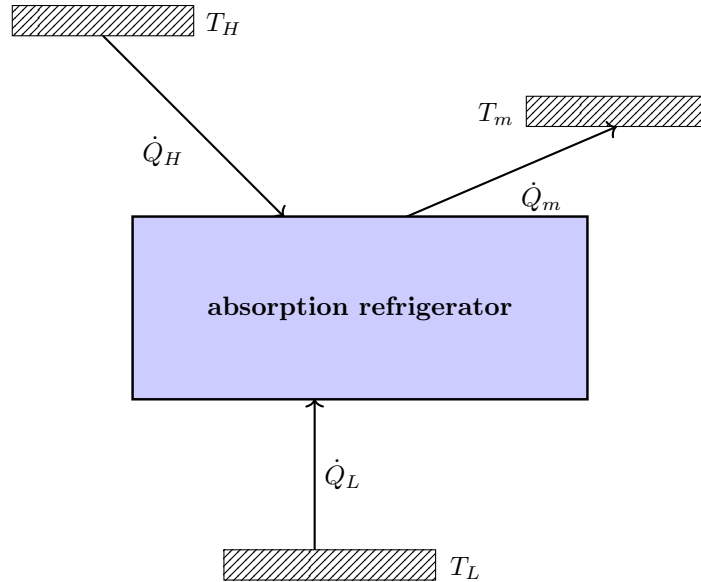


# CHEG231 Homework 7

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1.



**a**

Let the system be the absorption refrigerator. Assume it operates at steady—state and is reversible. the energy balance is

$$0 = \dot{Q}_H + \dot{Q}_L - \dot{Q}_m$$

and the entropy balance is

$$0 = \frac{\dot{Q}_H}{T_H} + \frac{\dot{Q}_L}{T_L} - \frac{\dot{Q}_m}{T_m}$$

now rearranging the energy balance

$$\dot{Q}_m = \dot{Q}_H + \dot{Q}_L$$

and slapping that into the entropy balance

$$0 = \frac{\dot{Q}_H}{T_H} + \frac{\dot{Q}_L}{T_L} - \frac{\dot{Q}_H + \dot{Q}_L}{T_m} = \frac{\dot{Q}_H}{T_H} - \frac{\dot{Q}_H}{T_m} + \frac{\dot{Q}_L}{T_L} + \frac{\dot{Q}_L}{T_m}$$

$$\dot{Q}_H \left( \frac{1}{T_m} - \frac{1}{T_H} \right) = \dot{Q}_L \left( \frac{1}{T_L} - \frac{1}{T_m} \right)$$

$$\boxed{\frac{\dot{Q}_L}{\dot{Q}_H} = \frac{\left( \frac{1}{T_m} - \frac{1}{T_H} \right)}{\left( \frac{1}{T_L} - \frac{1}{T_m} \right)}}$$

b.

First we need to convert the temperatures into Kelvin.

$$T_H = 1023 \text{ K}$$

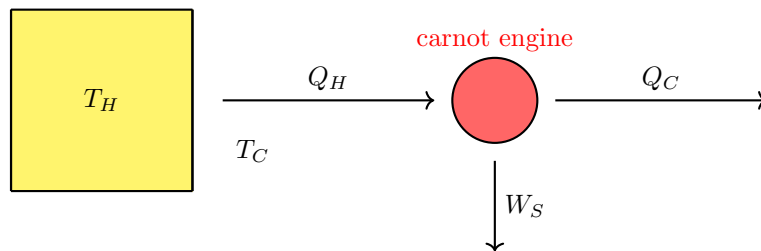
$$T_L = 270 \text{ K}$$

$$T_m = 300 \text{ K}$$

And now plugging these into the formula derived above

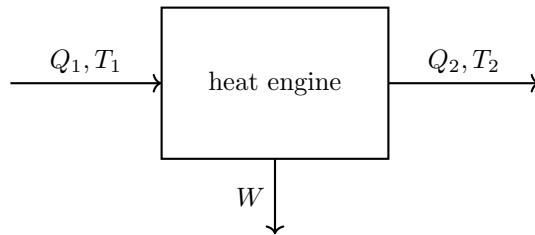
$$\frac{Q_L}{Q_H} = \frac{\left(\frac{1}{300} - \frac{1}{1023}\right)}{\left(\frac{1}{270} - \frac{1}{300}\right)} = \boxed{6.36}$$

2.



