

Kyle Decker  
Machine Learning  
HW # 3

Problem 1 /

$$\text{Set } \{x_n\} \Rightarrow \vec{x} = \sum_n a_n \vec{x}_n$$

$$\text{where } a_n \geq 0 \text{ and } \sum_n a_n = 1$$

$$\text{Set } \{y_m\} \Rightarrow \vec{y} = \sum_m b_m \vec{y}_m$$

$$\text{where } b_m \geq 0 \text{ and } \sum_m b_m = 1$$

$$f(\vec{x}) = w^T x_n + w_0 > 0$$

$$f(\vec{y}) = w^T y_m + w_0 < 0$$

$$\text{at point of intersection: } f(\vec{x} \text{ intersect } \vec{y}) = \sum_n a_n (w^T x_n + w_0) = \sum_m b_m (w^T y_m + w_0)$$

~~then~~  $a_n$  and  $b_m$  must be non-negative so this is not possible, as  $f(\vec{x} \text{ intersect } \vec{y})$  can not be negative and positive.

## Problem 2/

"Cost sensitive" AdaBoost

$$R^{\text{train}} = \sum_i g_i e^{-(m\lambda)_i}$$

Coordinate descent on  $R^{\text{train}}(\lambda)$

$$\begin{aligned} j_t &\in \operatorname{argmax}_j \left[ - \frac{\partial R^{\text{train}}(\lambda_t + \alpha e_j)}{\partial \alpha} \Big|_{\alpha=0} \right] \\ &= \operatorname{argmax}_j \left[ - \frac{\partial}{\partial \alpha} \left[ \sum_i g_i e^{-(m(\lambda_t + \alpha e_j))_i} \right] \Big|_{\alpha=0} \right] \\ &= \operatorname{argmax}_j \left[ - \frac{\partial}{\partial \alpha} \left[ \sum_i g_i e^{-(m\lambda_t)_i - \alpha m_{ij}} \right] \Big|_{\alpha=0} \right] \\ &= \operatorname{argmax}_j \left[ \sum_i g_i m_{ij} e^{-(m\lambda_t)_i} \right] \end{aligned}$$

$$d_{t,i} = g_i e^{-(m\lambda_t)_i} / z_t$$

where  $z_t = \sum_i g_i e^{-(m\lambda_t)_i}$

Line search along direction  $j$ :

$$0 = - \sum_{i: m_{ij}=1} d_{t,i} e^{-\alpha} - \sum_{i: m_{ij}=-1} -d_{t,i} e^{\alpha} \Big/ d_t$$

$$= -e^{-\alpha} \underbrace{\sum_{i: m_{ij}=1} d_{t,i}}_{d_+} + e^{\alpha} \underbrace{\sum_{i: m_{ij}=-1} d_{t,i}}_{d_-}$$

$$= -e^{-\alpha} d_+ + e^{\alpha} d_-$$

$$d_+ e^{\alpha} = e^{\alpha} d_-$$

$$\frac{d_+}{d_-} = e^{2\alpha}$$

$$\frac{1}{2} \ln \left( \frac{d_+}{d_-} \right) = \alpha \Rightarrow \alpha = \frac{1}{2} \ln \left( \frac{d_+}{d_-} \right)$$

Finding direction:

$$j_t \in \arg \max_j \sum_i g_i m_{ij} e^{-(m \lambda_t) i}$$

$$j_t \in \arg \max_j \sum_i m_{ij} d_{t,i}$$

$$j_t \in \arg \max_j (d_t^T m)_j$$

↓  
replace w/ weak learning algorithm

$$j_t \in \arg \min_j \sum_{i: m_{ij} = -1} d_{t,i}$$

Algorithm:

$$d_{1,i} = \frac{1}{n} g_i \quad \text{for } i = 1 \dots n$$

$$\lambda_1 = 0$$

for  $t = 1 \dots T$

$$\text{Step 1: } \arg \min_j \left[ \sum_{i: m_{ij} = -1} d_{t,i} \right]$$

$$\text{where } d_- = \sum_{m_{ij} = -1} d_{t,i}$$

$$\text{Step 2: } \alpha_t = \frac{1}{2} \ln \left( \frac{1 - d_-}{d_-} \right)$$

$$\text{Weight update: } d_{t+1,i} = g_i e^{-(m \lambda_t) i} / z_{t+1}$$

$$< d_{t,i} e^{-\alpha_t} \quad \text{if } m_{ij} = 1$$

$$< d_{t,i} e^{\alpha_t} \quad \text{if } m_{ij} = -1$$

Problem 3/ If weak learning assumption holds, AdaBoost's misclassification error decays exponentially fast:

$$\frac{1}{m} \sum_{i=1}^m \mathbb{1}(y_i \neq H(x_i)) \leq e^{-2\gamma_{wLA}^2 T}$$

There will be zero training error if:

$$e^{-2\gamma_{wLA}^2 T} \leq \frac{1}{m}$$

$$\ln(e^{-2\gamma_{wLA}^2 T}) \leq \ln\left(\frac{1}{m}\right)$$

$$-2\gamma_{wLA}^2 T \leq \ln\left(\frac{1}{m}\right)$$

$$2\gamma_{wLA}^2 T \geq \ln(m)$$

$$2\gamma_{wLA}^2 T \geq \ln(m)$$

$$T \geq \frac{\ln(m)}{2\gamma_{wLA}^2}$$

As long as there are at least  $\frac{\ln(m)}{2\gamma_{wLA}^2}$  iterations of updates, AdaBoost will find a linear classifier that classifies the dataset perfectly.

### Problem 3/

Symmetric data set, twin  $-h(x)$  of each classifier  $h(x)$  is included. If the weak learning assumption does not hold the AdaBoost can not find a classifier that classifies the symmetric dataset perfectly all of the time.

$$M_{ij} = \begin{bmatrix} 1 & -1 & -1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 & -1 & 1 \\ -1 & 1 & -1 & -1 & 1 & -1 \\ \hline 1 & -1 & 1 & 1 & -1 & 1 \\ -1 & 1 & 1 & 1 & -1 & 1 \\ 1 & -1 & 1 & 1 & -1 & 1 \\ -1 & 1 & -1 & -1 & 1 & -1 \end{bmatrix}$$

## Problem 4/

I decided to use the letter-recognition data from the UCI machine learning repository. I performed classification of B letters vs O letters.

Code: Please see attached matlab code and generated outputs

Preprocessing: Simply reading the data in, mapping the letter B to class 1 and letter O to class 0.  
Random permutation of examples.

Algorithm 5:

- 1) Logistic Regression
- 2) SVM  $\rightarrow$  optimize Kernel
- 3) CART
- 4) Random Forest  $\rightarrow$  optimize # of trees
- 5) Boosted Trees

Using nested cross-validation I evaluated the following parameters for SVM: Kernel = [Linear, Polynomial, Gaussian]

for RF: # trees = [3, 10, 30, 60, 100]

ROC Curves: Please see the generated matlab plots.

AUC values for features and algorithms : please see generated outputs in code.

### Discussion:

- (i) The algorithms performed similarly in terms of judging their performance from the AUC.  
SVM, RF, and Boosted trees performed slightly better than logistic regression and CART.
- (ii) The classification of B vs. D in the dataset I used was not terribly difficult, i.e., a linear decision boundary performed quite well in the classification. Thus the algorithms all performed very well in result.
- (iii) Combining features and training a model resulted in much better results than the individual features alone.

# Table of Contents

.....	1
Data Loading .....	1
Using Features themselves for ROC .....	1
For 5 different Algorithms .....	9
Nested CV to choose kernel for SVM .....	10
Nested CV to choose number of trees for RF .....	12
AUROC .....	20

```
% Kyle Decker
% Machine Learning
% HW 3
```

```
close all
clc
```

## Data Loading

```
f = fopen('letter-recognition.data.txt');  
data0 = textscan(f,'%s %f %f %f %f %f %f %f %f %f %f %f %f %f %f  
%f', 20000, 'Delimiter',' ');  
fclose(f);  
data_all = cell2mat(data0(:,2:end));  
letter = cell2mat(data0{1,1});  
class_all = zeros(size(letter,1),1);  
class_all_B = (letter=='B'); % Classify B vs D  
B_ind = find(class_all_B);  
class_all_D = (letter=='D');  
D_ind = find(class_all_D);  
  
data = cat(1,data_all(B_ind,:),data_all(D_ind,:));  
class = zeros(size(data,1),1);  
class(1:size(B_ind,1))=1; % B = class 1  
class = (class==1); % make logical  
  
% data0 = csvread('wine.data.txt');  
% data = data0(:,2:end);  
% class = data0(:,1)>2; % Make binary, wine 1 vs wine 2&3  
  
% Random permutation of the dataset  
rand_i = randperm(size(class,1));  
data_perm = data(rand_i,:);  
class_perm = class(rand i);
```

