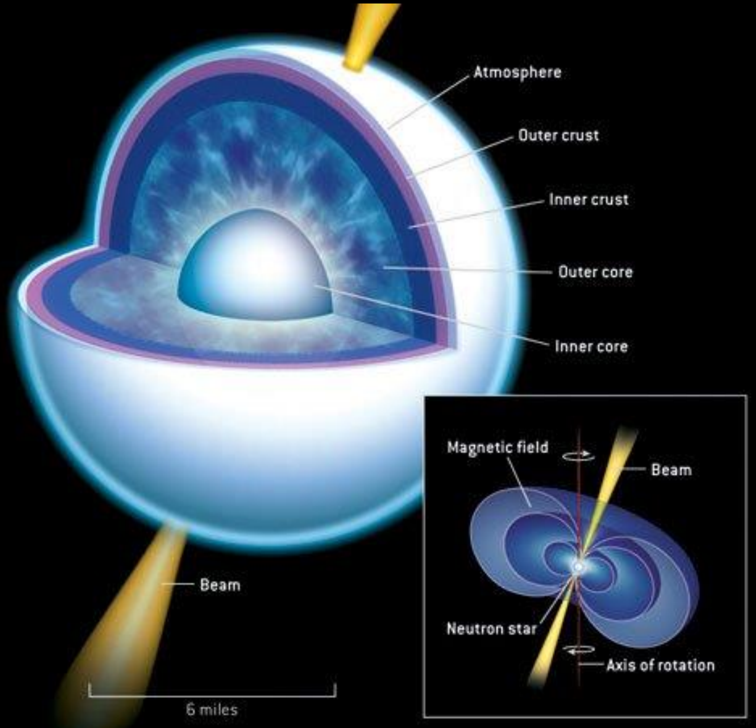
# NUCLEAR REACTION THEORY

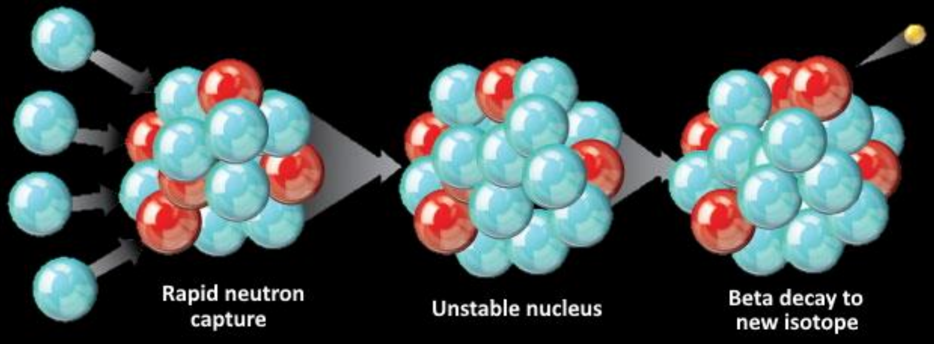
TAYLOR WHITEHEAD

# REACTION THEORY FOR RARE ISOTOPES

## Nuclear Astrophysics

The nuclear equation of state for neutron stars and core-collapse supernovae may be constrained by reaction theory



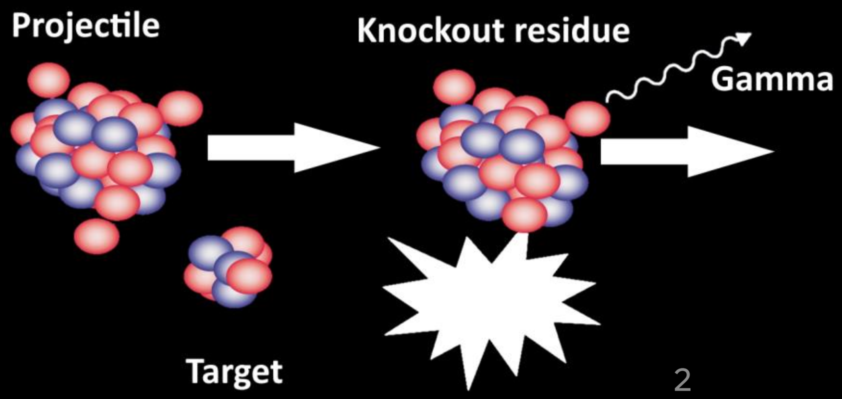




## Element Creation

Understanding the abundances of heavy elements requires the knowledge of neutron interactions with exotic neutron rich isotopes

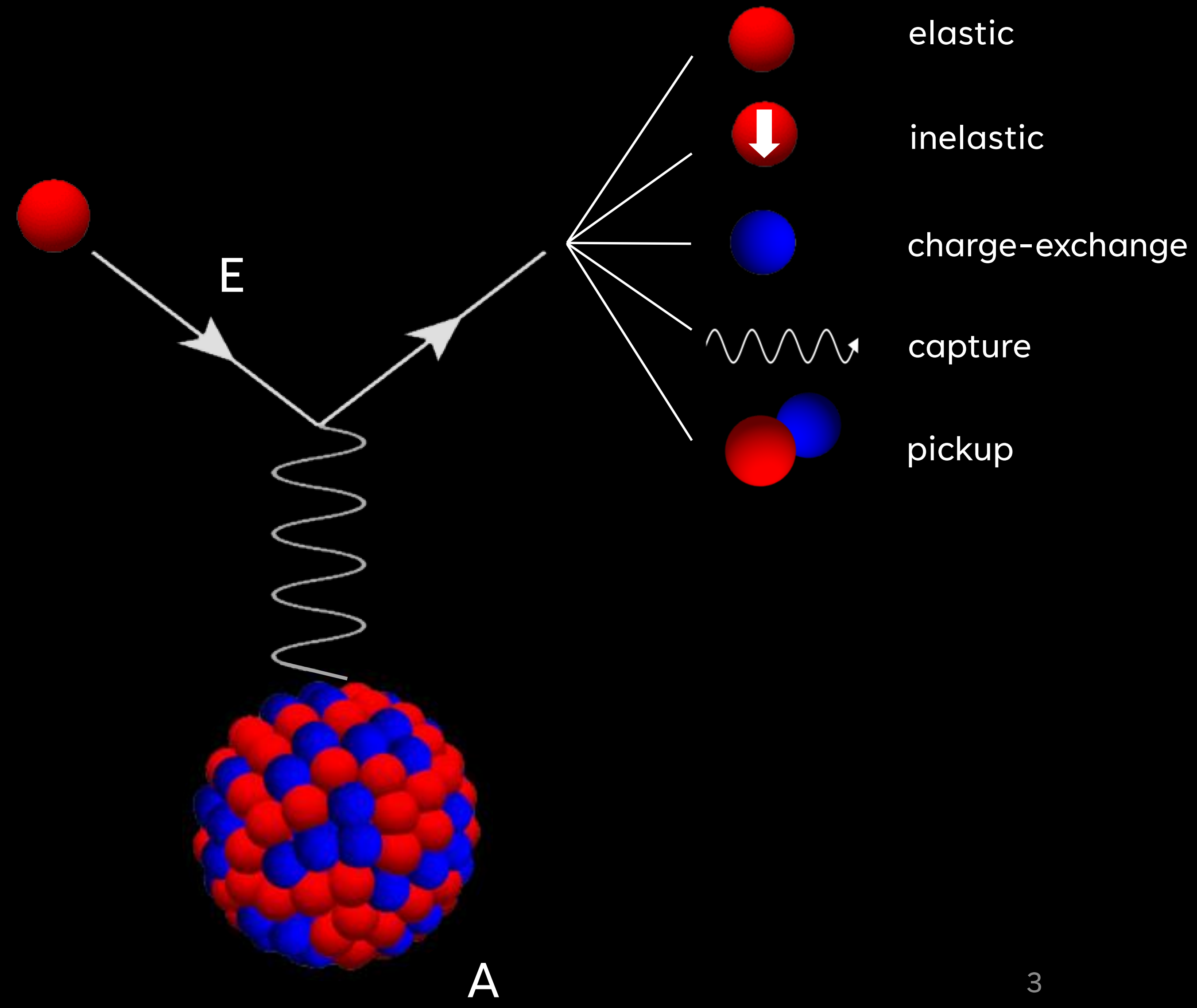
## Rare Isotope Experiments

Reaction theory for experiments with rare isotopes is untested so robust models with uncertainty estimates are crucial for experimental design and data analysis