### Carnot Efficiency Derivation

Consider a closed system of a heat engine where  $T_1 > T_2$



energy balance:

$$\cancel{\frac{dU}{dt}} = \dot{Q} + \dot{W}_s + \cancel{\sum \dot{m}_k \hat{u}_k}$$

$$0 = Q_1 + Q_2 + W$$

entropy balance:

$$\cancel{\frac{dS}{dt}} = \frac{\dot{Q}_1}{T_1} + \frac{\dot{Q}_2}{T_2} + \cancel{\sum_{k=1}^K \dot{m}_k \hat{s}_k} + \dot{S}_{\text{gen}}$$

$$0 = \frac{\dot{Q}_1}{T_1} + \frac{\dot{Q}_2}{T_2} + \dot{S}_{\text{gen}}$$

integrating, and remembering that reversible processes are more efficient than irreversible processes

$$0 = \frac{Q_1}{T_1} + \frac{Q_2}{T_2}$$

$$\frac{Q_1}{T_1} = \frac{-Q_2}{T_2}$$

$$Q_2 = \frac{-Q_1 T_2}{T_1}$$

and taking this into the energy balance

$$\begin{aligned} -W_s &= Q_1 + \frac{-Q_1 T_2}{T_1} = Q_1 \left( \frac{T_1 - T_2}{T_1} \right) \\ \eta &= \frac{-W_s}{Q_1} = \left( \frac{T_1 - T_2}{T_1} \right) \end{aligned}$$

### back to the actual problem

Let the system be the block (the yellow thingy). The energy balance in difference form simplified (please don't crucify me for skipping to simplified balances)

$$\Delta U = Q_H$$

$$dU = dQ_H$$

$$\frac{dU}{dT_H} = C_v$$

$$dU = C_v dT_H$$

Now for the carnot engine we know this relationship will hold

$$\frac{-W_s}{Q_H} = 1 - \frac{T_c}{T_H}$$

which, in terms of infinitesimals

$$\begin{aligned} \frac{-dW_s}{dQ_h} &= 1 - \frac{T_c}{T_H} \\ -dW_s &= dQ_h \left[ 1 - \frac{T_c}{T_H} \right] \end{aligned}$$

and applying the result of the energy balance

$$-dW_s = C_v dT + H \left[ 1 - \frac{T_c}{T_H} \right]$$

now integrating to sum up all the dQ

$$\begin{aligned} -W_s &= \int_{T_1}^{T_2} C_v dT_H - \int_{T_1}^{T_2} C_v \frac{T_c}{T_H} dT_H \\ -W_s &= C_v (T_2 - T_1) - C_v T_c \ln \left( \frac{T_2}{T_1} \right) \end{aligned}$$

and just turning the absolute heat capacity into a not so absolute heat capacity

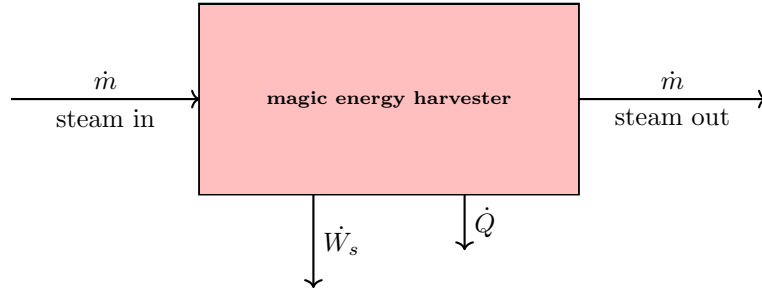
$$-W_s = m \hat{C}_v (T_2 - T_1) - m \hat{C}_v T_c \ln \left( \frac{T_2}{T_1} \right)$$

which now may be used to calculate out the two different exergies and compare them

Parameter	Case 1	Case 2
$T_H$	500 K	400 K
$T_C$	300 K	300 K
$m$	100 kg	200 kg
$\hat{C}_p$	500 J/kg K	500 J/kg K
$-W_S$	2.33 MJ	1.37 MJ

The block that starts at 500 K has more exergy.

