Performance Task

Modeling and Simulation

Normal Distribution

Formula:
$z = (x - \mu) / \sigma$

Problem:
The average height of adult men is 175 cm with a standard deviation of 10 cm. What is the probability that a randomly selected man is taller than 185 cm?

Solution:
$z = (185 - 175) / 10 = 1$
From a Z-table, $P(Z > 1) = 0.1587$
So, there's a 15.87% chance a man is taller than 185 cm.

Poisson Distribution

Formula:
$P(X = k) = (\lambda^k \times e^{(-\lambda)}) / k!$

Problem:
At 5 PM, an average of 3 cars pass per minute. What is the probability that exactly 5 cars pass in one minute?

Solution:
$P(5) = (3^5 \times e^{(-3)}) / 5! = 243 \times e^{-3} / 120 \approx 0.1008$

Exponential Distribution

Formula:
$P(T \leq t) = 1 - e^{(-\lambda t)}$

Problem:
On average, a bus comes every 10 minutes ($\lambda = 1/10$). What's the probability it arrives within 5 minutes?

Solution:
$P(T \leq 5) = 1 - e^{(-0.1 \times 5)} = 1 - e^{-0.5} \approx 0.3935$
So, there's a 39.35% chance.

So, there's a 10.08% chance.

Binomial Distribution

Formula:
$P(X = k) = C(n, k) \times p^k \times (1 - p)^{(n - k)}$

Problem:
What is the probability of getting exactly 3 heads in 5 coin flips ($p = 0.5$)?

Solution:
$P(3 \text{ heads}) = C(5, 3) \times 0.5^3 \times 0.5^2 = 10 \times 0.125 \times 0.25 = 0.3125$
So, there's a 31.25% chance.

Beta Distribution

Formula:
$\text{Beta}(\alpha, \beta)$ where $\alpha = \text{heads} + 1$, $\beta = \text{tails} + 1$

Problem:
You flip a coin 10 times and get 7 heads and 3 tails. What is your estimate of the probability of heads?

Solution:
$\text{Beta}(8, 4) \rightarrow \text{mean} = \alpha / (\alpha + \beta) = 8 / (8 + 4) = 0.6667$
Estimated probability of heads ≈ 67%.

Logarithmic Distribution

Formula:
$P(X = k) = -1/\ln(1 - p) \times (p^k / k)$, for $0 < p < 1$

Problem:
You model rare sightings of a rare owl with $p = 0.3$. What's the probability of spotting exactly 2 in a year?

Solution:
$P(2) = -1 / \ln(1 - 0.3) \times (0.3^2 / 2) = 1.357 \times 0.045 = 0.061$
About 6.1% chance.

Trigonometric Distribution

Use: von Mises distribution (circular normal)

Problem:
If crime happens most at 2 AM and follows a circular pattern, what's the chance it happens close to 2 AM?

Solution:
Use von Mises centered at θ = 2 AM with a concentration κ. The mode is at 2 AM, and probability falls symmetrically.
Highest probability is near 2 AM, lowest near 2 PM.
Formula: $f(\theta) = (1 / 2\pi I_0(\kappa)) \times e^{(\kappa \cos(\theta - \mu))}$

Weibull Distribution

Formula:
$P(t) = (k/\lambda) \times (t/\lambda)^{(k - 1)} \times e^{-(t/\lambda)^k}$

Problem:
A phone model has λ = 2 years and k = 1.5. What is the probability it fails before 1 year?

Solution:
$P(T < 1) \approx 1 - e^{-(1/2)^{1.5}} \approx 1 - e^{-0.353} \approx 1 - 0.702 = 0.298$
About 29.8% chance of failing in the first year.

Gamma Distribution

Formula:
$f(x; \alpha, \beta) = (\beta^\alpha x^{(\alpha-1)} e^{-\beta x}) / \Gamma(\alpha)$

Problem:
You wait for 3 buses. Each bus arrives randomly at an average of 10 minutes (β = 0.1). What's the probability they all come within 20 minutes?

Solution:
Use $\Gamma(3) = 2! = 2$
$P(x < 20) \approx$ use gamma calculator → About 0.857
85.7% chance all buses arrive within 20 minutes.

Uniform Distribution

Formula:
$P(x) = 1 / (b - a + 1)$

Problem:
What is the probability of rolling a 4 on a fair six-sided die?

Solution:
$P(4) = 1 / 6 = 0.1667 \rightarrow$ 16.67% chance.