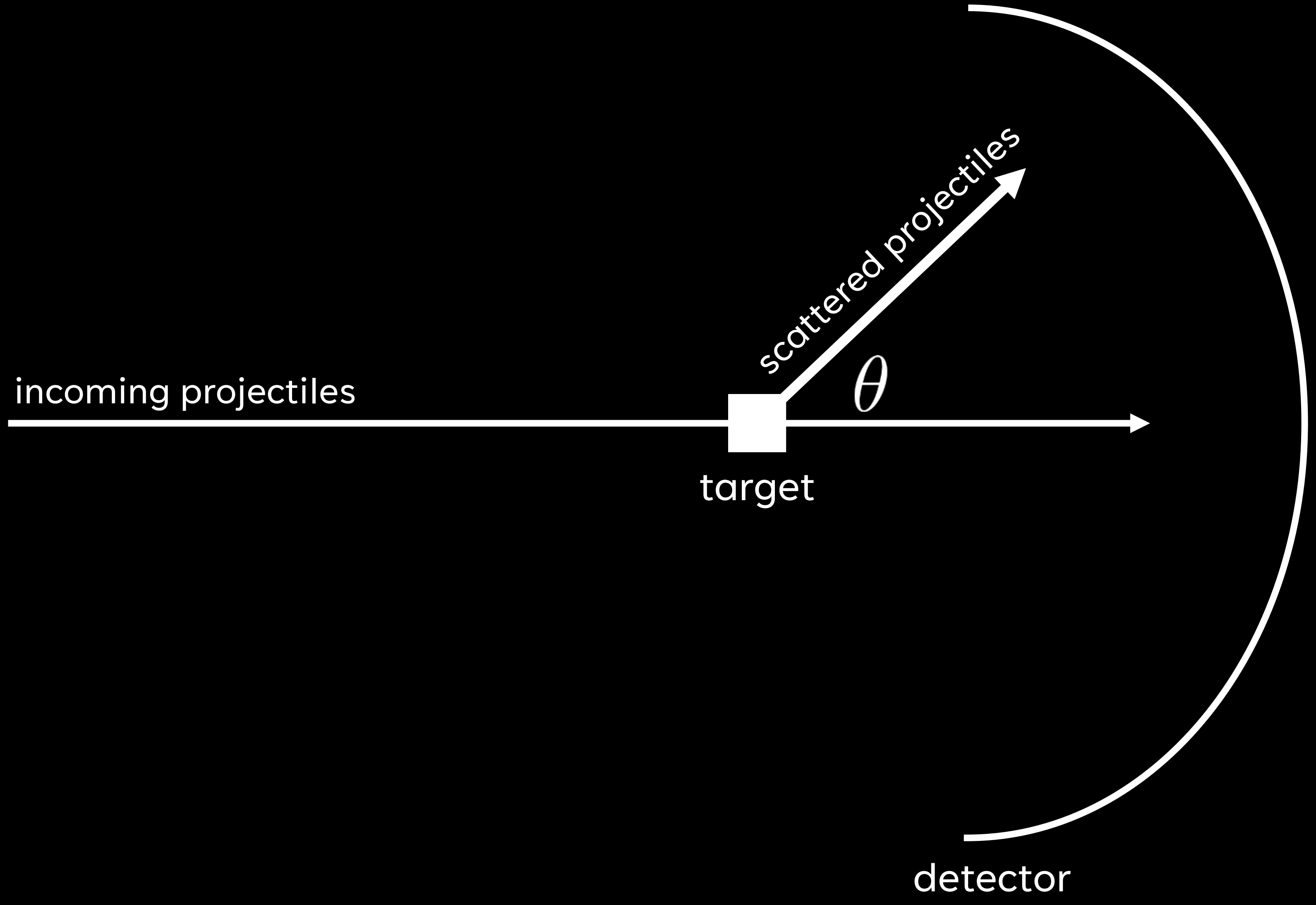


# REACTION MODELS NEEDED!

Cutting edge research in nuclear physics requires reaction models for a wide range of isotopes (A) at energies (E)

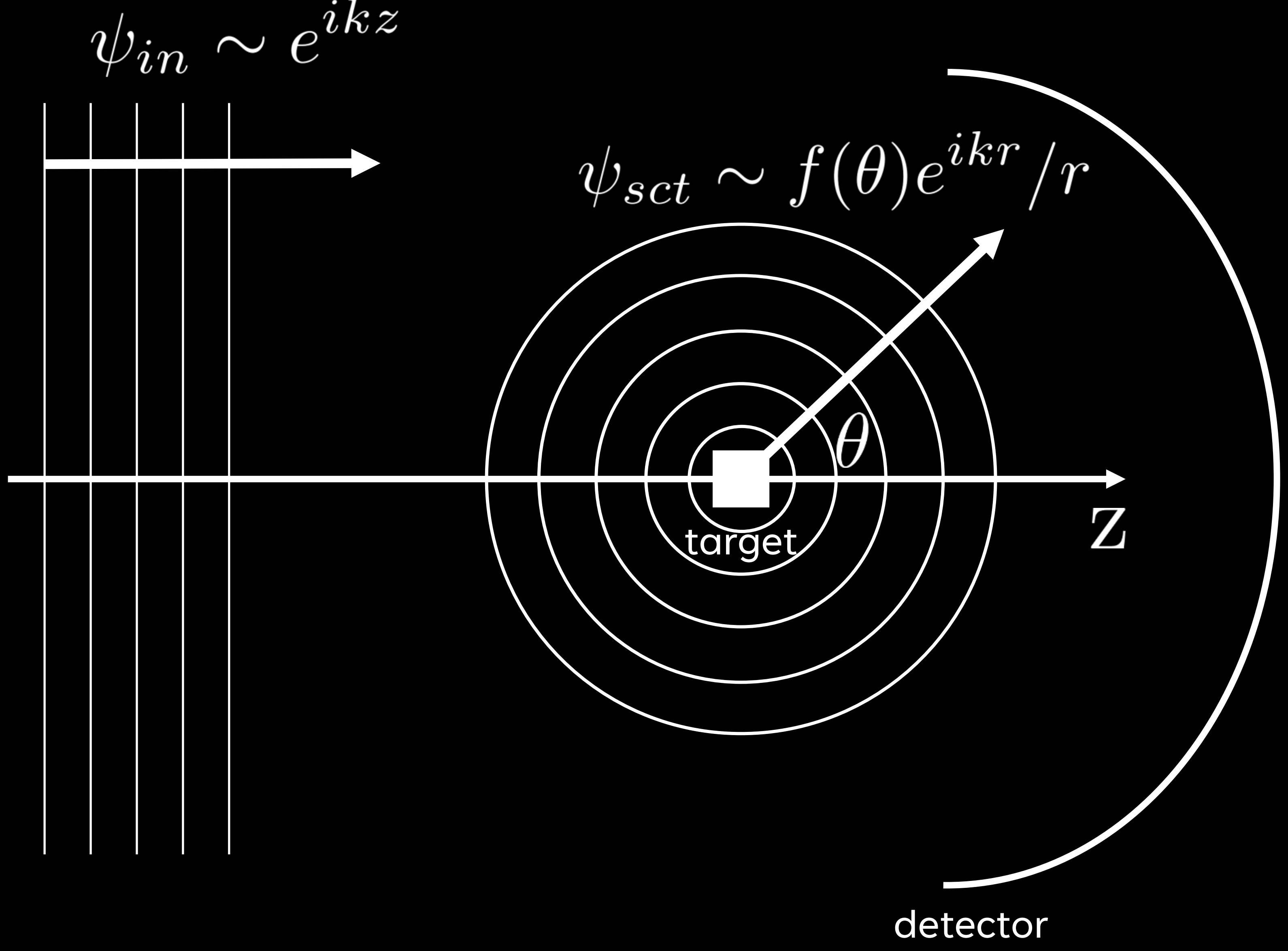


# NUCLEAR REACTION EXPERIMENTS




Traditional nuclear scattering experiments scatter a beam of projectiles on a stable target

# QUANTUM MECHANICS OF SCATTERING

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$$\sigma(\theta) \propto |f(\theta)|^2$$

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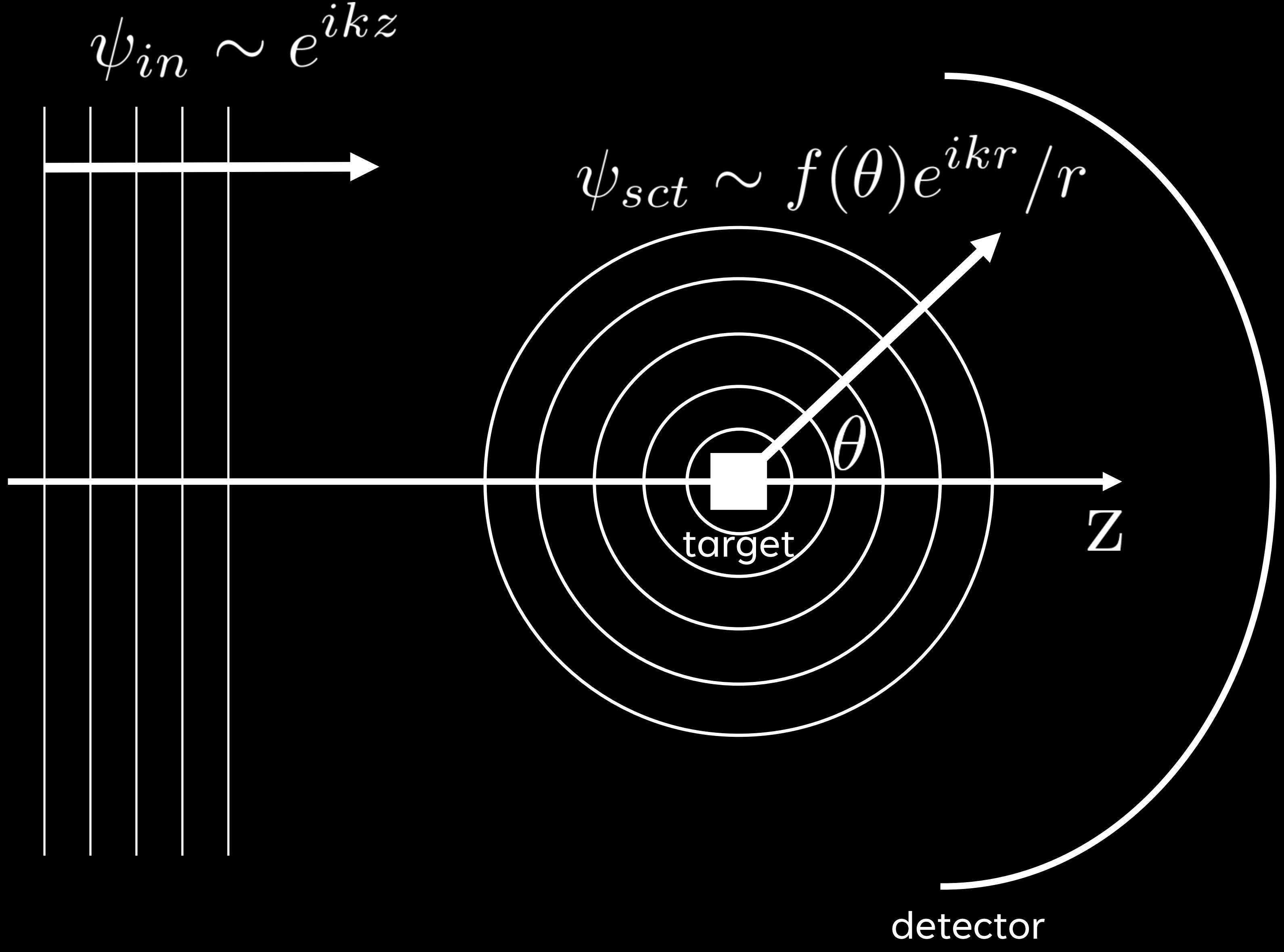