## Using Features themselves for ROC

```
AUROC_all = zeros(10,size(data_perm,2));
```



---

```

for test_ind = 1:10
    p = 0.9; % proportion of the dataset for training
    m = size(data,1);

    % Circ shift to create new test and training set
    data_perm = circshift(data_perm,round((1-p)*m),1);
    class_perm = circshift(class_perm,round((1-p)*m),1);

    X = data_perm(1:round(p*m),:);
    Y = class_perm(1:round(p*m));
    Xtest = data_perm(round(p*m)+1:end,:);
    Ytest = class_perm(round(p*m)+1:end);

    figure;
    Yscores = Xtest(:,1);
    [Xfeat,Yfeat,Tfeat,AUCfeat] = perfcurve(Ytest,Yscores,'true');
    AUROC_all(test_ind,1) = AUCfeat;
    plot(Xfeat, Yfeat)
    hold on
    for feat_ind = 2:size(Xtest,2)
        %feat_ind = 1;
        Yscores = Xtest(:,feat_ind);
        [Xfeat,Yfeat,Tfeat,AUCfeat] = perfcurve(Ytest,Yscores,'true');
        AUROC_all(test_ind,feat_ind) = AUCfeat;
        plot(Xfeat, Yfeat)
    end
    xlabel('false positive rate');
    ylabel('true positive rate');
    title_str = ['ROC Curves for Individual Features Test Set:
',num2str(test_ind)];
    title(title_str)
    axis([-0.01,1.01,0,1.01])
    legend('F1', 'F2', 'F3', 'F4', 'F5', 'F6', 'F7', 'F8', ...
        'F9', 'F10', 'F11', 'F12', 'F13', 'F14', 'F15', 'F16')
    hold off
end

fprintf('AUROC (mean +/- standard deviation) for\n');
fprintf('Feature 1: %f +/- %f\n',
    mean(AUROC_all(:,1)),std(AUROC_all(:,1)) );
fprintf('Feature 2: %f +/- %f\n',
    mean(AUROC_all(:,2)),std(AUROC_all(:,2)) );
fprintf('Feature 3: %f +/- %f\n',
    mean(AUROC_all(:,3)),std(AUROC_all(:,3)) );
fprintf('Feature 4: %f +/- %f\n',
    mean(AUROC_all(:,4)),std(AUROC_all(:,4)) );
fprintf('Feature 5: %f +/- %f\n',
    mean(AUROC_all(:,5)),std(AUROC_all(:,5)) );
fprintf('Feature 6: %f +/- %f\n',
    mean(AUROC_all(:,6)),std(AUROC_all(:,6)) );
fprintf('Feature 7: %f +/- %f\n',
    mean(AUROC_all(:,7)),std(AUROC_all(:,7)) );

```

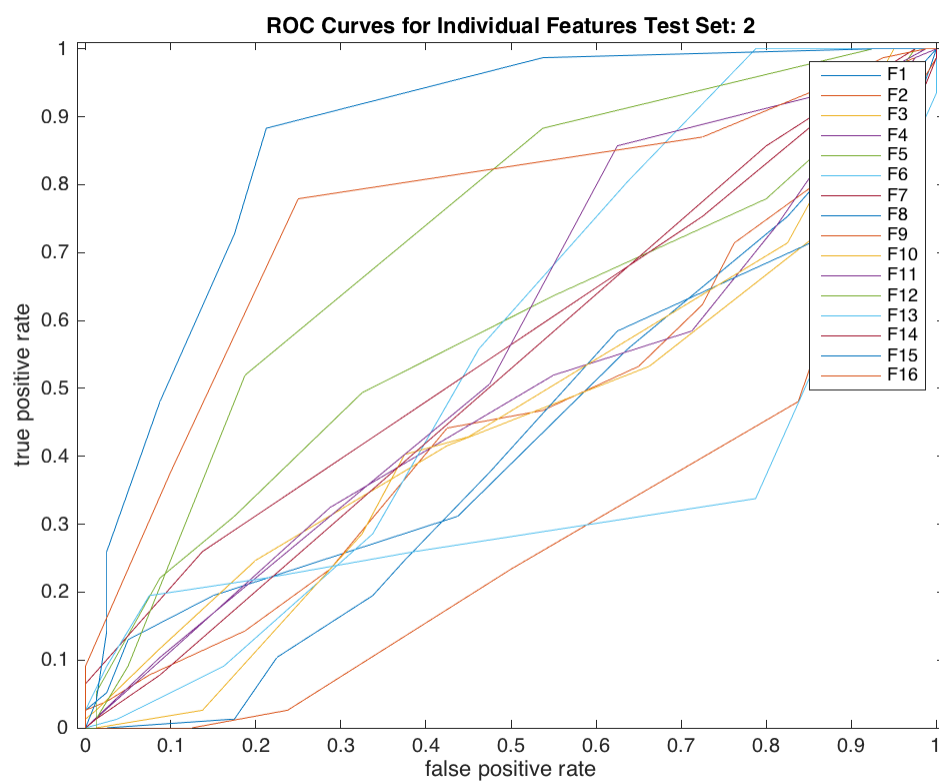
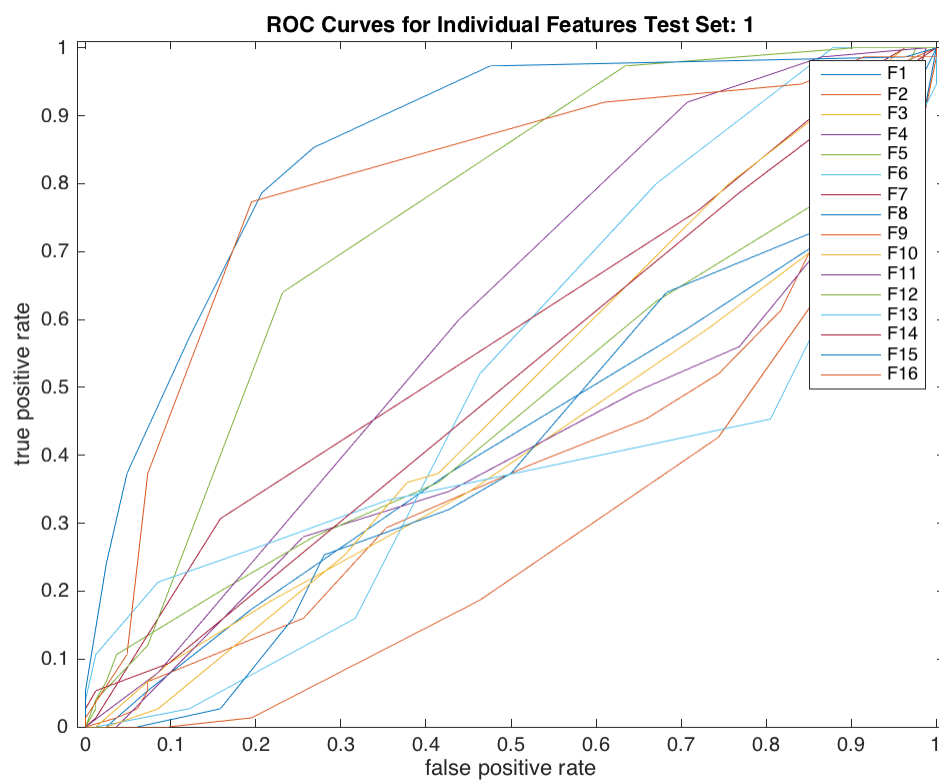
---

