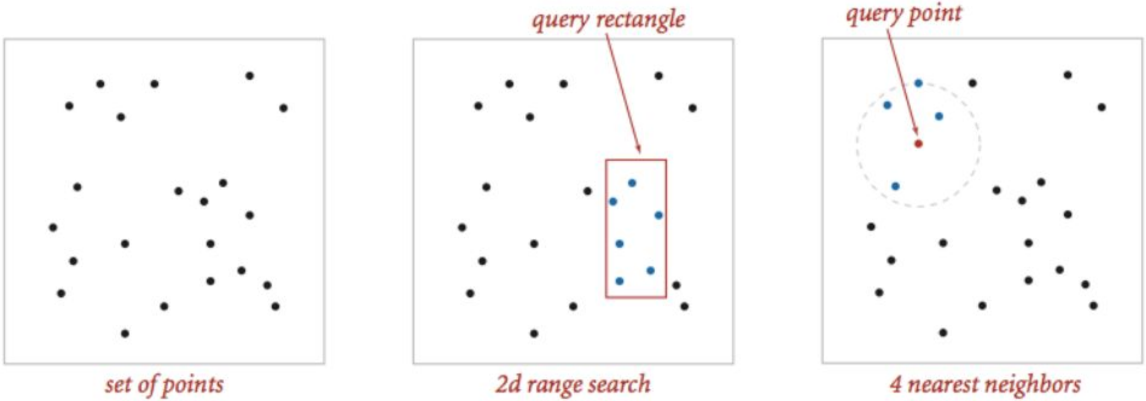
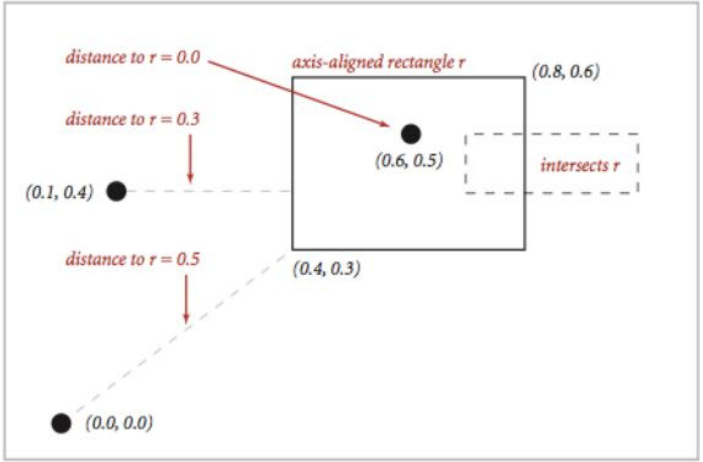
Discussion 10: KD Trees Project

The purpose of this project is to create a symbol table data type whose keys are two-dimensional points. We'll use a 2dTree to support efficient range search (find all the points contained in a query rectangle) and k-nearest neighbor search (find k points that are closest to a query point). 2dTrees have numerous applications, ranging from classifying astronomical objects to computer animation to speeding up neural networks to mining data to image retrieval.



Geometric Primitives We will use the data types `dsa.Point2D` and `dsa.RectHV` to represent points and axis-aligned rectangles in the plane.



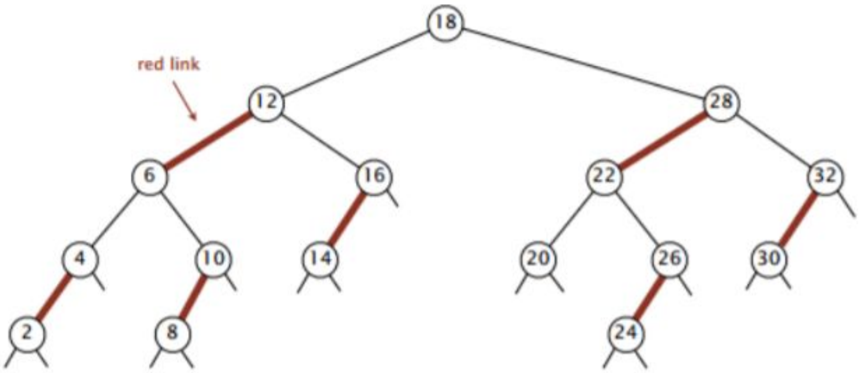
Symbol Table API Here is a Java interface `PointST<Value>` specifying the API for a symbol table data type whose keys ar `Point2D` objects and values are generic objects:

PointST<Value>	
`boolean isEmpty()`	returns `true` if this symbol table is empty, and `false` otherwise
`int size()`	returns the number of key-value pairs in this symbol table
`void put(Point2D p, Value value)`	inserts the given point and value into this symbol table
`Value get(Point2D p)`	returns the value associated with the given point in this symbol table, or `null`
`boolean contains(Point2D p)`	returns `true` if this symbol table contains the given point, and `false` otherwise
`Iterable<Point2D> points()`	returns all the points in this symbol table
`Iterable<Point2D> range(RectHV rect)`	returns all the points in this symbol table that are inside the given rectangle
`Point2D nearest(Point2D p)`	returns the point in this symbol table that is different from and closest to the given point, or `null`
`Iterable<Point2D> nearest(Point2D p, int k)`	returns up to `k` points from this symbol table that are different from and closest to the given point

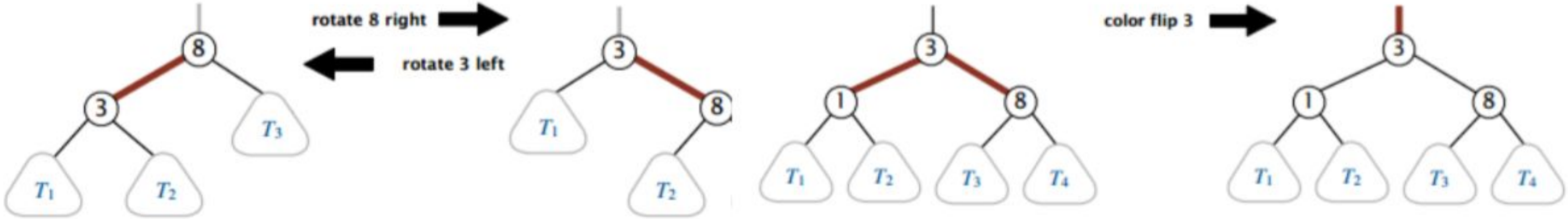
Problem 1. (*Brute-force Implementation*) Develop a data type called `BrutePointST` that implements the above API using a red-black BST (`RedBlackBST`) as the underlying data structure.

`BrutePointST<Value> implements PointST<Value>`	
`BrutePointST()`	constructs an empty symbol table

- Instance variable
  - An underlying data structure to store the 2d points (keys) and their corresponding values, `RedBlackBST<Point2D, Value> bst`



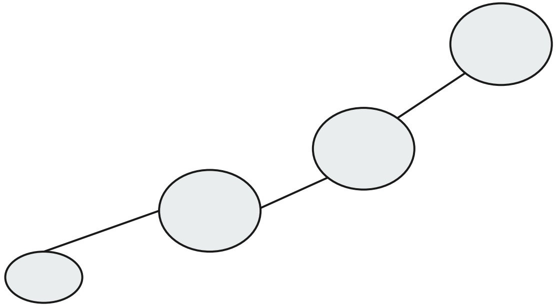
Allowed operations (rotations and color flip):



Red-Black-BST

- You do not need a deep understanding of it for this project.
- One of many types “Self balancing trees”
  - Likened to AVL tree or 2-3 Tree or B-Tree

Prevents occurrences such as this:



What you need to take away is that it is a Binary Search Tree, that Balances itself upon insertion and deletions to maintain a time complexity of O(logn) at worst and O(logn) at best