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*The scattering cross section is the measurable quantity of interest!*

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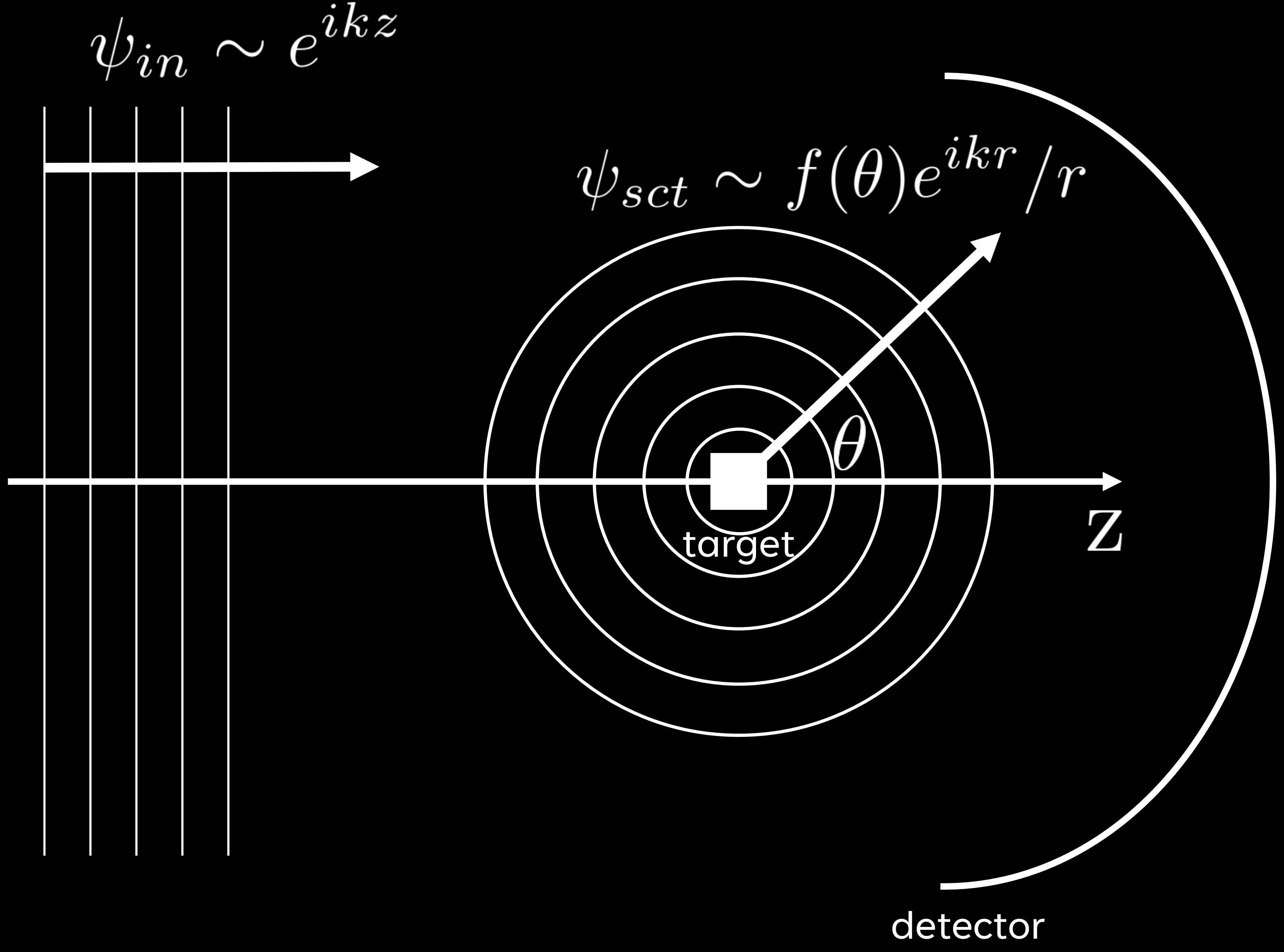


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*How does the scattering amplitude relate to the projectile-target interaction?*

# BORN APPROXIMATION

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Taking a Yukawa potential:

$$U(r) = g\frac{e^{-\mu r}}{r}$$

$$f(\theta) = -\frac{2mg}{\mu^2 + \mathbf{k}^2} \rightarrow \frac{d\sigma}{d\Omega} = \frac{4m^2g^2}{(\mu^2 + 4k^2\sin^2(\theta/2))^2}$$

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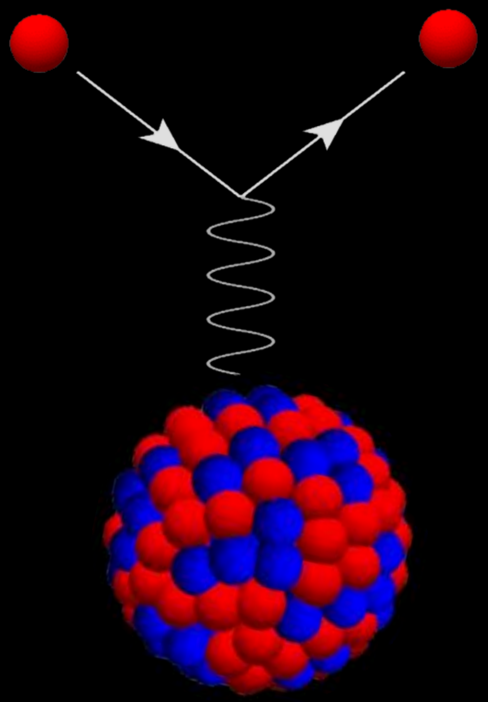
This is the Rutherford cross section!

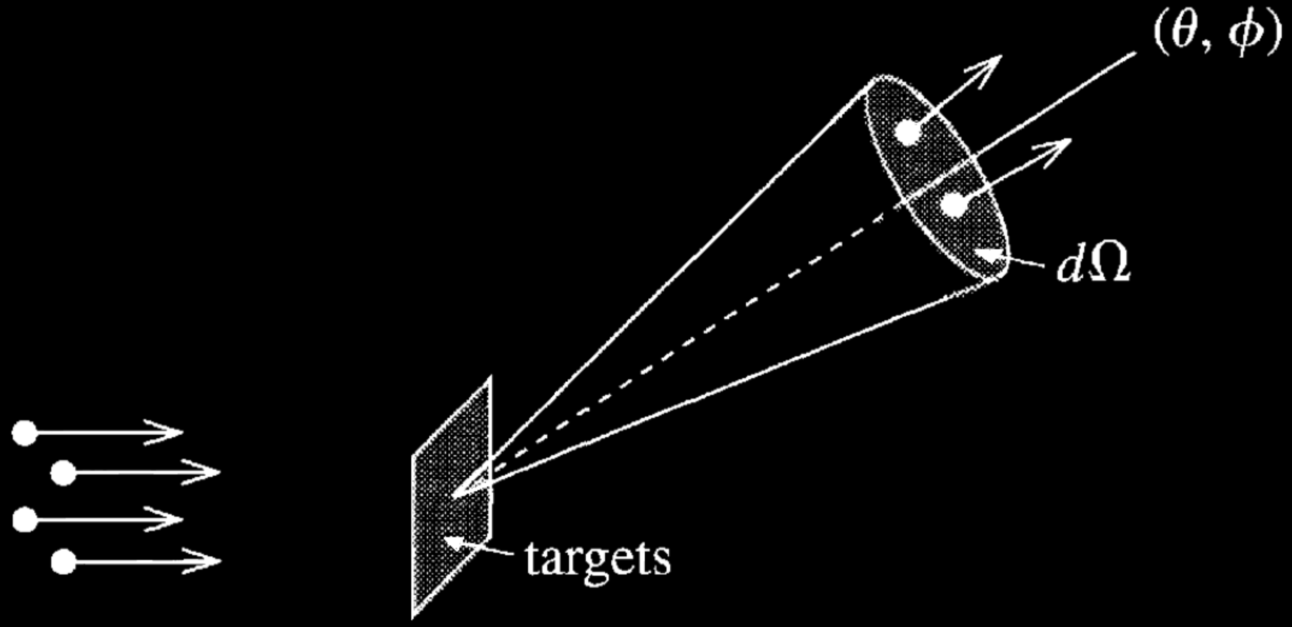
# NUCLEAR REACTION THEORY

**Goal:** Construct a simple and reliable reaction model from a theoretical description of the nuclear force to predict reaction observables measured by experimentalists

