

Speed and Simplicity for Incremental Sequence Computation

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What is Incremental Computation?

4 2 5 1 4 9 5 7 8 3 3 6

What is Incremental Computation?

$$\max(4 \ 2 \ 5 \ 1 \ 4 \ 9 \ 5 \ 7 \ 8 \ 3 \ 3 \ 6) = 9$$

What is Incremental Computation?

$\max(4 \ 2 \ 5 \ 1 \ 4 \ 9 \ 5 \ 7 \ 8 \ 3 \ 3 \ 6) = 9$

Change Data



4 2 5 1 4 3 5 7 8 3 3 6

What is Incremental Computation?

$\max(4 \ 2 \ 5 \ 1 \ 4 \ 9 \ 5 \ 7 \ 8 \ 3 \ 3 \ 6) = 9$

Change Data

Update Result?

$\max(4 \ 2 \ 5 \ 1 \ 4 \ 3 \ 5 \ 7 \ 8 \ 3 \ 3 \ 6) = 8$

What is Incremental Computation?

$$\max(4 \ 2 \ 5 \ 1 \ 4 \ 9 \ 5 \ 7 \ 8 \ 3 \ 3 \ 6) = 9$$

Change Data

Update Result?

$$\max(4 \ 2 \ 5 \ 1 \ 4 \ 3 \ 5 \ 7 \ 8 \ 3 \ 3 \ 6) = 8$$

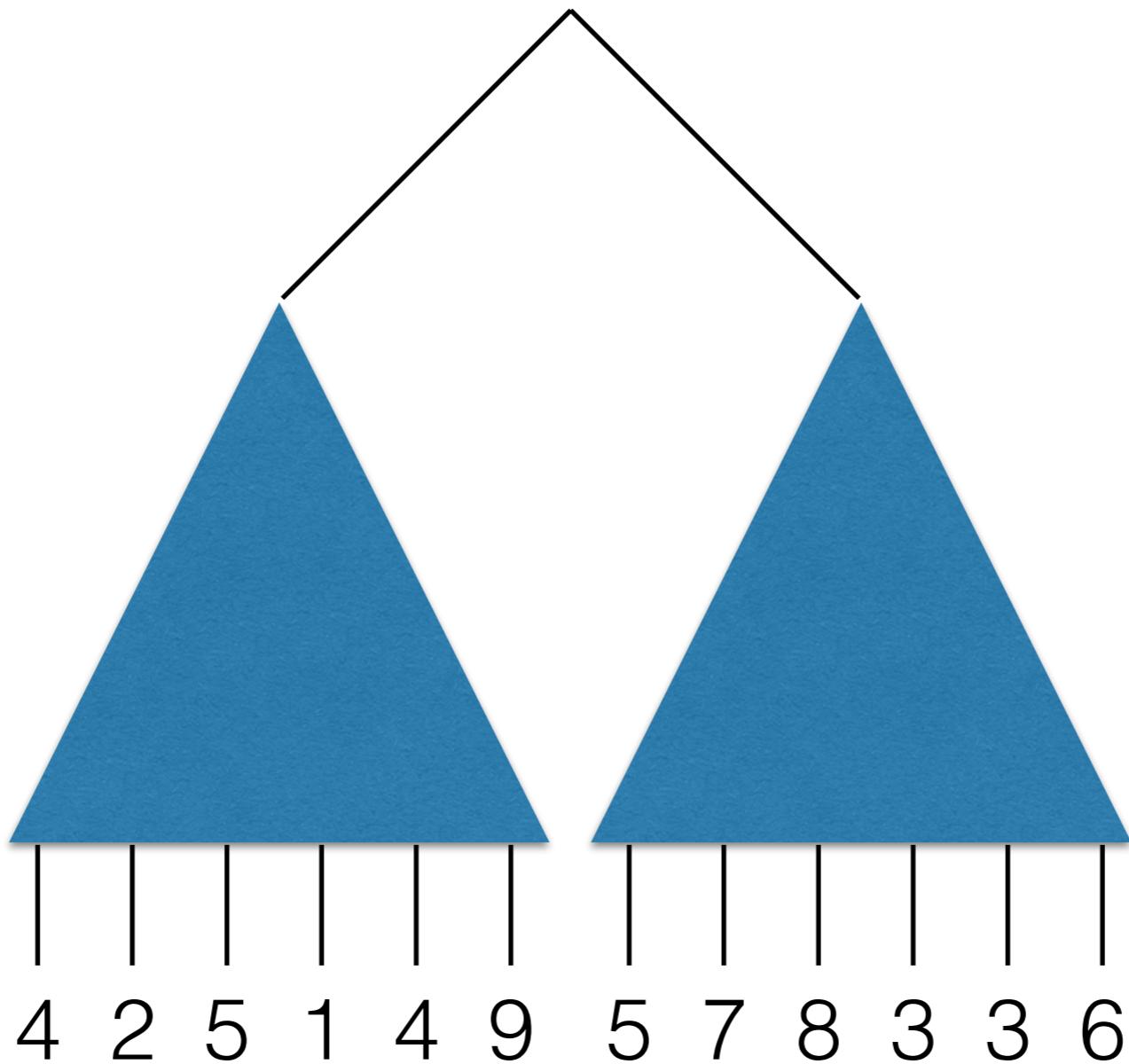
Not incremental: requires additional full scan of data