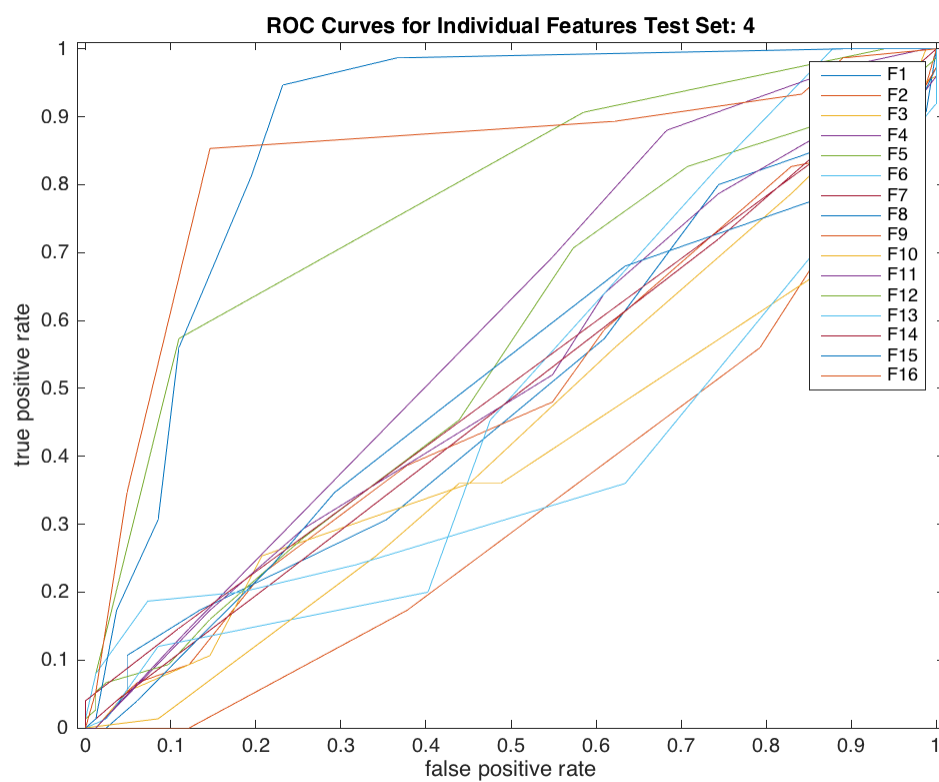
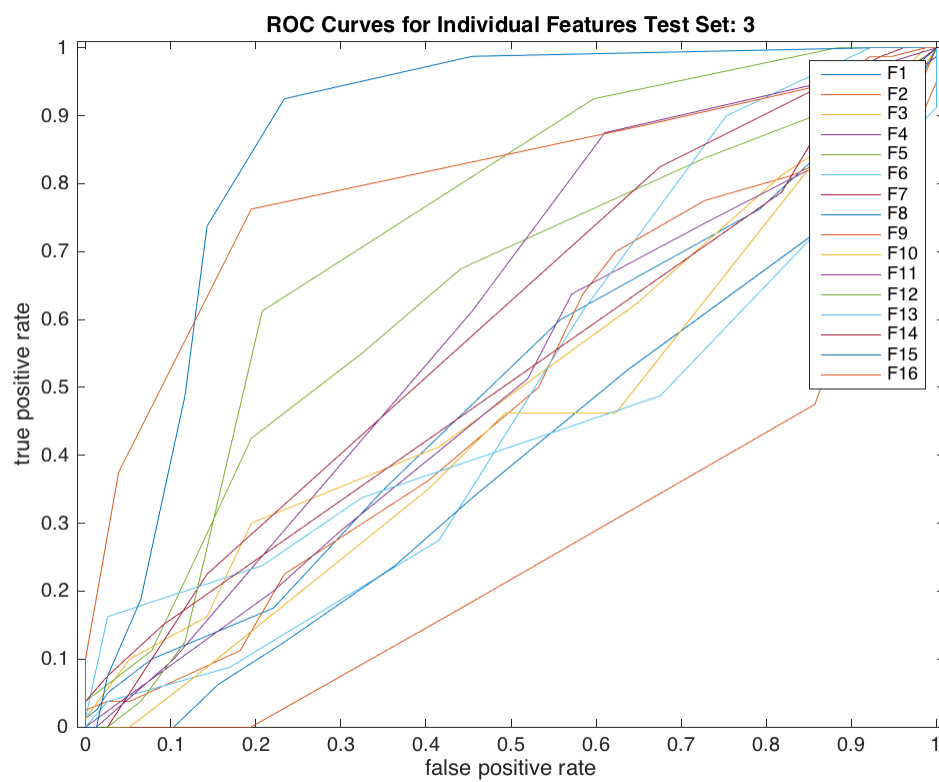
---

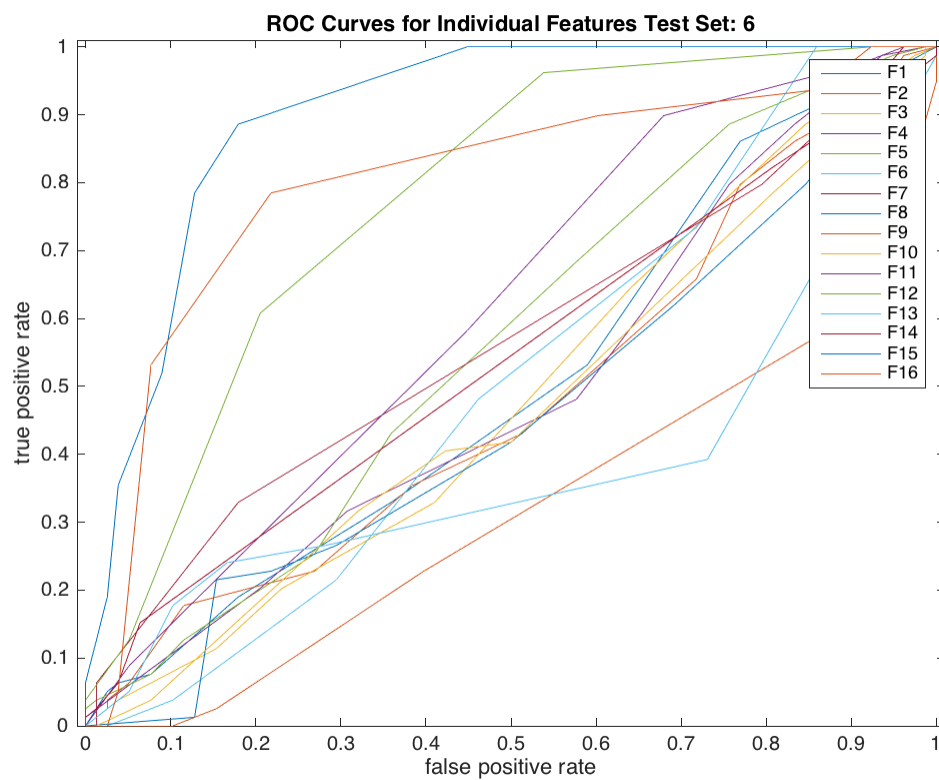
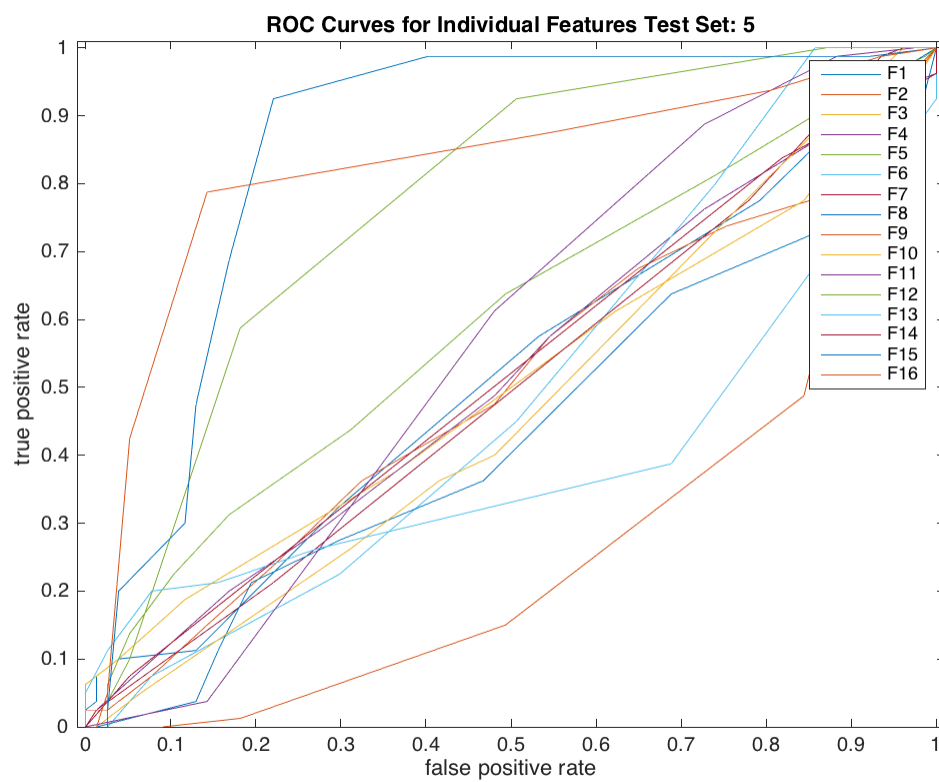
```
fprintf('Feature 8: %f +/- %f\n',
    mean(AUROC_all(:,8)),std(AUROC_all(:,8)) );
fprintf('Feature 9: %f +/- %f\n',
    mean(AUROC_all(:,9)),std(AUROC_all(:,9)) );
fprintf('Feature 10: %f +/- %f\n',
    mean(AUROC_all(:,10)),std(AUROC_all(:,10)) );
fprintf('Feature 11: %f +/- %f\n',
    mean(AUROC_all(:,11)),std(AUROC_all(:,11)) );
fprintf('Feature 12: %f +/- %f\n',
    mean(AUROC_all(:,12)),std(AUROC_all(:,12)) );
fprintf('Feature 13: %f +/- %f\n',
    mean(AUROC_all(:,13)),std(AUROC_all(:,13)) );
fprintf('Feature 14: %f +/- %f\n',
    mean(AUROC_all(:,14)),std(AUROC_all(:,14)) );
fprintf('Feature 15: %f +/- %f\n',
    mean(AUROC_all(:,15)),std(AUROC_all(:,15)) );
fprintf('Feature 16: %f +/- %f\n',
    mean(AUROC_all(:,16)),std(AUROC_all(:,16)) );
```