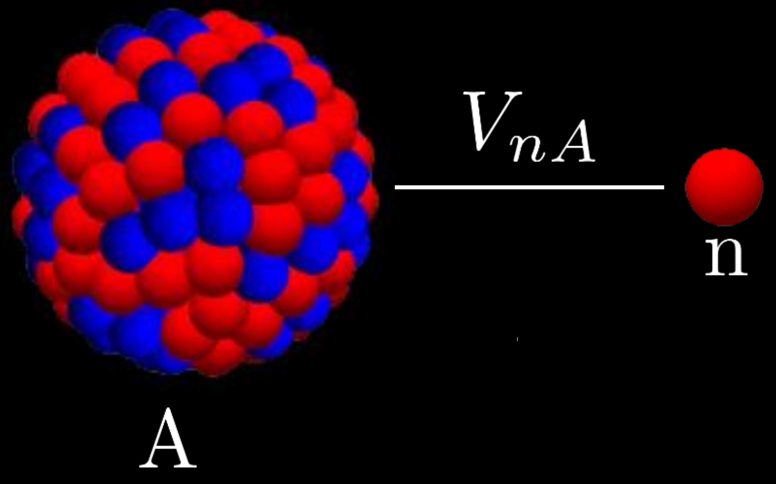
Theory



Experiment



# NUCLEON-NUCLEUS INTERACTION



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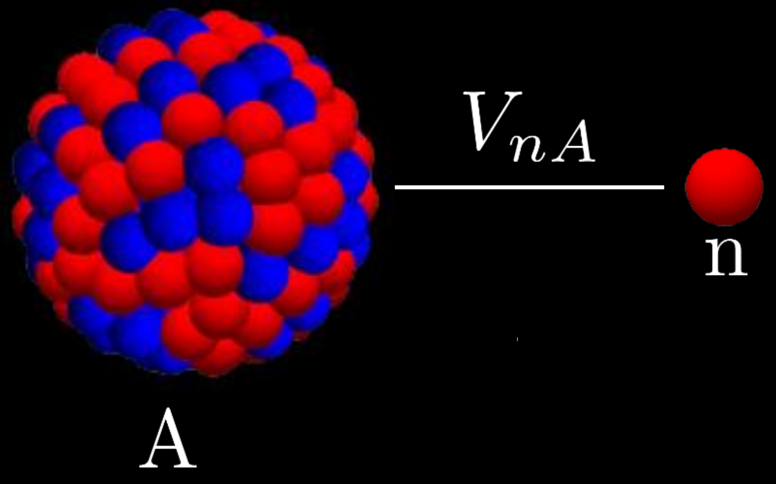


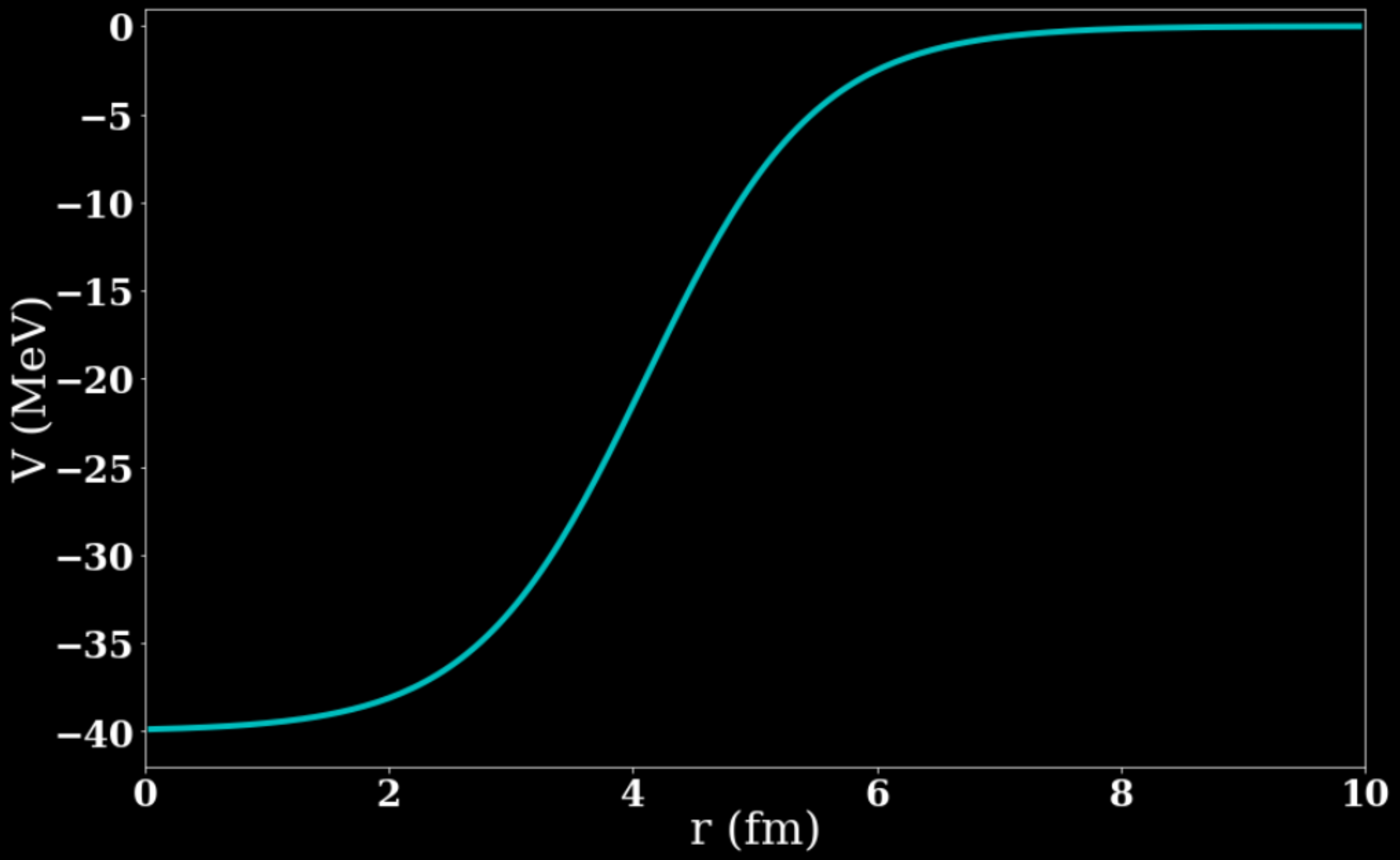


Could a single-particle potential work?

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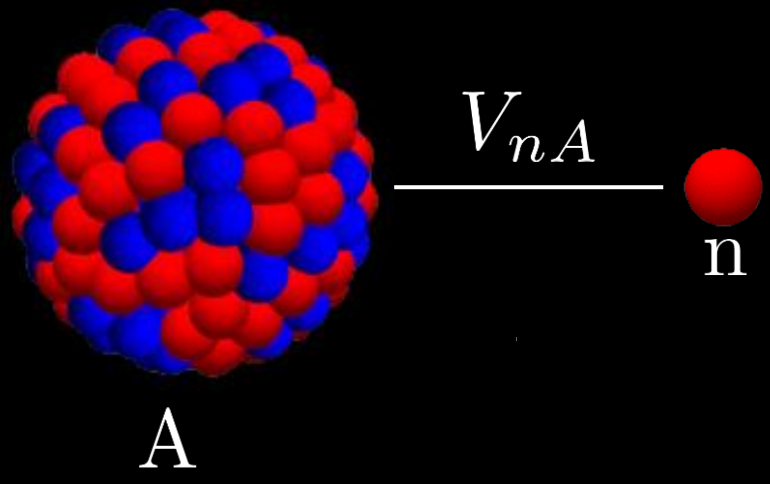


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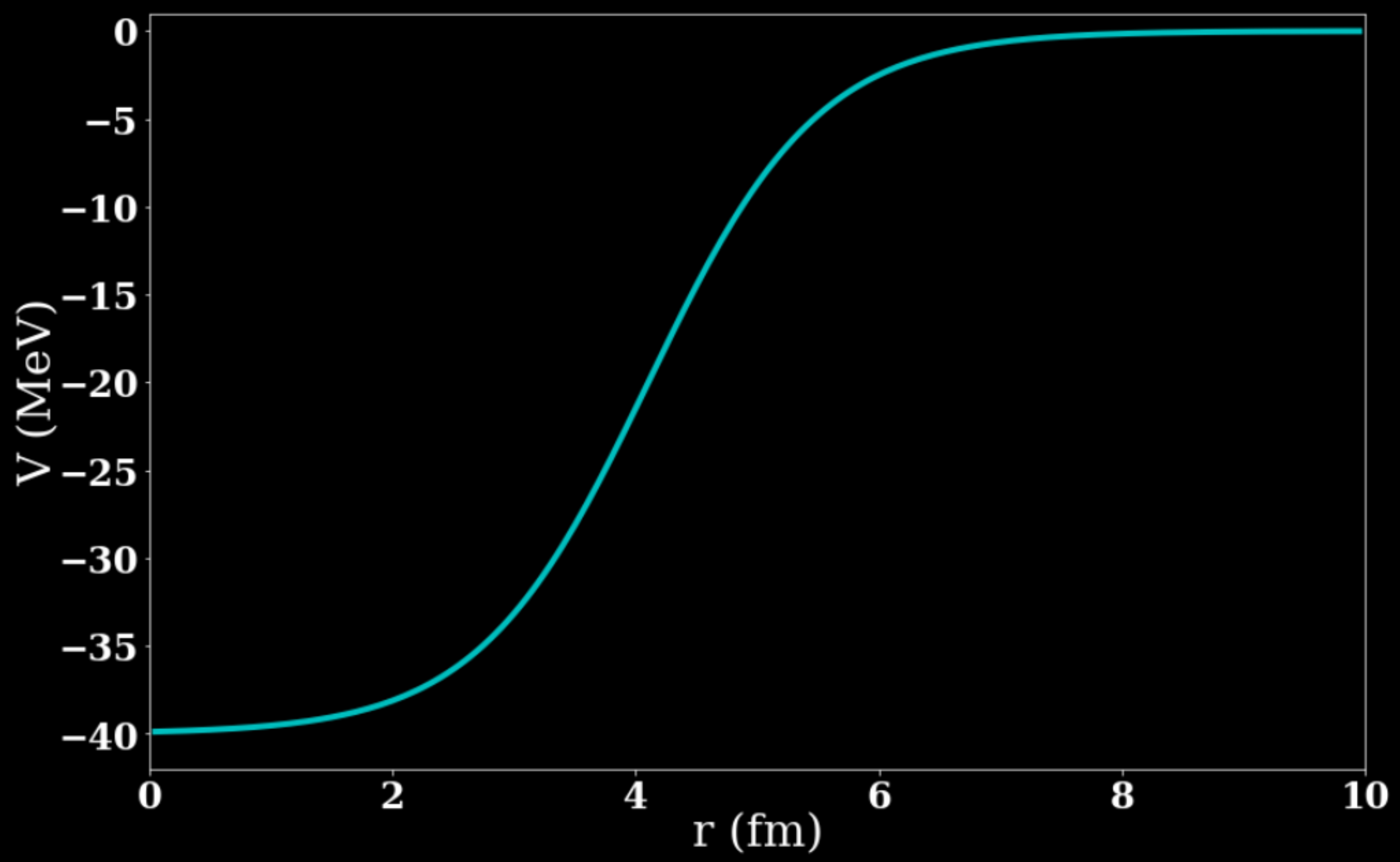
Nuclear scattering is complicated: many-body system with internal degrees of freedom, excitations, and resonances to account for.