3.



the system is the carnot engine. this system is at steady state. the mass balance is

$$\frac{dm}{dt} = 0 \rightarrow \dot{m}_{in} = \dot{m}_{out} = \dot{m}$$

energy balance

$$0 = \dot{Q} + \dot{W}_s + \sum_{k=1}^K \dot{m}_k \hat{H}_k = \dot{Q} + \dot{W}_s + \dot{m} (\hat{H}_{in} - \hat{H}_{out})$$

entropy balance

$$0 = \dot{m} (\hat{S}_{in} - \hat{S}_{out}) + \frac{\dot{Q}}{T} + \dot{S}_{gen}$$

but a reversible process is more efficient than an irreversible process, so,

$$0 = \dot{m} (\hat{S}_{in} - \hat{S}_{out}) + \frac{\dot{Q}}{T}$$

and rearranging the entropy balance and slap—smacking it into the energy balance

$$\begin{aligned} \dot{Q} &= T_{out} \dot{m} (\hat{S}_{out} - \hat{S}_{in}) \\ \dot{m} \left[ (\hat{H}_{in} - T_{out} \hat{S}_{in}) - (\hat{H}_{out} - T_{out} \hat{S}_{out}) \right] + \dot{W}_s &= 0 \\ -\dot{W}_s &= \dot{m} \left[ (\hat{H}_{in} - T_{out} \hat{S}_{in}) - (\hat{H}_{out} - T_{out} \hat{S}_{out}) \right] \end{aligned}$$

now going into the depths of the steam tables to look up the enthalpys and entropys

- $\hat{H}$  (200C, 1 bar) = 2875.3
- $\hat{H}$  (25C, 1 bar) = 104.89
- $\hat{S}$  (200C, 1 bar) = 7.8343
- $\hat{S}$  (25C, 1 bar) = 0.3674

also converting the flow rate

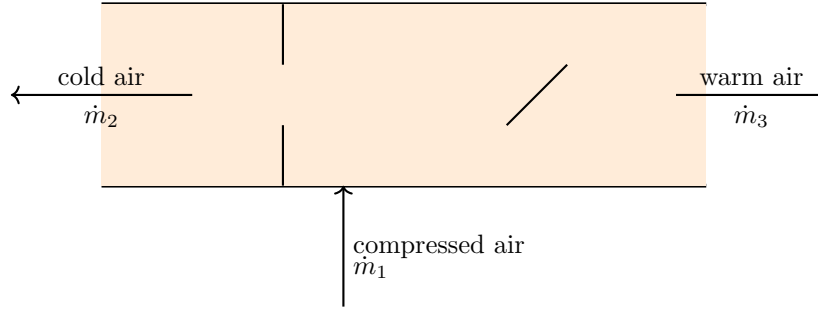
$$3.6 \frac{\text{tons}}{\text{hr}} \times \frac{1000 \text{ kg}}{1 \text{ ton}} \times \frac{1 \text{ hr}}{3600 \text{ sec}} = 1.0 \text{ kg/s}$$

plugging everything into the derived equation

$$\begin{aligned} -\dot{W}_s &= 1 (2875.3 - 298 \times 7.8343 - (104.89 - 298 \times 0.3674)) \\ -\dot{W}_s &= 545.3 \text{ kJ/s} \end{aligned}$$

#### 4.

To tell if a process is thermodynamically possible it needs to satisfy all the equations and it needs to have non-negative  $\dot{S}_{gen}$ . Let's take a look at this hilsch tube I've been hearing about...



the mass balance looks like

$$0 = \dot{n}_1 - \dot{n}_2 - \dot{n}_3$$

and we know mass in = mass out and that its a 50-50 split

$$\dot{n}_1 = 2\dot{n}_2 = 2\dot{n}_3 = \dot{n}$$

energy balance

$$0 = \dot{n}_1 \underline{H}_1 - \dot{n}_2 \underline{H}_2 - \dot{n}_3 \underline{H}_3$$

$$0 = 2\underline{H}_1 - \underline{H}_2 - \underline{H}_3$$

$$0 = (\underline{H}_1 - \underline{H}_2) + (\underline{H}_1 - \underline{H}_3)$$

and remembering that  $\Delta \underline{H} = C_p^* \Delta T$  for constant  $C_p^*$

$$0 = \hat{C}_p (T_1 - T_2) + \hat{C}_p (T_1 - T_3)$$

$$0 = (T_1 - T_2) + (T_1 - T_3)$$

$$T_1 = \frac{1}{2} (T_2 + T_3) = 300 \text{ K}$$

now we toss this into the entropy balance to see if  $\dot{S}_{gen} > 0$

$$0 = \sum n_k \underline{S}_k + \dot{S}_{gen} = n \left( \underline{S}_1 + \frac{1}{2} \underline{S}_2 + \frac{1}{2} \underline{S}_3 \right) + \dot{S}_{gen}$$