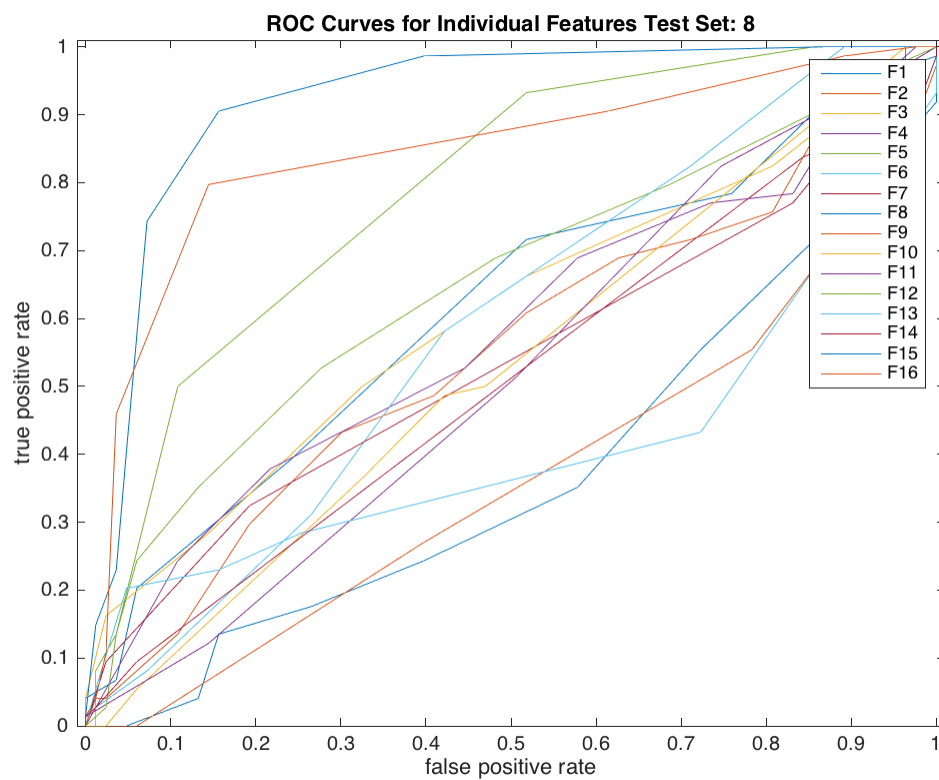
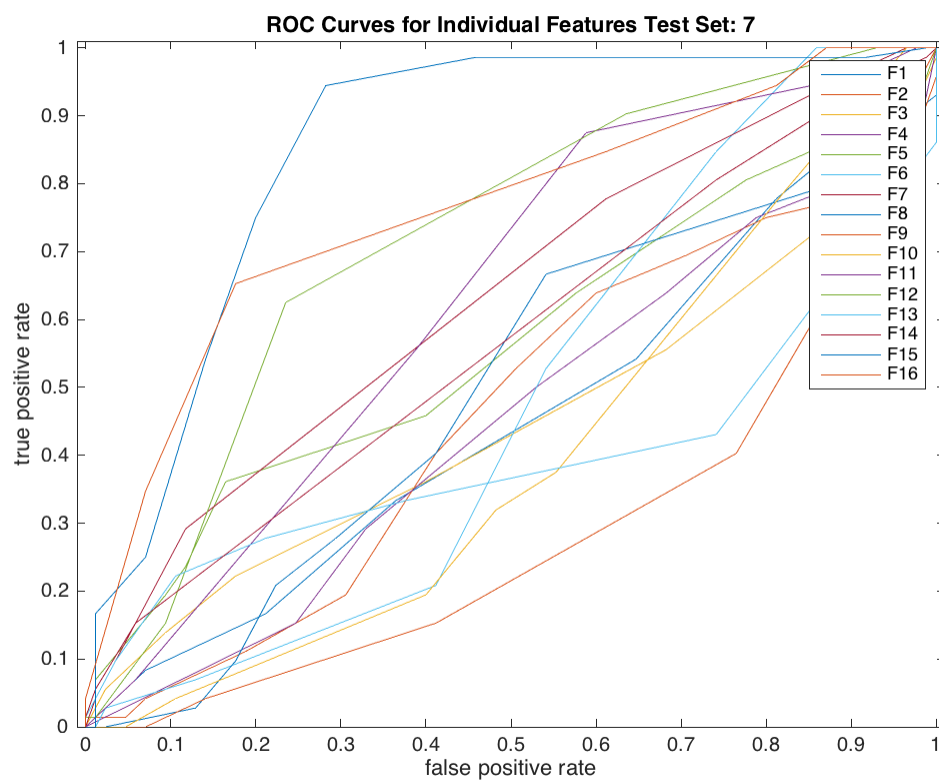
*AUROC (mean +/- standard deviation) for*

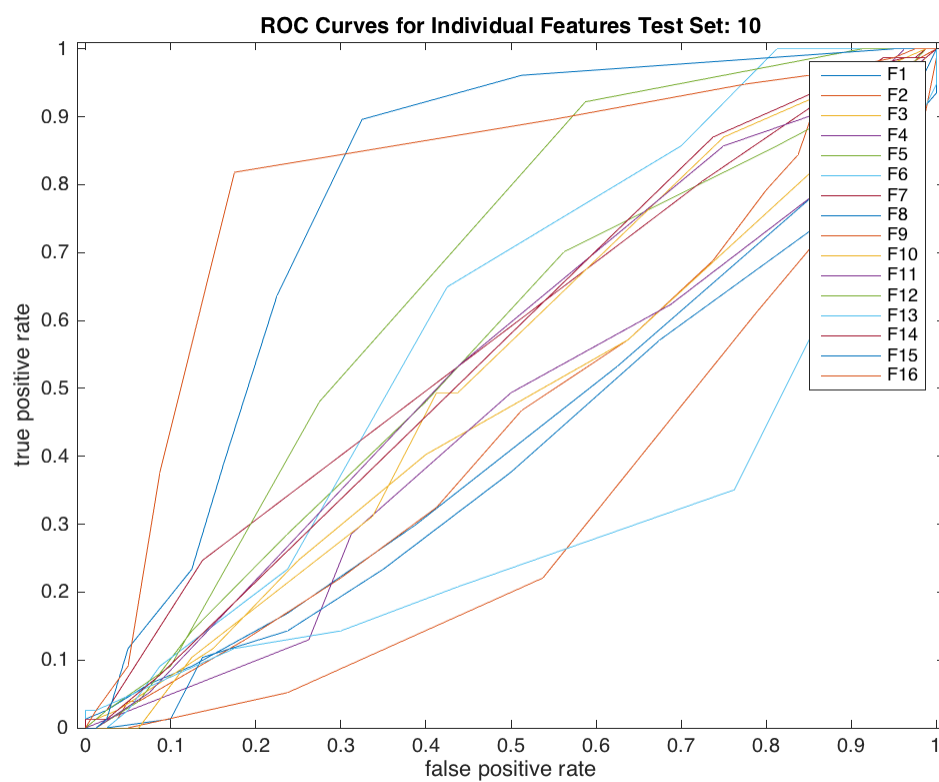
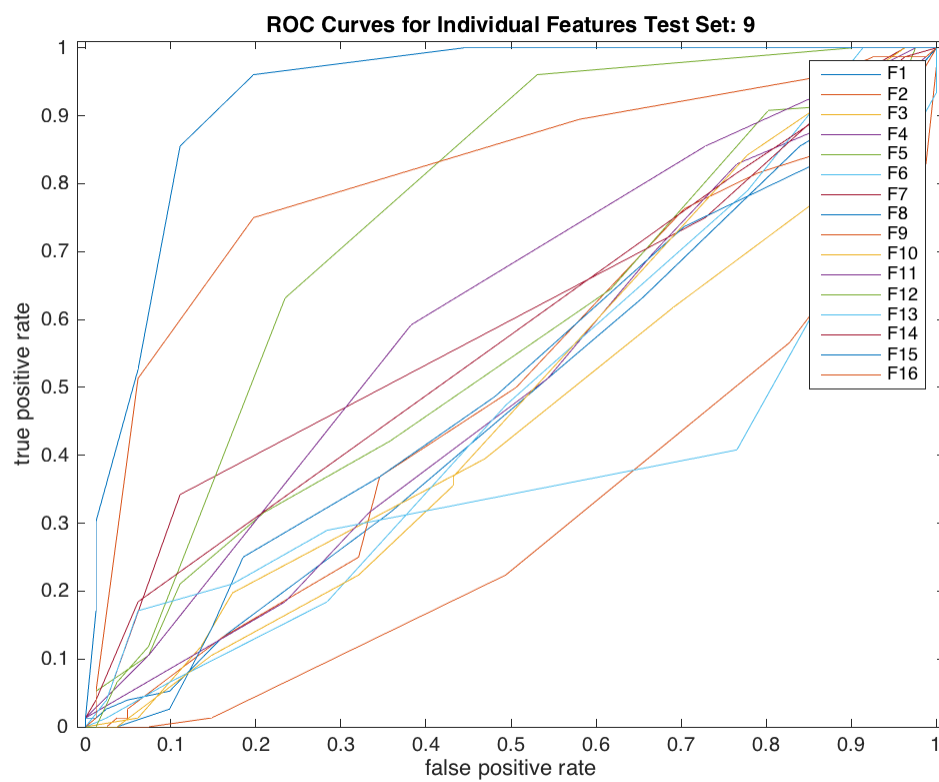
Feature 1:	0.487335 +/- 0.052990
Feature 2:	0.477438 +/- 0.040073
Feature 3:	0.481606 +/- 0.051506
Feature 4:	0.488458 +/- 0.044363
Feature 5:	0.571973 +/- 0.048383
Feature 6:	0.522496 +/- 0.045899
Feature 7:	0.563577 +/- 0.043376
Feature 8:	0.436689 +/- 0.048794
Feature 9:	0.299221 +/- 0.032979
Feature 10:	0.466499 +/- 0.048161
Feature 11:	0.587595 +/- 0.035790
Feature 12:	0.748445 +/- 0.035816
Feature 13:	0.382636 +/- 0.040675
Feature 14:	0.531480 +/- 0.025521
Feature 15:	0.872255 +/- 0.040407
Feature 16:	0.804822 +/- 0.027788