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$$n = \frac{c}{v}, \;\; n(\lambda) = n_R(\lambda) + i n_I(\lambda)$$

Complex index of refraction:

- real part $n_R$: refraction
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$$U(E) = V(E) + iW(E)$$

Complex index of refraction:

- real part $n_R$: refraction
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By analogy, for a complex nuclear potential:

- real part V: elastic scattering
- imaginary part W: inelastic processes
- energy dependent

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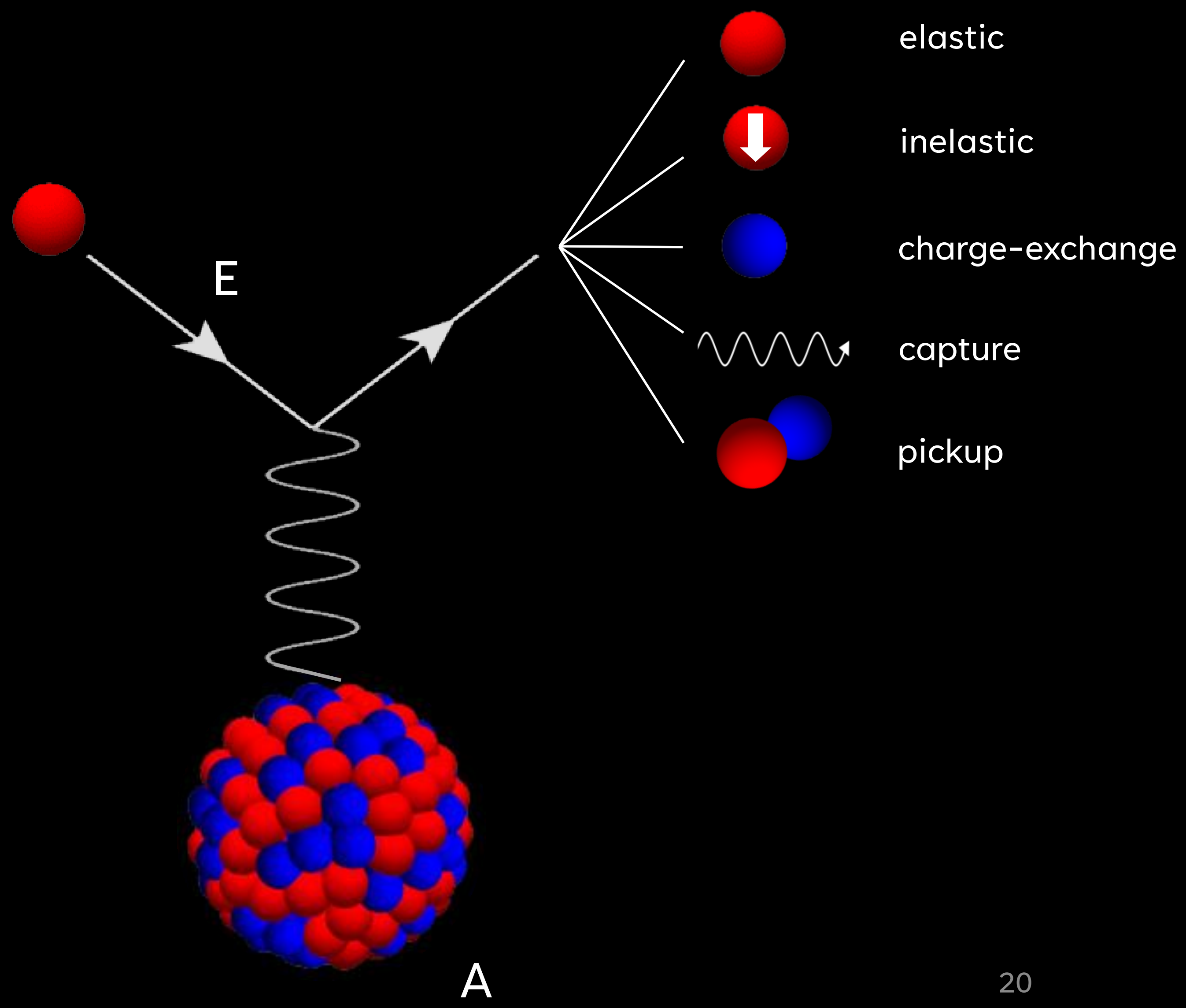
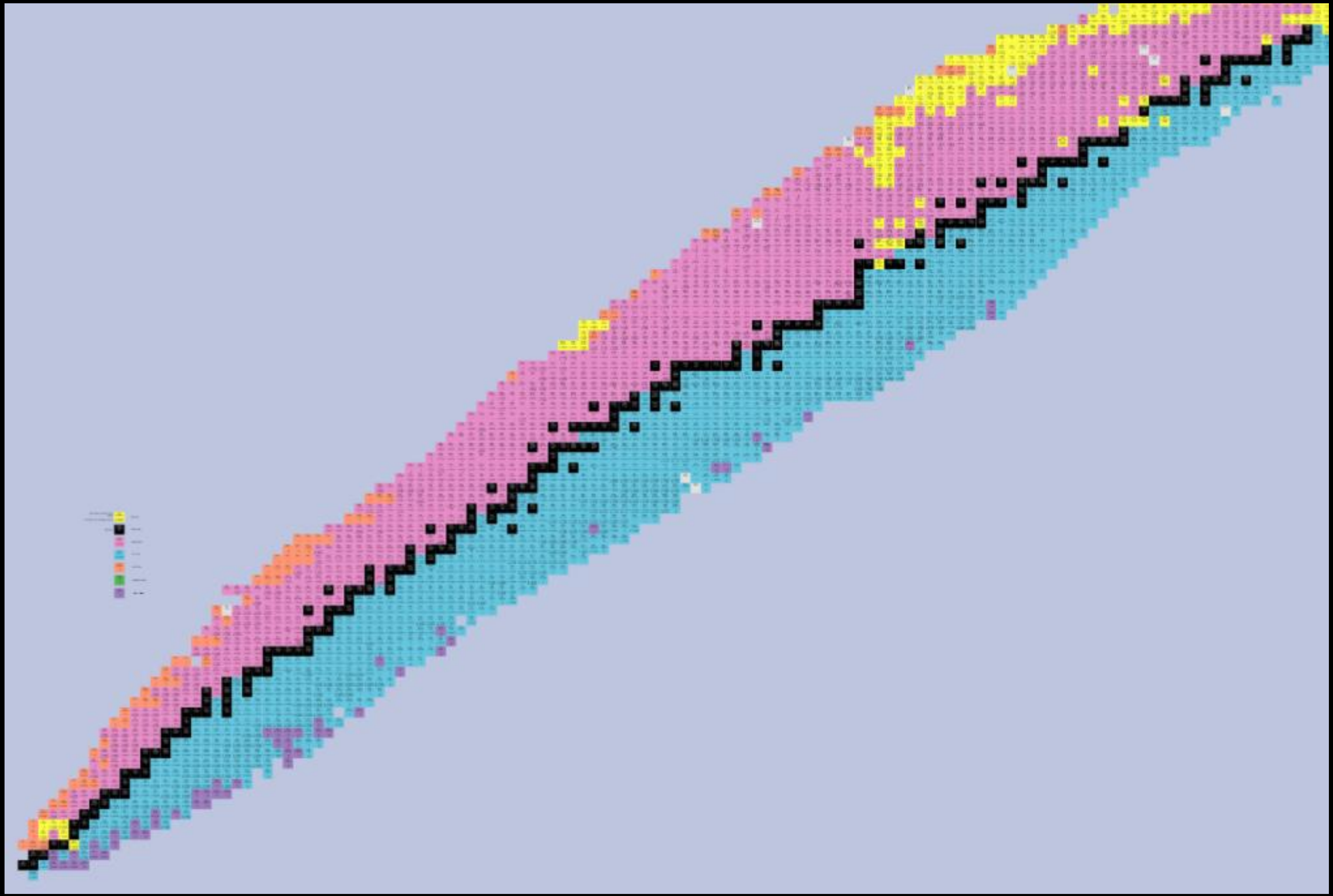


# APPROACHES TO GLOBAL OPTICAL POTENTIALS

**Phenomenological**

Assume form of reaction model then fit to experimental data

- ❖ Successful at describing reactions for nuclei near stability
- ❖ Descriptions of rare isotope reactions are unreliable



