{RT}x_2^2\right)$$

$$\gamma_1^{\mathrm{S}}(x^{\mathrm{S}}) = \exp\left(\frac{\Omega_S}{RT}x_2^2\right) \qquad \gamma_2^{\mathrm{S}}(x^{\mathrm{S}}) = \exp\left(\frac{\Omega_S}{RT}x_1^2\right)$$

$$x_1^{\mathrm{L}} = \frac{\left(1 - \exp\left[\frac{\Omega_L x_1^2 - \Omega_S x_1^2 + \Delta_{\mathrm{fus}} H_2(T_{m,2})\left(1 - \frac{T}{T_{m,2}}\right)}{RT}\right]\right)}{\exp\left[\frac{\Omega_L x_2^2 - \Omega_S x_2^2 + \Delta_{\mathrm{fus}} H_1(T_{m,1})\left(1 - \frac{T}{T_{m,1}}\right)}{RT}\right] - \exp\left[\frac{\Omega_L x_1^2 - \Omega_S x_1^2 + \Delta_{\mathrm{fus}} H_2(T_{m,2})\left(1 - \frac{T}{T_{m,2}}\right)}{RT}\right]}$$

$$x_1^{\mathrm{S}} = \frac{\left(1 - \exp\left[\frac{\Omega_S x_1^2 - \Omega_L x_1^2 - \Delta_{\mathrm{fus}} H_2(T_{m,2})\left(1 - \frac{T}{T_{m,2}}\right)}{RT}\right]\right)}{\exp\left[\frac{\Omega_S x_2^2 - \Omega_L x_2^2 - \Delta_{\mathrm{fus}} H_1(T_{m,1})\left(1 - \frac{T}{T_{m,1}}\right)}{RT}\right] - \exp\left[\frac{\Omega_S x_1^2 - \Omega_L x_1^2 - \Delta_{\mathrm{fus}} H_2(T_{m,2})\left(1 - \frac{T}{T_{m,2}}\right)}{RT}\right]}$$