---

## For 5 different Algorithms

```
AUCglm = zeros(10,1);
AUCsvm = zeros(10,1);
AUCcart = zeros(10,1);
AUCrf = zeros(10,1);
AUCbt = zeros(10,1);

for test_ind = 1:10

    fprintf('Performing Testing using Fold %d of 10 \n',test_ind);
    p = 0.9; % proportion of the dataset for training
    m = size(data,1);

    % Circ shift to create new test and training set
    data_perm = circshift(data_perm,round((1-p)*m),1);
    class_perm = circshift(class_perm,round((1-p)*m),1);

    X = data_perm(1:round(p*m),:);
    Y = class_perm(1:round(p*m));
    Xtest = data_perm(round(p*m)+1:end,:);
    Ytest = class_perm(round(p*m)+1:end);
    % Generalized Linear Model (Logistic Regression)

    glmModel = fitglm(X,
Y, 'Distribution', 'binomial', 'Link', 'logit');
    Yscores = predict(glmModel, Xtest); % these are the posterior
probabilities
    % of class 1 for the test data

    % ... compute the standard ROC curve and the AUROC
    [Xglm, Yglm, Tglm, AUCglm(test_ind)] = perfcurve(Ytest,
Yscores, 'true');

    Performing Testing using Fold 1 of 10

    Performing Testing using Fold 2 of 10

    Performing Testing using Fold 3 of 10

    Performing Testing using Fold 4 of 10

    Performing Testing using Fold 5 of 10

    Performing Testing using Fold 6 of 10

    Performing Testing using Fold 7 of 10

    Performing Testing using Fold 8 of 10

    Performing Testing using Fold 9 of 10

    Performing Testing using Fold 10 of 10
```



---

## Nested CV to choose kernel for SVM

```
AUCsvm_nested = zeros(10,3);
for val_ind = 1:10
    pn = 0.9; % proportion of data for training
    mn = size(X,1);

    X = circshift(X,round((1-pn)*mn),1);
    Y = circshift(Y,round((1-pn)*mn),1);

    X_nested = X(1:round(pn*mn),:);
    Y_nested = Y(1:round(pn*mn));
    X_nested_test = X(round(pn*mn)+1:end,:);
    Y_nested_test = Y(round(pn*mn)+1:end);

    for k_index = 1:3
        % Define K parameter values
        if k_index == 1
            K = 'linear';
        elseif k_index == 2
            K = 'polynomial';
        elseif k_index == 3
            K = 'rbf';
        end

        % Support Vector Machine (SVM)

        svmModel = fitcsvm(X_nested, Y_nested, 'Standardize',
true, 'KernelFunction', K);
        svmModel = fitPosterior(svmModel);
        [~, Yscores] = predict(svmModel, X_nested_test);

        % ... compute the standard ROC curve and the AUROC
        [Xsvm_nested, Ysvm_nested, Tsvm_nested,
AUCsvm_nested(val_ind,k_index)] = perfcurve(Y_nested_test, Yscores(:,
2), 'true');

    end
end

% Pick the Paramter with Highest Mean AUROC across validation
folds
AUCsvm_nested_mean = mean(AUCsvm_nested,1);
k_best_index = find(AUCsvm_nested_mean ==
max(AUCsvm_nested_mean));

if k_best_index == 1
    K_best_svm = 'linear'
elseif k_best_index == 2
    K_best_svm = 'polynomial'
elseif k_best_index == 3
```

---

```

        K_best_svm = 'rbf'
    end

    % Support Vector Machine (SVM) for Test Set

    svmModel = fitcsvm(X, Y, 'Standardize', true, 'KernelFunction',
        K_best_svm);
    svmModel = fitPosterior(svmModel);
    [~, Yscores] = predict(svmModel, Xtest);

    % ... compute the standard ROC curve and the AUROC
    [Xsvm, Ysvm, Tsvm, AUCsvm(test_ind)] = perfcurve(Ytest, Yscores(:,
        2), 'true');

K_best_svm =

rbf

K_best_svm =

rbf

K_best_svm =

rbf

K_best_svm =

rbf

K_best_svm =

rbf

K_best_svm =

rbf

K_best_svm =

rbf

K_best_svm =

rbf

```

---

---

*polynomial*

*K\_best\_svm =*

*rbf*

*K\_best\_svm =*

*rbf*

*K\_best\_svm =*

*rbf*

Classification Tree (CART)

```
ctreeModel = fitctree(X, Y);
[~, Yscores, ~, ~] = predict(ctreeModel, Xtest);

% ... compute the standard ROC curve and the AUROC
[Xcart, Ycart, Tcart, AUCcart(test_ind)] = perfcurve(Ytest,
Yscores(:, 2), 'true');
```

## Nested CV to choose number of trees for RF

```
AUCrf_nested = zeros(10,5);
k_values = [3,10,30,60,100];
for val_ind = 1:10
    pn = 0.9; % proportion of data for training
    mn = size(X,1);

    X = circshift(X,round((1-pn)*mn),1);
    Y = circshift(Y,round((1-pn)*mn),1);

    X_nested = X(1:round(pn*mn),:);
    Y_nested = Y(1:round(pn*mn));
    X_nested_test = X(round(pn*mn)+1:end,:);
    Y_nested_test = Y(round(pn*mn)+1:end);

    for k_index = 1:length(k_values)

        % Random Forest (RF)

        rfModel = fitensemble(X_nested, Y_nested, 'Bag',
k_values(k_index), 'Tree', 'Type', 'Classification');
```

---

```

        [~, Yscores] = predict(rfModel, X_nested_test);

        % ... compute the standard ROC curve and the AUROC
        [Xrf_nested, Yrf_nested, Trf_nested,
AUCrf_nested(val_ind,k_index)] = perfcurve(Y_nested_test, Yscores(:,
2), 'true');

        end
    end

    % Pick the Paramter with Highest Mean AUROC across validation
    folds
    AUCrf_nested_mean = mean(AUCrf_nested,1);
    k_best_index = find(AUCrf_nested_mean == max(AUCrf_nested_mean));

    K_best_rf = k_values(k_best_index)

    % Random Forest (RF)

    rfModel = fitensemble(X, Y, 'Bag',
K_best_rf, 'Tree', 'Type', 'Classification');
    [~, Yscores] = predict(rfModel, Xtest);

    % ... compute the standard ROC curve and the AUROC
    [Xrf, Yrf, Trf, AUCrf(test_ind)] = perfcurve(Ytest, Yscores(:,
2), 'true');

K_best_rf =

    100

K_best_rf =

    100

K_best_rf =

    100

K_best_rf =

    100

K_best_rf =

    100