$$0 = \frac{1}{2} n ((\underline{S}_1 - \underline{S}_2) + (\underline{S}_1 - \underline{S}_3)) + \dot{S}_{gen}$$

and now using that silly ideal gas adiabatic change in entropy equation derived from the gibbs equation

$$-\dot{S}_{gen} = \frac{1}{2} n \left( \left( C_p \ln \left( \frac{T_2}{T_1} \right) - R \ln \left( \frac{P_2}{P_1} \right) \right) + \left( C_p \ln \left( \frac{T_3}{T_1} \right) - R \ln \left( \frac{P_3}{P_1} \right) \right) \right)$$

$$\dot{S}_{gen} = -\frac{1}{2} n \left( \left( 35.026 \ln \left( \frac{100}{300} \right) - 8.3145 \ln \left( \frac{1}{3.33} \right) \right) + \left( 35.026 \ln \left( \frac{500}{300} \right) - 8.3145 \ln \left( \frac{1}{3.33} \right) \right) \right)$$

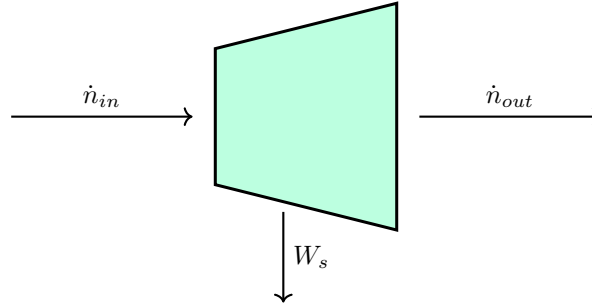
$$\dot{S}_{gen} = -0.292n$$

since the entropy generation is negative, this process is impossible.

#### 5.

Assume air is a diatomic gas in ideal state s.t.  $C_p = \frac{7}{2} R$  and  $C_v = \frac{5}{2} R$ .

a. adiabatic, reversible turbine



Let the system be the contents of the turbine highlighted in mint green. Mass balance:

$$0 = \dot{n}_{in} - \dot{n}_{out}$$

energy balance

$$0 = -\dot{W}_s + \dot{n}_{in}\hat{H}_{in} - \dot{n}_{out}\hat{H}_{out}$$

$$-\dot{W}_s = \dot{n}(\hat{H}_{out} - \hat{H}_{in})$$

entropy balance

$$0 = \dot{n}(\underline{S}_{in} - \underline{S}_{out})$$

isentropic!

$$\underline{S}_{in} = \underline{S}_{out}$$

now bringing in that one fun equation derived from the gibbs equation

$$\Delta \underline{S} = 0 = C_p \ln \left( \frac{T_2}{T_1} \right) - R \ln \left( \frac{P_2}{P_1} \right)$$

$$\left( \frac{T_2}{T_1} \right)^{C_p} = \left( \frac{P_2}{P_1} \right)^R$$

$$T_2 = T_1 \left( \frac{P_2}{P_1} \right)^{R/C_p}$$

$$T_2 = 400 \left( \frac{1.01325}{10} \right)^{8.3145/(3.5 \times 8.3145)}$$

$$T_2 = 208 \text{ K}$$

remembering  $\Delta H = C_p \Delta T$  for constant  $C_p$

$$-\dot{W}_s = \dot{n} C_p (T_{out} - T_{in})$$

and now because this question is annoyingly ideal i'll use the ideal gas law to find the molar volume

$$\underline{V} = \frac{RT}{P} = \frac{8.3145 \times 400}{10 \times 10^5} \times 1000 \frac{\text{L}}{\text{m}^2} = 3.33 \frac{\text{L}}{\text{mol}}$$