`BrutePointST()`

- Initialize the instance variable `bst` appropriately

`int size()`

- Return the size of `bst`

`boolean isEmpty()`

- Return whether `bst` is empty

`void put(Point2D p, Value value)`

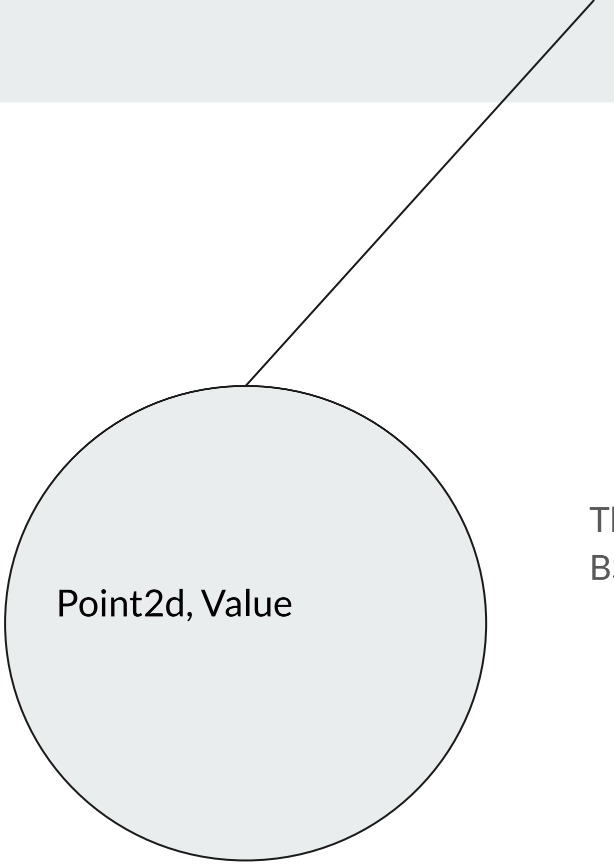
- Insert the given point and value into `bst`

`Value get(Point2D p)`

- Return the value associated with the given point in `bst`, or `null`

`boolean contains(Point2D p)`

- Return whether `bst` contains the given point



This structure will be generic since the values in the BST will be of type Value!

`Iterable<Point2D> range(RectHV rect)`

- Return an iterable object containing all the points in `bst` that are inside the given rectangle

`Point2D nearest(Point2D p)`

- Return a point from `bst` that is different from and closest to the given point, or `null`

`Iterable<Point2D> nearest(Point2D p, int k)`

- Return up to `k` points from `bst` that are different from and closest to the given point

Corner Cases

- The `put()` method should throw a `NullPointerException()` with the message `"p is null"` if p is `null` and the message `"value is null"` if $value$ is `null`.
- The `get()`, `contains()`, and `nearest()` methods should throw a `NullPointerException()` with the message `"p is null"` if p is `null`.
- The `rect()` method should throw a `NullPointerException()` with the message `"rect is null"` if $rect$ is `null`.