```

---

---

```
100

K_best_rf =

    60

K_best_rf =

    30

K_best_rf =

    60

K_best_rf =

    100

K_best_rf =

    100

% Boosted Trees

btModel = fitensemble(X, Y, 'AdaBoostM1', 100, 'Tree');
[~, Yscores] = predict(btModel, Xtest);

% ... compute the standard ROC curve and the AUROC
[Xbt, Ybt, Tbt, AUCbt(test_ind)] = perfcurve(Ytest,
sigmf(Yscores(:, 2), [1 0]), ...
    'true');

% ROC Curves
figure;
plot(Xglm, Yglm, '--')
hold on
plot(Xsvm, Ysvm, '--.')
plot(Xcart, Ycart, '--')
plot(Xrf, Yrf, '--')
plot(Xbt, Ybt)
legend('Logistic Regression', 'Support Vector
Machine', 'CART', ...
```

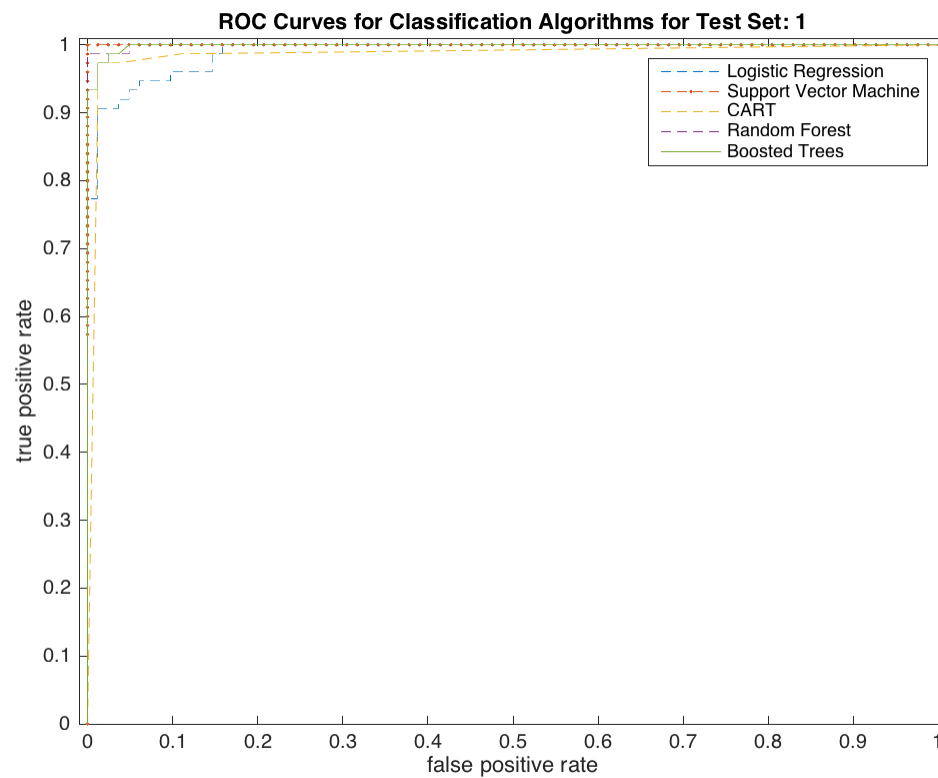
---

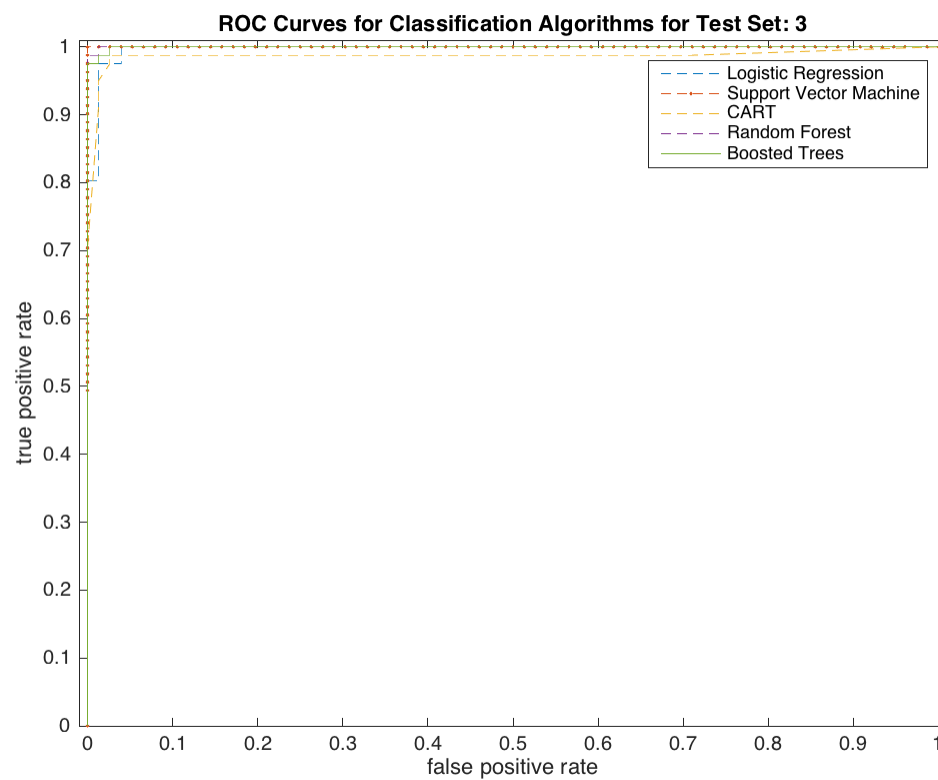
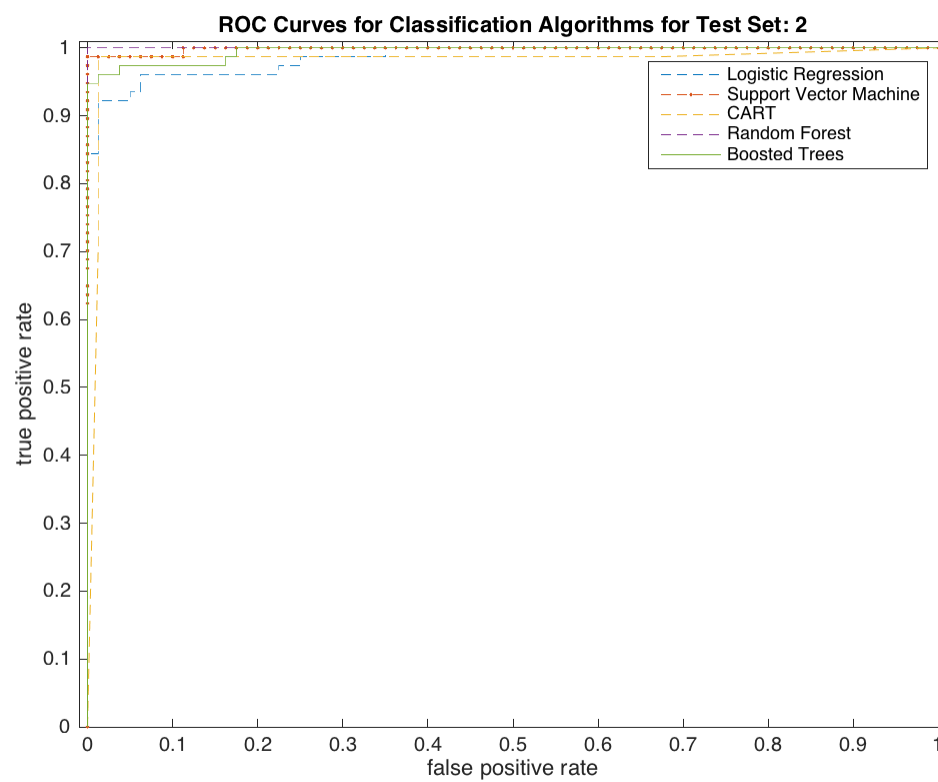
---