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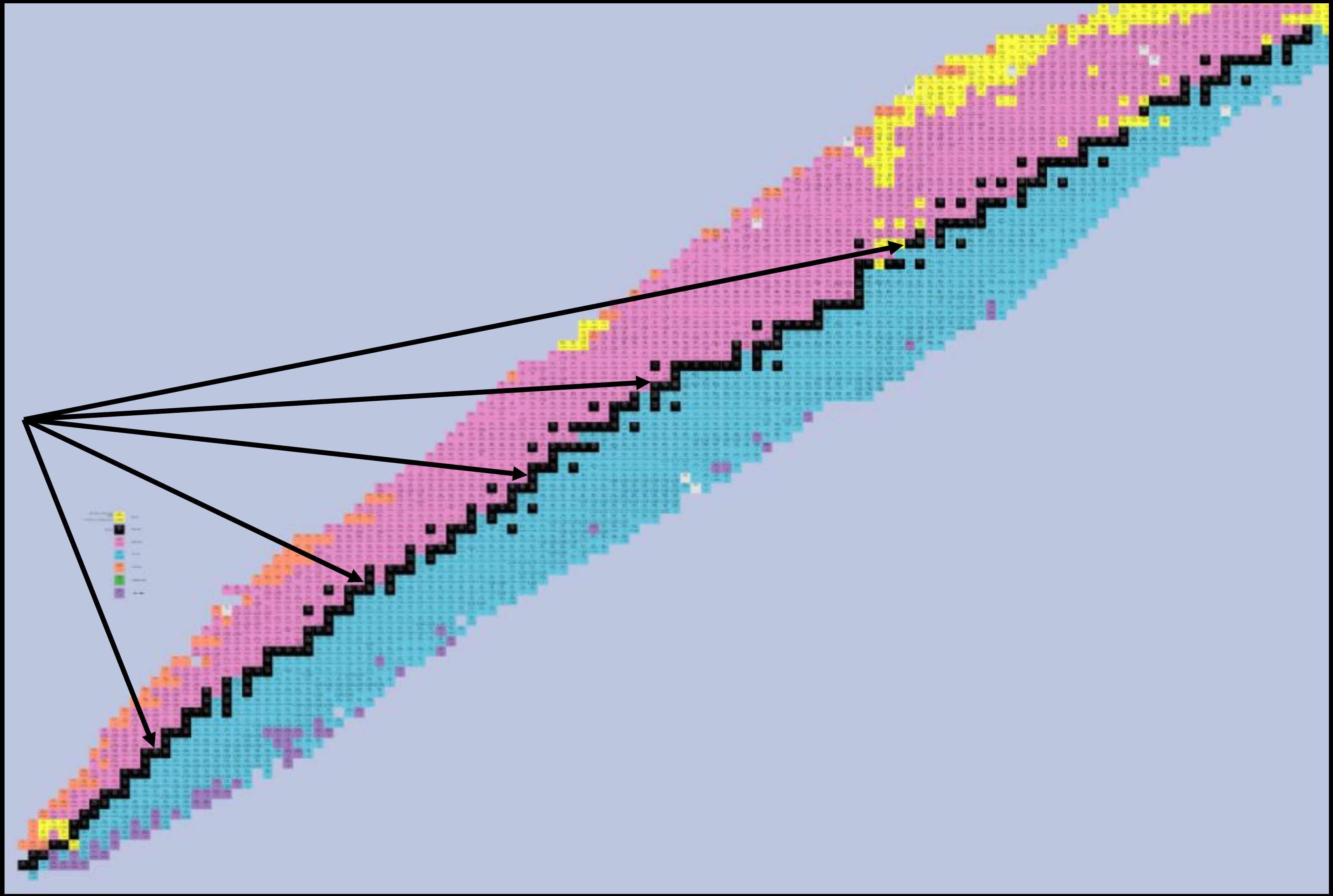

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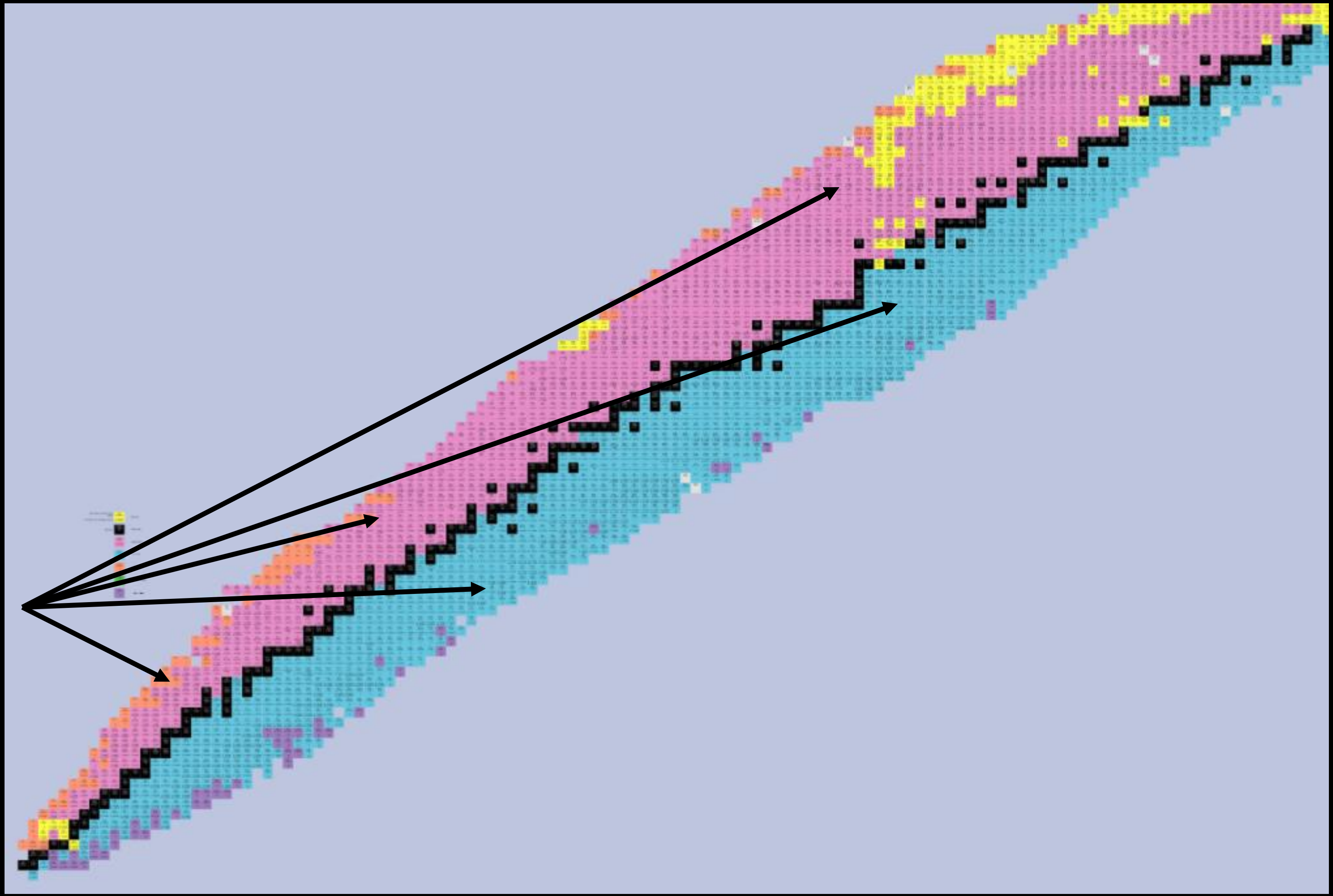

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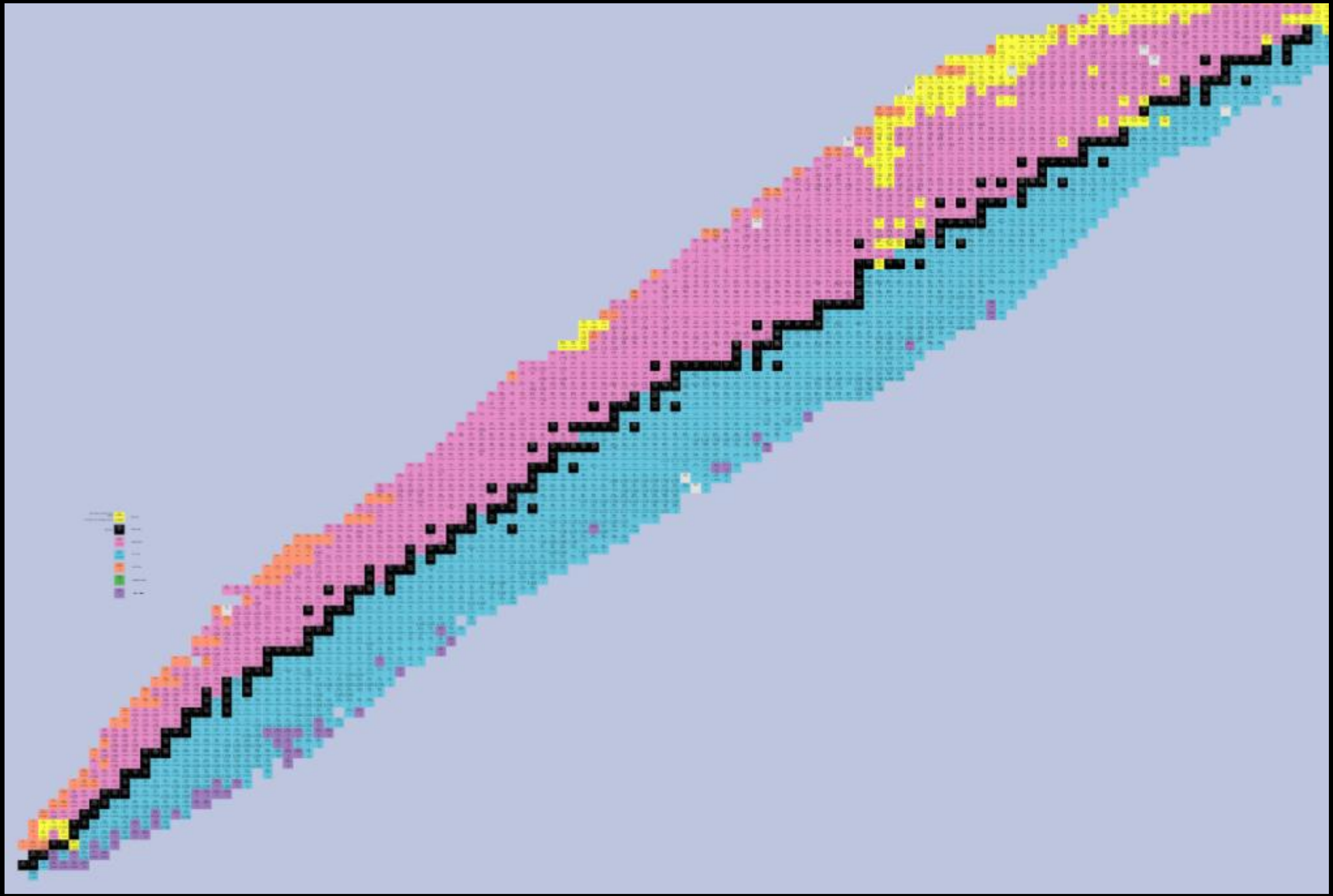
**Microscopic**