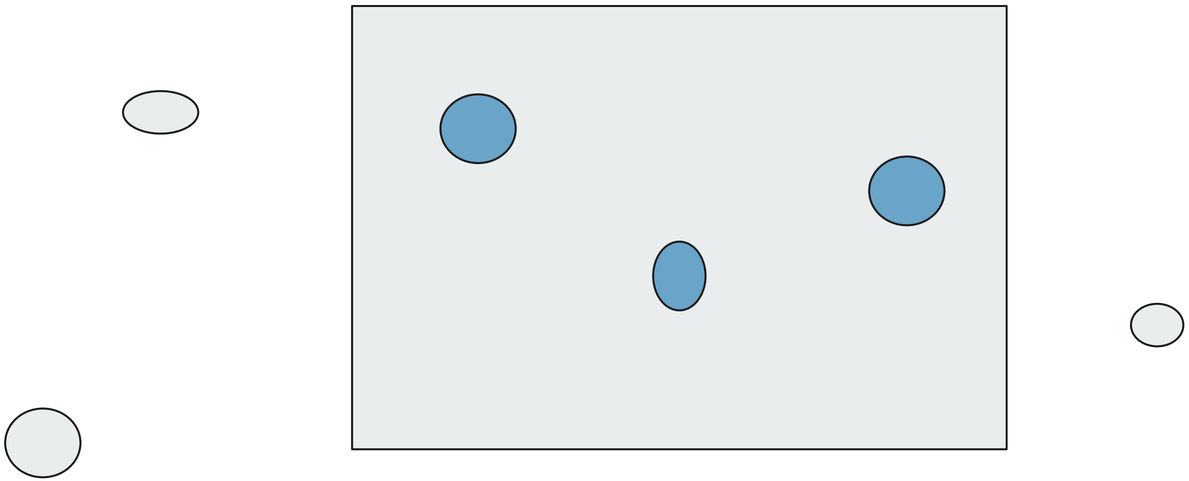
Performance Requirements

- The `isEmpty()` and `size()` methods should run in time $T(n) \sim 1$, where n is the number of key-value pairs in the symbol table.
- The `put()`, `get()`, and `contains()` methods should run in time $T(n) \sim \log n$.
- The `points()`, `range()`, and `nearest()` methods should run in time $T(n) \sim n$.

Range

RectHV is the input parameter.

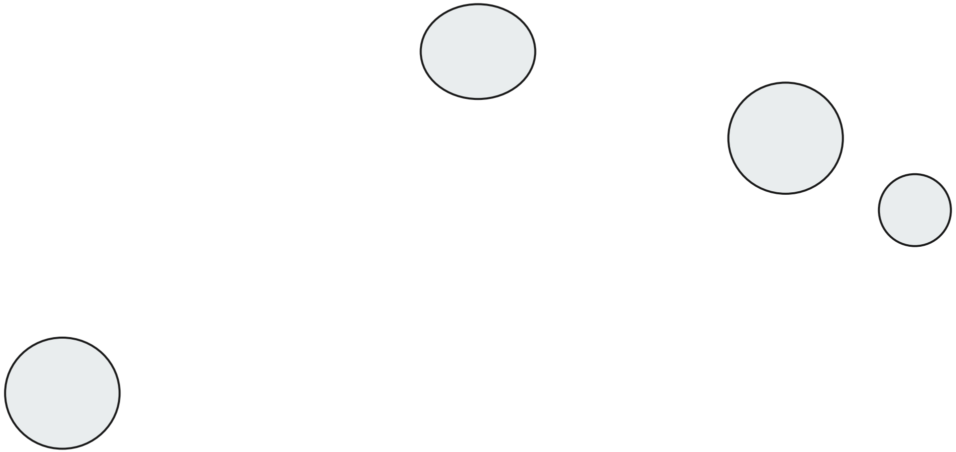
The range will be all the points that live within the rectangle!



Nearest (1 or k)