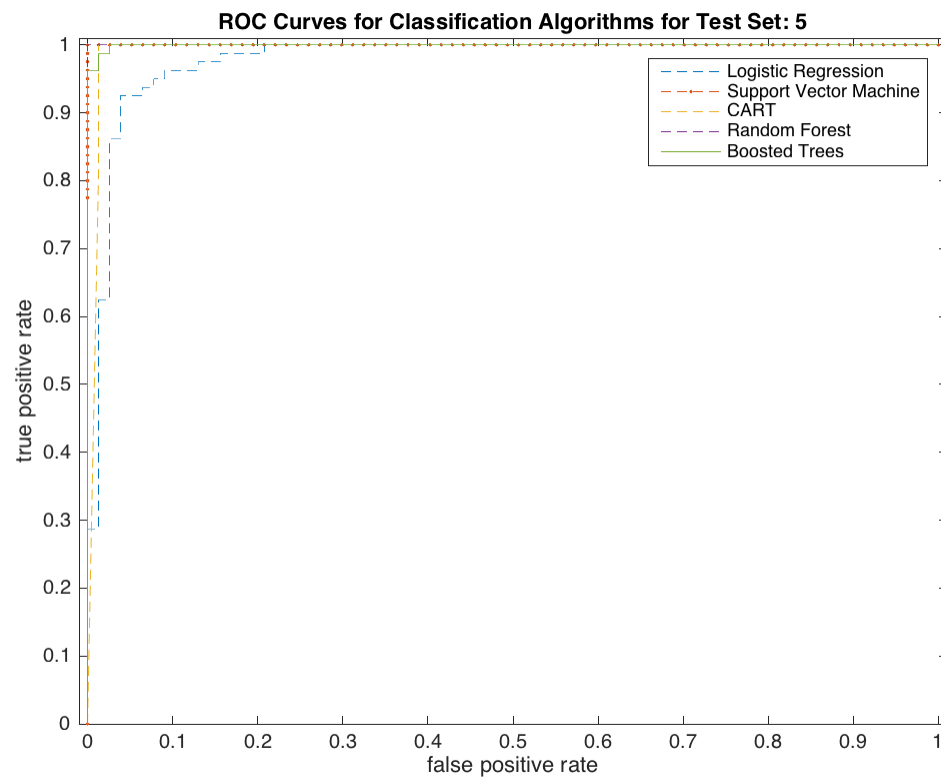
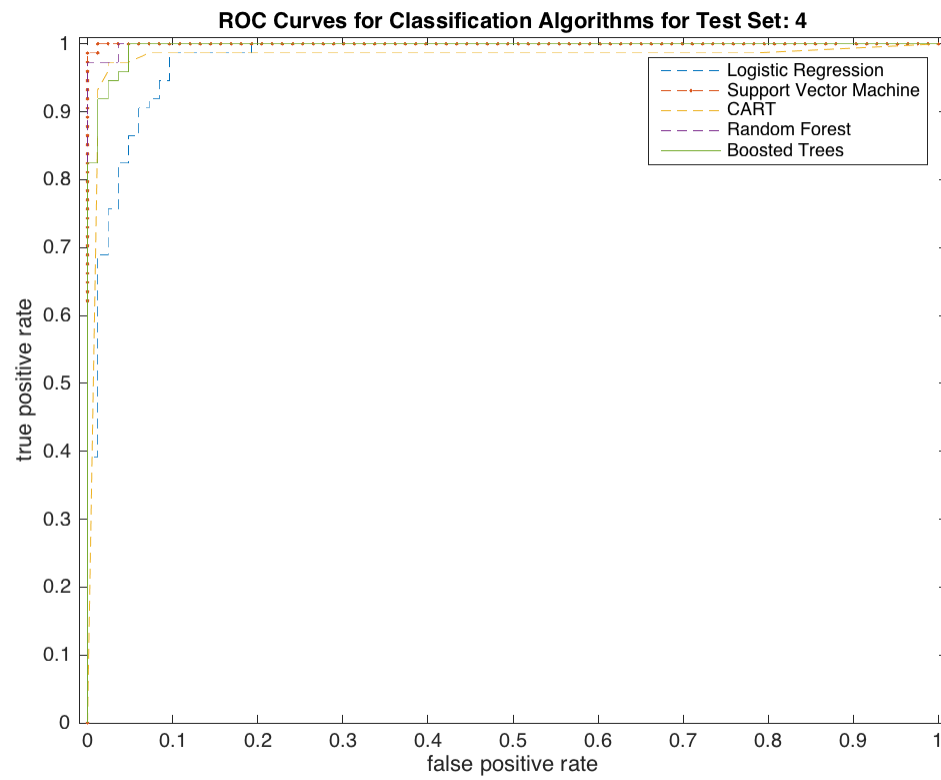
```

        'Random Forest', 'Boosted Trees')
xlabel('false positive rate');
ylabel('true positive rate');
title_str = ['ROC Curves for Classification Algorithms for Test
Set: ', num2str(test_ind)];
title(title_str)
axis([-0.01,1.01,0,1.01])
hold off

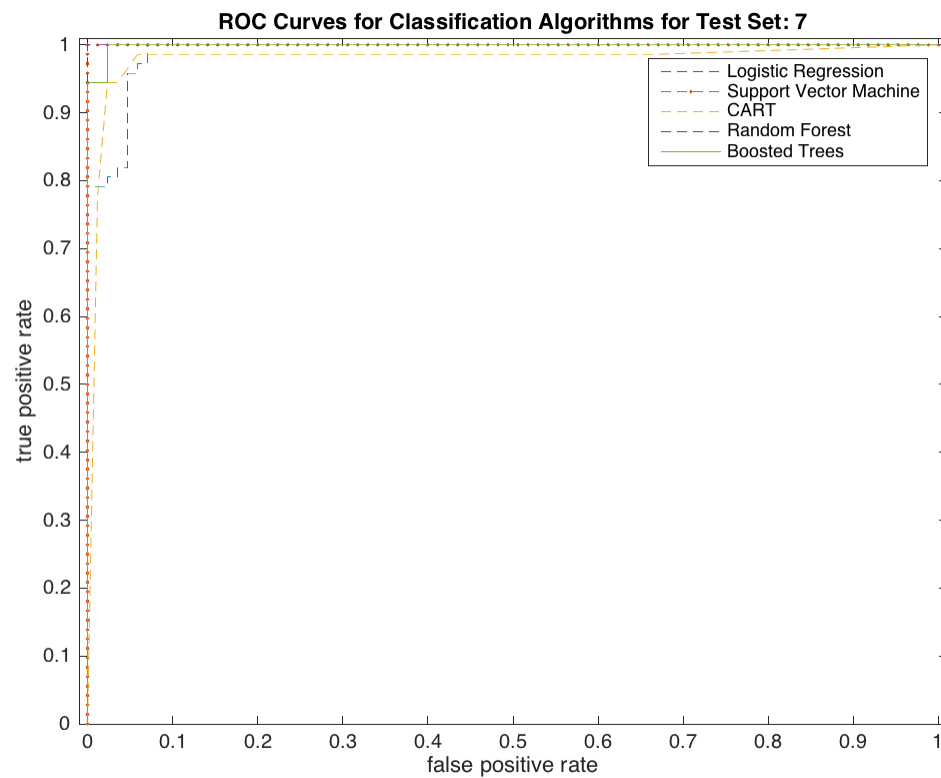
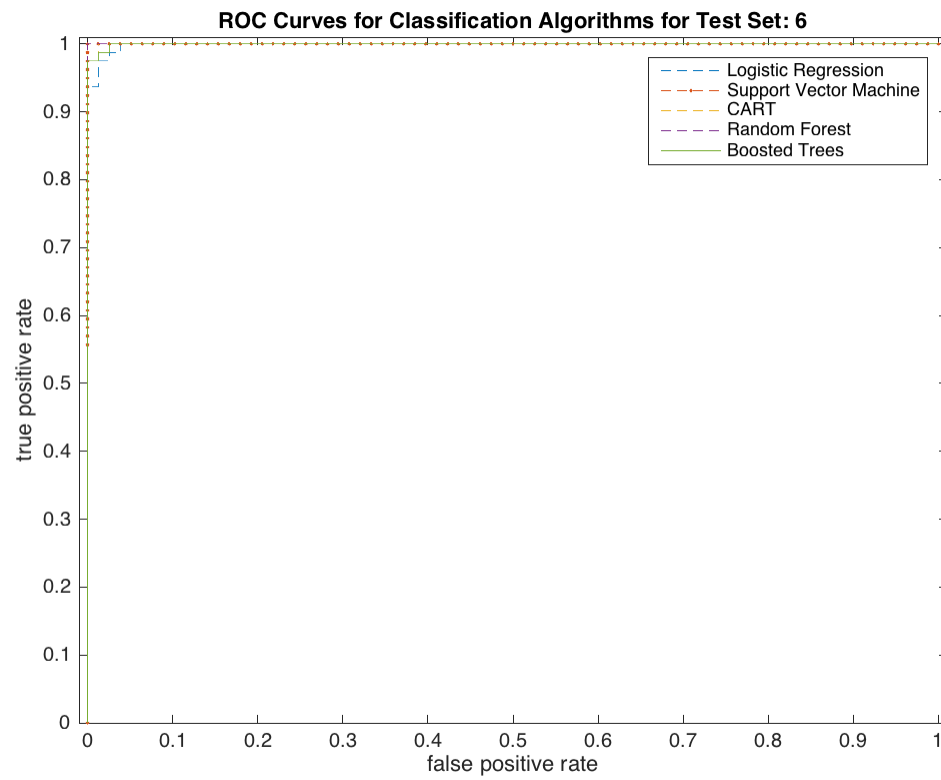
```