Calculate in many-body framework with realistic nuclear forces

- ❖ Robust predictions for unstable nuclei where experiments are difficult



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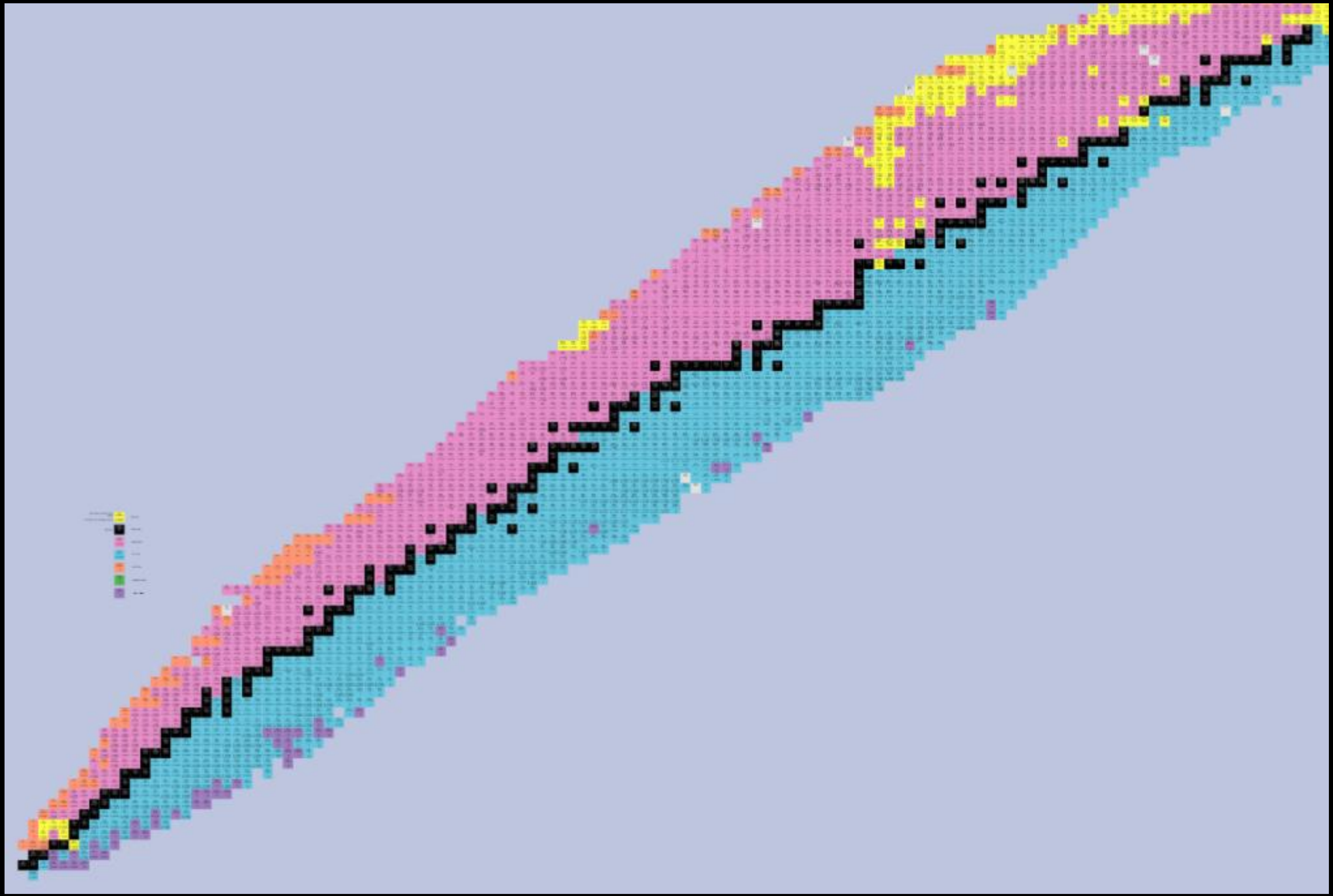
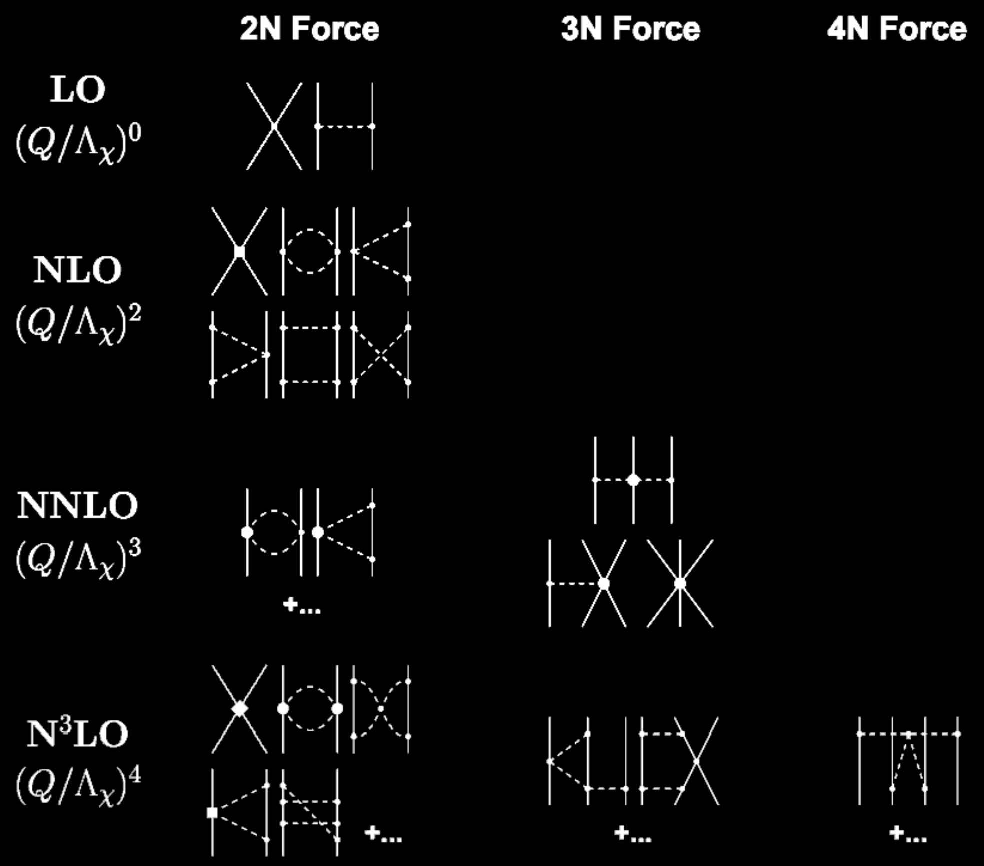
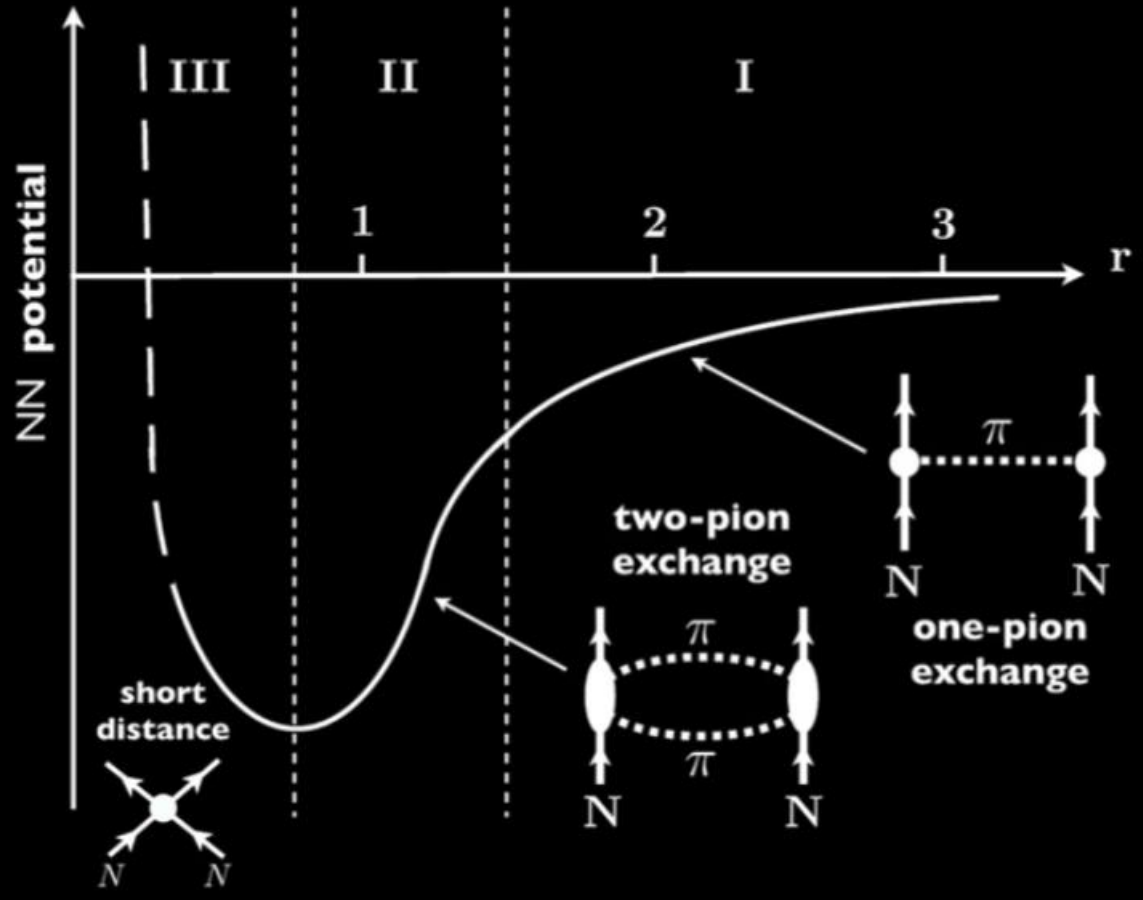
*Realistic nuclear forces?*