If 1 nearest, the nearest point is the one to the right

If 2 nearest, the nearest points are the 2 closer ones



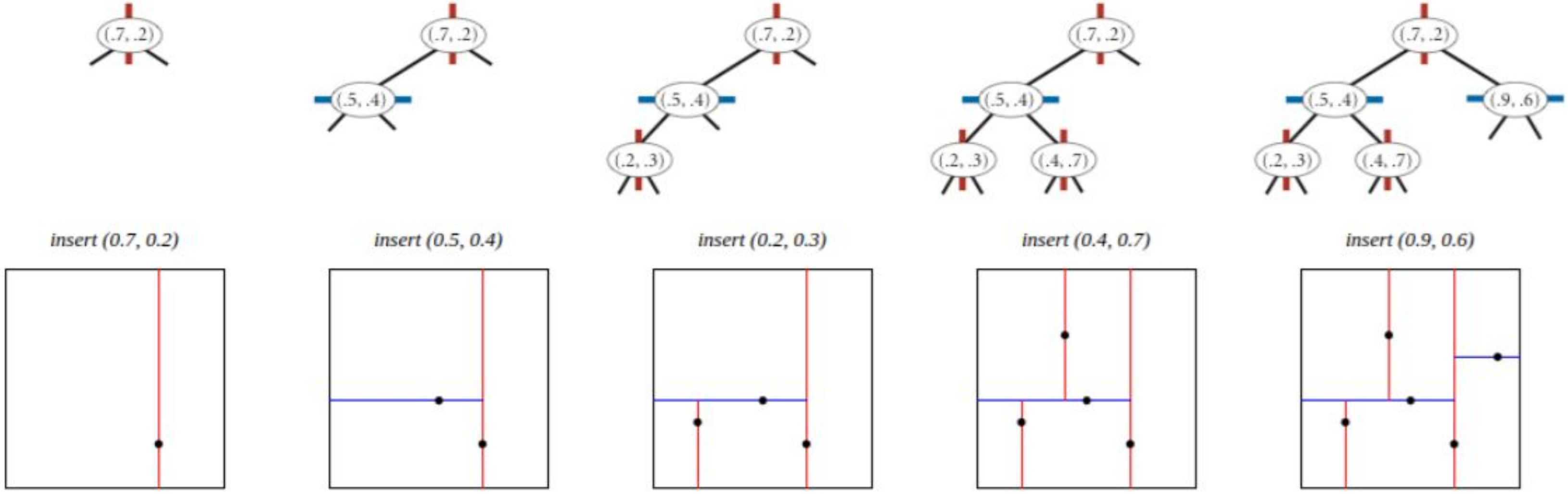
```
$ java BrutePointST 0.975528 0.345492 5 < data/circle10.txt
st.size() = 10
st.contains((0.975528, 0.345492))? true
st.range([-1.0, -1.0] x [1.0, 1.0]):
  (0.5, 0.0)
  (0.206107, 0.095492)
  (0.793893, 0.095492)
  (0.024472, 0.345492)
  (0.975528, 0.345492)
  (0.024472, 0.654508)
  (0.975528, 0.654508)
  (0.206107, 0.904508)
  (0.793893, 0.904508)
  (0.5, 1.0)
st.nearest((0.975528, 0.345492)) = (0.975528, 0.654508)
st.nearest((0.975528, 0.345492), 5):
  (0.975528, 0.654508)
  (0.793893, 0.095492)
  (0.793893, 0.904508)
  (0.5, 0.0)
  (0.5, 1.0)
```

Problem 2. (*2dTree Implementation*) Develop a data type called `KdTreePointST` that uses a 2dTree to implement the above symbol table API.

`KdTreePointST<Value> implements PointST<Value>`	
`KdTreePointST()`	constructs an empty symbol table

A 2dTree is a generalization of a BST to two-dimensional keys. The idea is to build a BST with points in the nodes, using the x- and y-coordinates of the points as keys in strictly alternating sequence, starting with the x-coordinates.

Search and insert. The algorithms for search and insert are similar to those for BSTs, but at the root we use the x-coordinate (if the point to be inserted has a smaller x-coordinate than the point at the root, go left; otherwise go right); then at the next level, we use the y-coordinate (if the point to be inserted has a smaller y-coordinate than the point in the node, go left; otherwise go right); then at the next level the x-coordinate, and so forth.



Level-order traversal. The `points()` method should return the points in level-order: first the root, then all children of the root (from left/bottom to right/top), then all grandchildren of the root (from left to right), and so forth. The level-order traversal of the 2dTree above is $(.7, .2)$, $(.5, .4)$, $(.9, .6)$, $(.2, .3)$, $(.4, .7)$.

The prime advantage of a 2dTree over a BST is that it supports efficient implementation of range search, nearest neighbor, and k-nearest neighbor search. Each node corresponds to an axis-aligned rectangle, which encloses all of the points in its subtree. The root corresponds to the infinitely large square from $[(-\infty, -\infty), (+\infty, +\infty)]$; the left and right children of the root correspond to the two rectangles split by the x-coordinate of the point at the root; and so forth.

- *Range search.* To find all points contained in a given query rectangle, start at the root and recursively search for points in both subtrees using the following pruning rule: if the query rectangle does not intersect the rectangle corresponding to a node, there is no need to explore that node (or its subtrees). That is, you should search a subtree only if it might contain a point contained in the query rectangle.