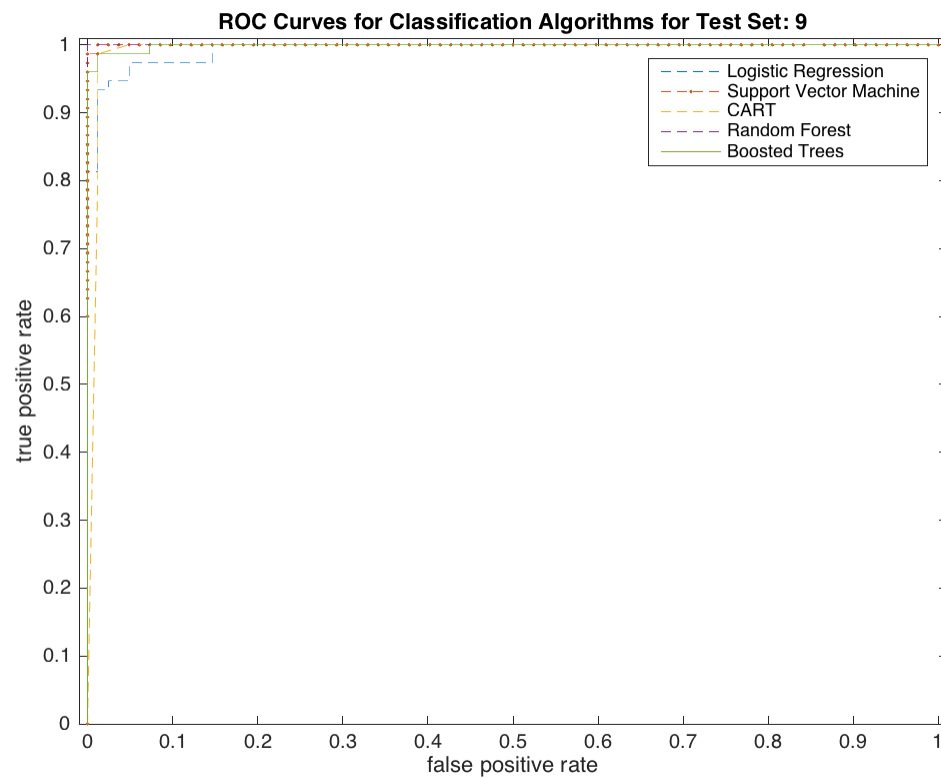
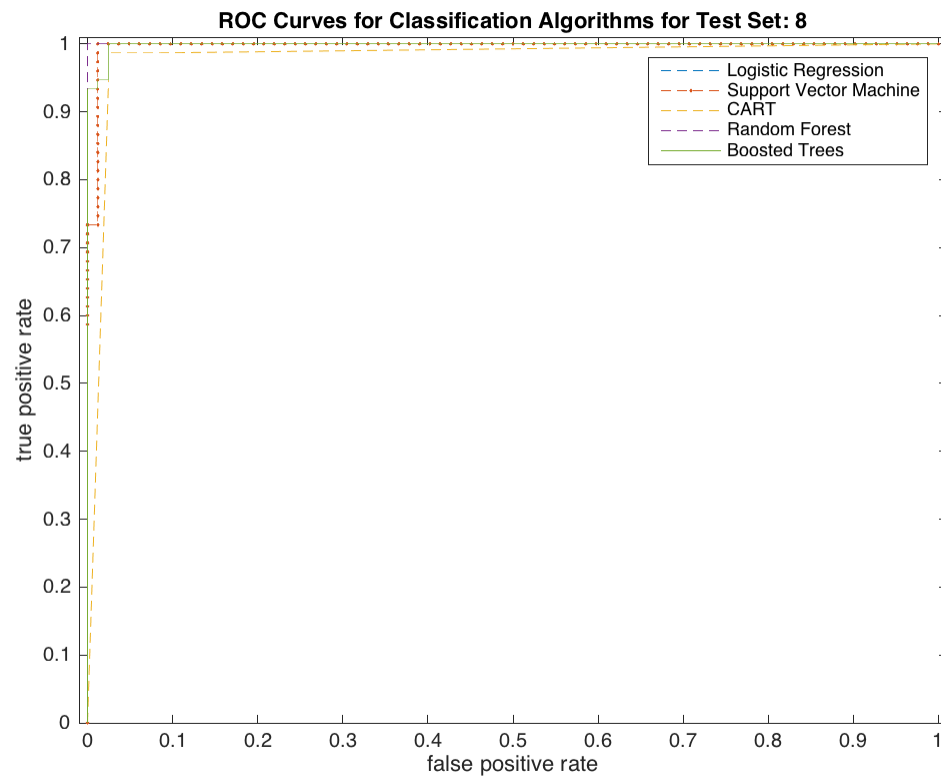


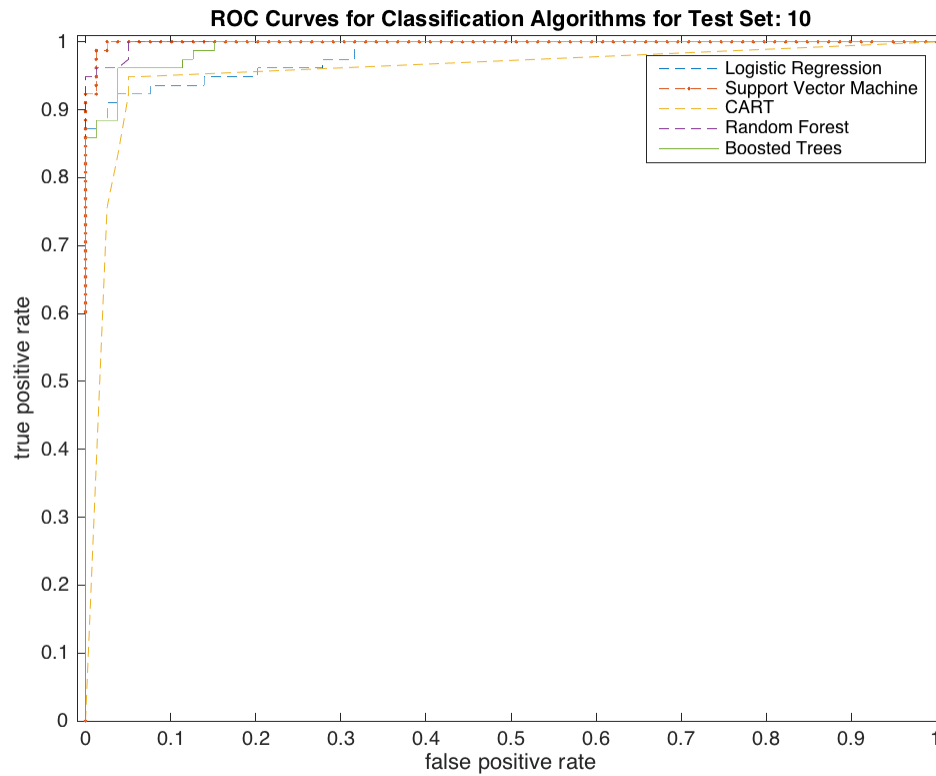












end

## AUROC

```
fprintf('AUROC (mean +/- standard deviation) for\n');
fprintf('Logistic Regression: %f +/- %f\n', mean(AUCglm),
    std(AUCglm));
fprintf('Support Vector Machine: %f +/- %f\n', mean(AUCsvm),
    std(AUCsvm));
fprintf('CART: %f +/- %f\n', mean(AUCcart),std(AUCcart));
fprintf('Random Forest: %f +/- %f\n', mean(AUCrf),std(AUCrf));
fprintf('Boosted Trees: %f +/- %f\n', mean(AUCbt),std(AUCbt));

return
```

```
AUROC (mean +/- standard deviation) for
Logistic Regression: 0.989124 +/- 0.008126
Support Vector Machine: 0.999383 +/- 0.001067
CART: 0.983270 +/- 0.011873
Random Forest: 0.999618 +/- 0.000672
Boosted Trees: 0.997538 +/- 0.002571
```

*Published with MATLAB® R2015b*