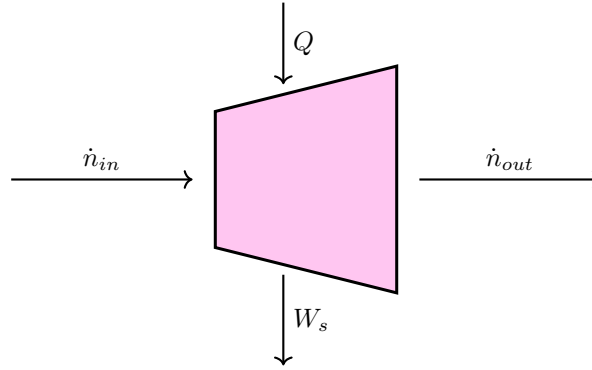
$$\dot{n} = \frac{10 \text{ L/s}}{3.33 \text{ L/mol}} = 3.01 \text{ mol/s}$$

back to the equation from the energy and mass balance

$$-\dot{W}_s = 3.01 \times 29 (400 - 208)$$

$$-\dot{W}_s = 16782 \text{ kJ/s}$$

## b. isothermal turbine



mass balance

$$\dot{m}_{in} = \dot{m}_{out} = \dot{m}$$

energy balance

$$0 = W_s + Q - m(\hat{H}_{out} - \hat{H}_{in})$$

however, we know that for an ideal gas the enthalpy is only a function of temperature so  $\Delta H = 0$

$$-W_s = Q$$

now looking at the entropy balance differential flavor

$$0 = \frac{\dot{Q}}{T} + \dot{n}(\underline{S}_{in} - \underline{S}_{out}) + \dot{S}_{gen}$$

since the problem statement said 100% efficient, no entropy is generated

$$0 = \frac{\dot{Q}}{T} + \dot{n}(\underline{S}_{in} - \underline{S}_{out})$$

$$\dot{Q} = T\dot{n}(\underline{S}_{out} - \underline{S}_{in})$$

remembering the  $\Delta S$  equation that comes from the gibbs equation

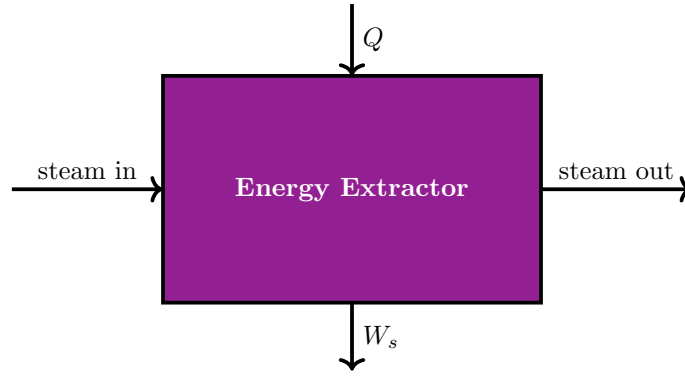
$$\begin{aligned}\Delta \underline{S} &= C_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right) \\ &= -R \ln\left(\frac{P_2}{P_1}\right) \\ &= -R \ln\left(\frac{1.01325 \text{ bar}}{10 \text{ bar}}\right) \\ &= 19.04 \frac{\text{kJ}}{\text{mol} \cdot \text{K}}\end{aligned}$$

and going back to the energy balance

$$-W_s = Q = T\dot{n}\Delta \underline{S} = 400 \times 3 \times 19.04 = 22865 \text{ kJ/s}$$

## c. maximum theoretical

consider the steam going through a magical black box that extracts all the energy from the steam until it is in the dead state.



the system is the contents of the energy extractor. mass balance

$$\frac{dn}{dt} = 0 = \dot{n}_{in} - \dot{n}_{out}$$

$$\dot{n}_{in} = \dot{n}_{out} = \dot{n}$$

energy balance

$$\frac{dU}{dt} = 0 = W_s + Q + \dot{n}_{in}\underline{H}_{in} - \dot{n}_{out}\underline{H}_{out}$$

entropy balance, noting that reversible processes are more efficient than irreversible processes

$$\frac{dS}{dt} = 0 = \dot{n}_{in}\underline{S}_{in} - \dot{n}_{out}\underline{S}_{out} + \frac{Q}{T_{\text{ambient}}}$$

$$\dot{Q} = \dot{n}(\underline{S}_{out} - \underline{S}_{in})$$

now substituting that into the energy balance

$$0 = W_s + \dot{n}(\underline{S}_{out} - \underline{S}_{in}) + \dot{n}\underline{H}_{in} - \dot{n}_{out}\underline{H}_{out}$$

$$-W_s = \dot{n}[(\underline{H}_{in} - T_{\text{ambient}}\underline{S}_{in}) - (\underline{H}_{out} - T_{\text{ambient}}\underline{S}_{out})]$$