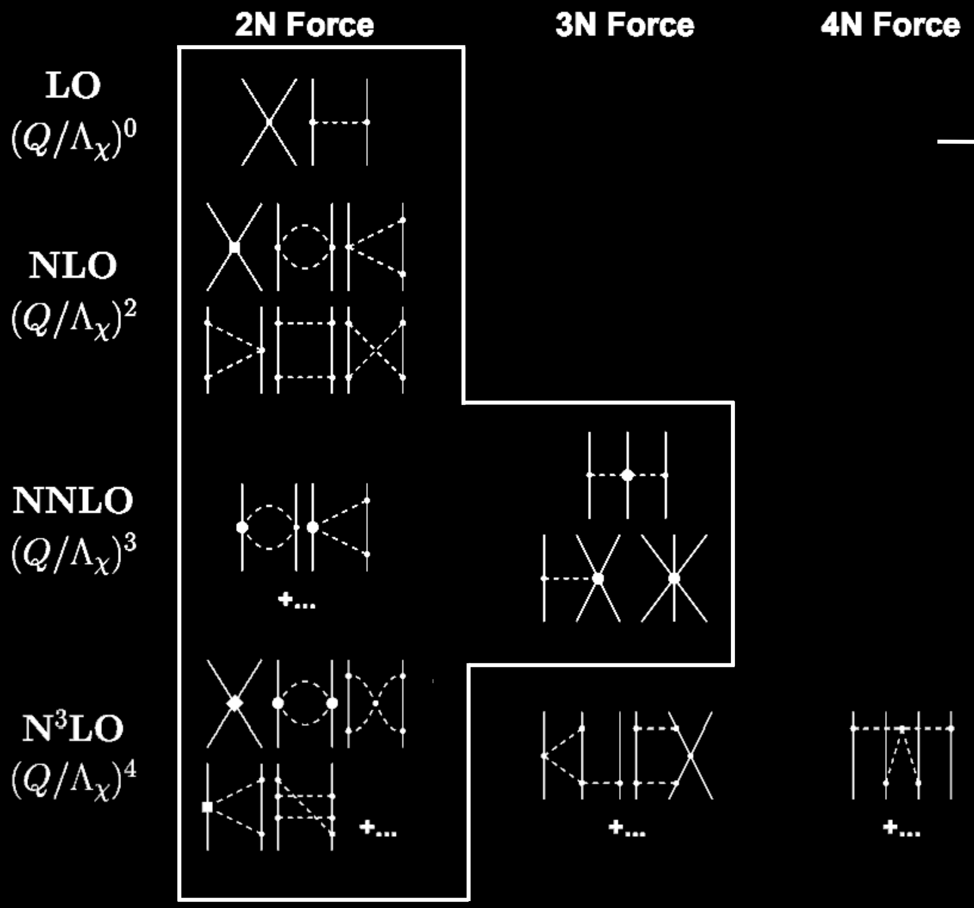
# NUCLEAR FORCES FROM CHIRAL EFFECTIVE FIELD THEORY



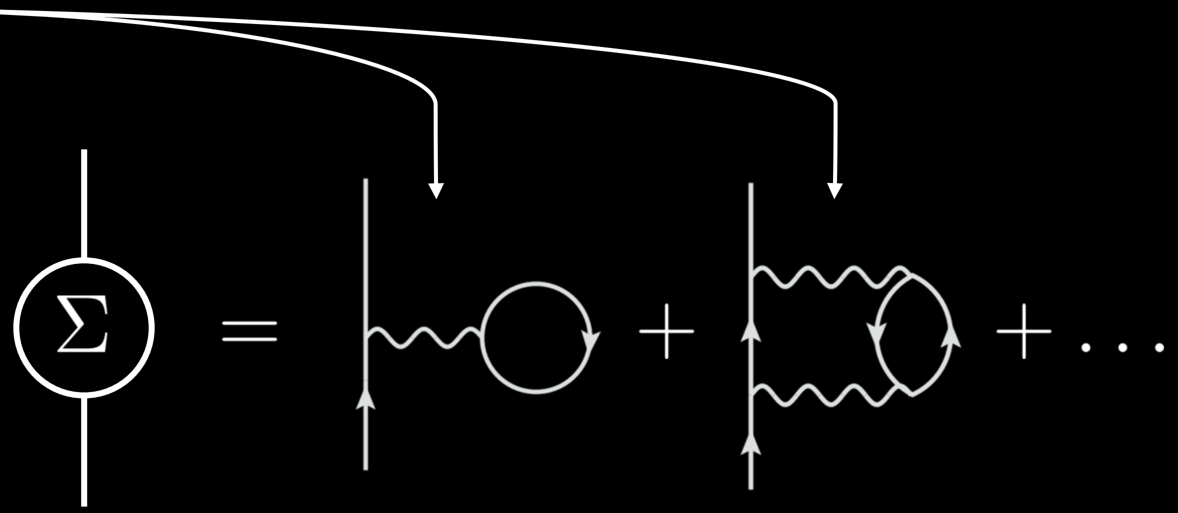



# MICROSCOPIC OPTICAL POTENTIAL

## Chiral Effective Field Theory

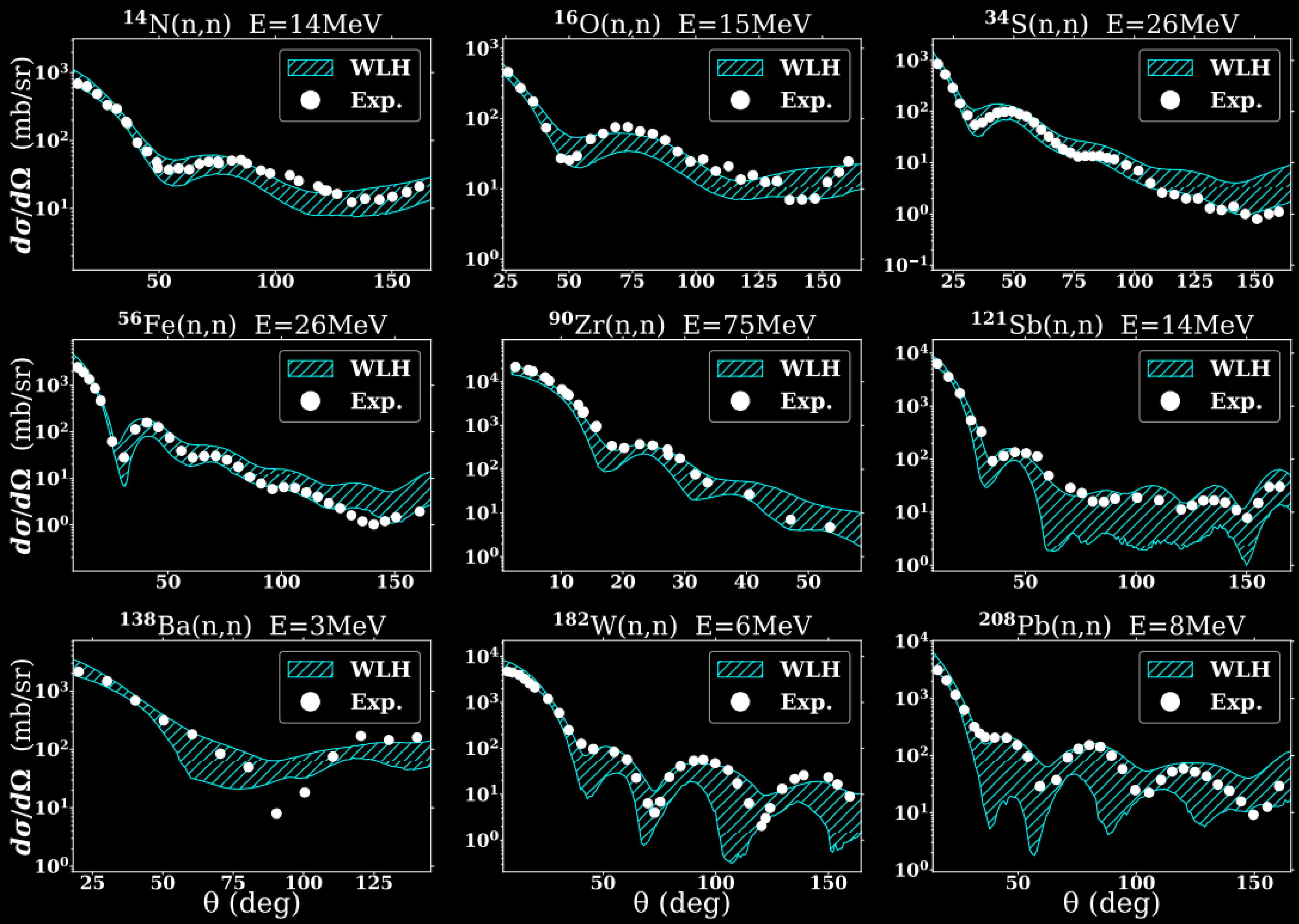



## Many-Body Perturbation Theory




*"The self-energy (Σ) is equivalent to the optical potential"*
-Bell, Squires; PRL (1959)

# ELASTIC NEUTRON SCATTERING




T.R. Whitehead et al., Phys. Rev. Lett. **127** (2021)

# RECAP

Scattering theory

Nuclear reactions