- *Nearest neighbor search.* To find a closest point to a given query point, start at the root and recursively search in both subtrees using the following pruning rule: if the closest point discovered so far is closer than the distance between the query point and the rectangle corresponding to a node, there is no need to explore that node (or its subtrees). That is, you should search a node only if it might contain a point that is closer than the best one found so far. The effectiveness of the pruning rule depends on quickly finding a nearby point. To do this, organize your recursive method so that when there are two possible subtrees to go down, you choose first the subtree that is on the same side of the splitting line as the query point; the closest point found while exploring the first subtree may enable pruning of the second subtree.

- *k-nearest neighbor search.* Use the nearest neighbor search described above.

Instance variables

- Reference to the root of a 2dTree, `Node root`
- Number of nodes in the tree, `int n`

`KdTreePointST()`

- Initialize instance variables `root` and `n` appropriately

`int size()`

- Return the number of nodes in the 2dTree

`boolean isEmpty()`

- Return whether the 2dTree is empty

`void put(Point2D p, Value value)`

- Call the private `put()` method with appropriate arguments to insert the given point and value into the 2dTree; the parameter `lr` in this and other helper methods represents if the currrent node is x-aligned (`lr = true`) or y-aligned (`lr = false`)

`Node put(Node x, Point2D p, Value value, RectHV rect, boolean lr)`

- If `x = null`, return a new `Node` object built appropriately
- If the point in `x` is the same as the given point, update the value in `x` to the given value
- Otherwise, make a recursive call to `put()` with appropriate arguments to insert the given point and value into the left subtree `x.lb` or the right subtree `x.rt` depending on how `x.p` and `p` compare (use `lr` to decide which coordinate to consider)
- Return `x`

`Value get(Point2D p)`

- Call the private `get()` method with appropriate arguments to find the value corresponding to the given point

`Value get(Node x, Point2D p, boolean lr)`

- If `x = null`, return `null`
- If the point in `x` is the same as the given point, return the value in `x`
- Make a recursive call to `get()` with appropriate arguments to find the value corresponding to the given point in the left subtree `x.lb` or the right subtree `x.rt` depending on how `x.p` and `p` compare

`boolean contains(Point2D p)`

- Return whether the given point is in the 2dTree

`Iterable<Point2D> points()`

- Return all the points in the 2dTree, collected using a level-order traversal of the tree; use two queues, one to aid in the traversal and the other to collect the points

`Iterable<Point2D> range(RectHV rect)`

- Call the private `range()` method with appropriate arguments, the last one being an empty queue of `Point2D` objects, and return the queue

`void range(Node x, RectHV rect, Queue<Point2D> q)`

- If `x = null`, simply return
- If `rect` contains the point in `x`, enqueue the point into `q`
- Make recursive calls to `range()` on the left subtree `x.lb` and on the right subtree `x.rt`
- Incorporate the *range search* pruning rule mentioned in the project writeup

`Point2D nearest(Point2D p)`

- Return a point from the 2dTree that is different from and closest to the given point by calling the private method `nearest()` with appropriate arguments

`Point2D nearest(Node x, Point2D p, Point2D nearest, boolean lr)`

- If `x = null`, return `nearest`
- If the point `x.p` is different from the given point `p` and the squared distance between the two is smaller than the squared distance between `nearest` and `p`, update `nearest` to `x.p`
- Make a recursive call to `nearest()` on the left subtree `x.lb`
- Make a recursive call to `nearest()` on the right subtree `x.rt`, using the value returned by the first call in an appropriate manner
- Incorporate the *nearest neighbor* pruning rules mentioned in the project writeup

`Iterable<Point2D> nearest(Point2D p, int k)`

- Call the private `nearest()` method passing it an empty maxPQ of `Point2D` objects (built with a suitable comparator from `Point2D`) as one of the arguments, and return the PQ

`void nearest(Node x, Point2D p, int k, MaxPQ<Point2D> pq, boolean lr)`

- If `x = null` or if the size of `pq` is greater than `k`, simply return
- If the point in `x` is different from the given point, insert it into `pq`
- If the size of `pq` exceeds `k`, remove the maximum point from the `pq`
- Make recursive calls to `nearest()` on the left subtree `x.lb` and on the right subtree `x.rt`
- Incorporate the *nearest neighbor* pruning rules mentioned in the project writeup

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