$$-W_s = \dot{n}[(\underline{H}_{in} - \underline{H}_{out}) - T_{\text{ambient}}(\underline{S}_{in} - \underline{S}_{out})]$$

$$-W_s = \dot{n} \left[ (-C_p^* \Delta T) + T_{\text{ambient}} \left( C_p^* \ln \left( \frac{T_2}{T_1} \right) - R \ln \left( \frac{P_2}{P_1} \right) \right) \right]$$

$$-W_s = 3 \left[ ((29)(400 - 298)) + 298 \left( 29 \ln \left( \frac{298}{400} \right) - 8.3145 \ln \left( \frac{1.013}{10} \right) \right) \right]$$

$$-W_s = 18266 \text{ kW}$$

both  $\Delta$   
are negative

**6.**

**a.**

**a.**

$$d\underline{U} = \left( \frac{\partial \underline{U}}{\partial \underline{S}} \right)_{\underline{V}} d\underline{S} + \left( \frac{\partial \underline{U}}{\partial \underline{V}} \right)_{\underline{S}} d\underline{V}$$

**b.**

$$d\underline{H} = \left( \frac{\partial \underline{H}}{\partial \underline{S}} \right)_P d\underline{S} + \left( \frac{\partial \underline{H}}{\partial P} \right)_{\underline{S}} dP$$

**c.**

$$d\underline{H} = \left( \frac{\partial \underline{H}}{\partial T} \right)_P dT + \left( \frac{\partial \underline{H}}{\partial P} \right)_T dP$$



**b.**

restating the fundamental relationship

$$d\underline{U} = T d\underline{S} - P d\underline{V}$$

and now turning this into a complete/total differential

$$d\underline{U} = \left( \frac{\partial \underline{U}}{\partial \underline{S}} \right)_{\underline{V}} d\underline{S} - \left( \frac{\partial \underline{U}}{\partial \underline{V}} \right)_{\underline{S}} d\underline{V}$$

we see this

$$\begin{aligned} d\underline{U} &= T d\underline{S} - P d\underline{V} \\ d\underline{U} &= \left( \frac{\partial \underline{U}}{\partial \underline{S}} \right)_{\underline{V}} d\underline{S} - \left( \frac{\partial \underline{U}}{\partial \underline{V}} \right)_{\underline{S}} d\underline{V} \end{aligned}$$

so now just putting this as an equation so it feels all formalized

$$\left( \frac{\partial \underline{U}}{\partial \underline{S}} \right)_{\underline{V}} = T$$

$$\left( \frac{\partial \underline{U}}{\partial \underline{V}} \right)_{\underline{S}} = -P$$

**7.**

Starting off by writing the total differential

$$\begin{aligned} d\underline{H} &= \left( \frac{\partial \underline{H}}{\partial T} \right)_P dT + \left( \frac{\partial \underline{H}}{\partial P} \right)_T dP \\ \frac{\partial}{\partial T} \Big|_{\underline{H}} d\underline{H} &= \left( \frac{\partial \underline{H}}{\partial T} \right)_{\underline{H}} = \left( \frac{\partial \underline{H}}{\partial T} \right)_P \left( \frac{\partial T}{\partial T} \right)_{\underline{H}} + \left( \frac{\partial \underline{H}}{\partial P} \right)_T \left( \frac{\partial P}{\partial T} \right)_{\underline{H}} \end{aligned}$$

remembering the rules tells us that the red term is 0 and the green term is 1

$$\begin{aligned} 0 &= \left( \frac{\partial \underline{H}}{\partial T} \right)_P + \left( \frac{\partial \underline{H}}{\partial P} \right)_T \left( \frac{\partial P}{\partial T} \right)_{\underline{H}} \\ \left( \frac{\partial \underline{H}}{\partial T} \right)_P &= - \left( \frac{\partial \underline{H}}{\partial P} \right)_T \left( \frac{\partial P}{\partial T} \right)_{\underline{H}} \end{aligned}$$

and now diving over the ones on the right since we can do that thanks to rule 3

$$\left( \frac{\partial \underline{H}}{\partial T} \right)_P \left( \frac{\partial P}{\partial \underline{H}} \right)_T \left( \frac{\partial T}{\partial P} \right)_{\underline{H}} = -1$$