A computation is incremental if repeating it with a changed input is faster than re-computation

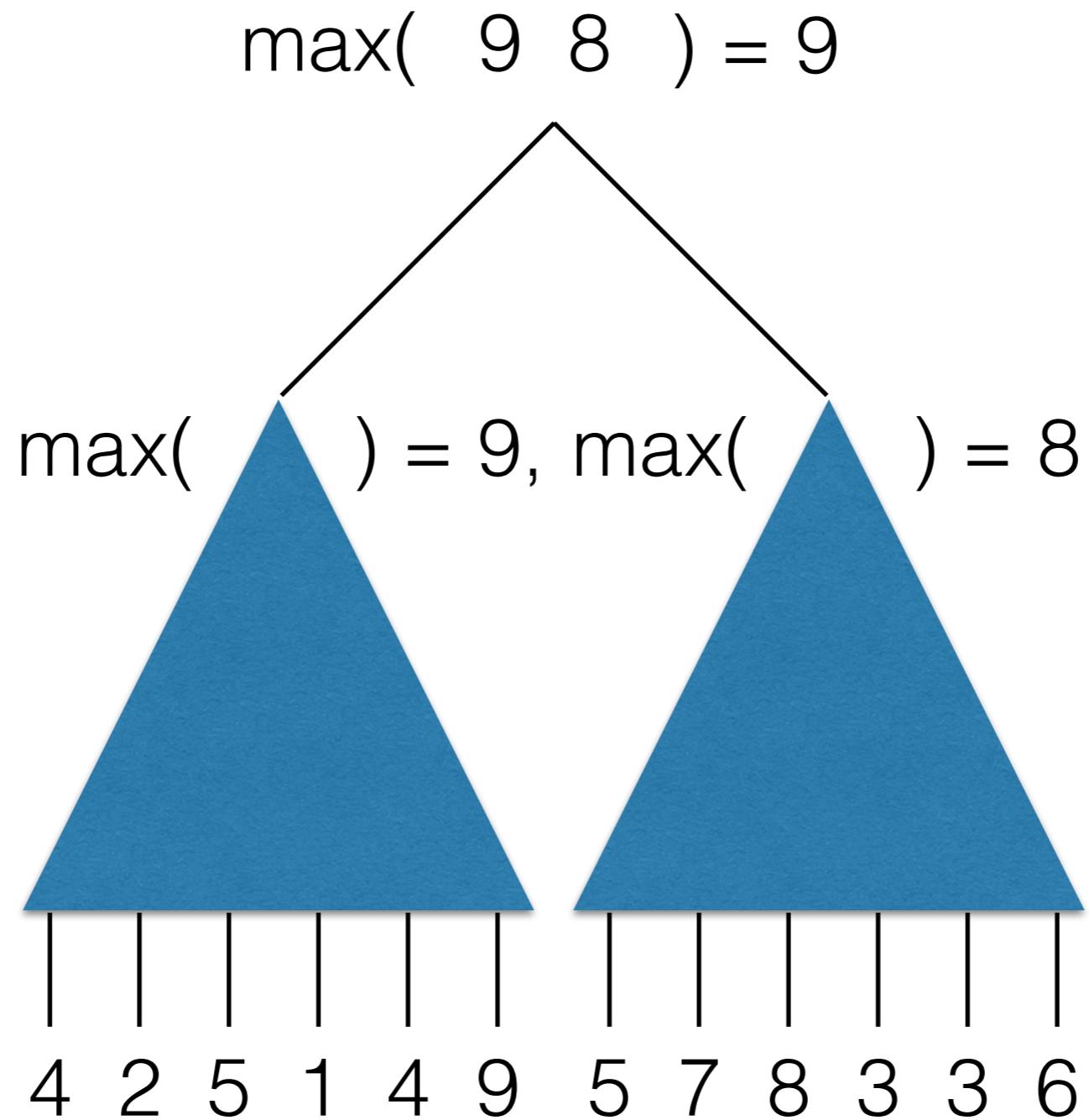
Using Memo Tables

4 2 5 1 4 9 5 7 8 3 3 6

Using Memo Tables

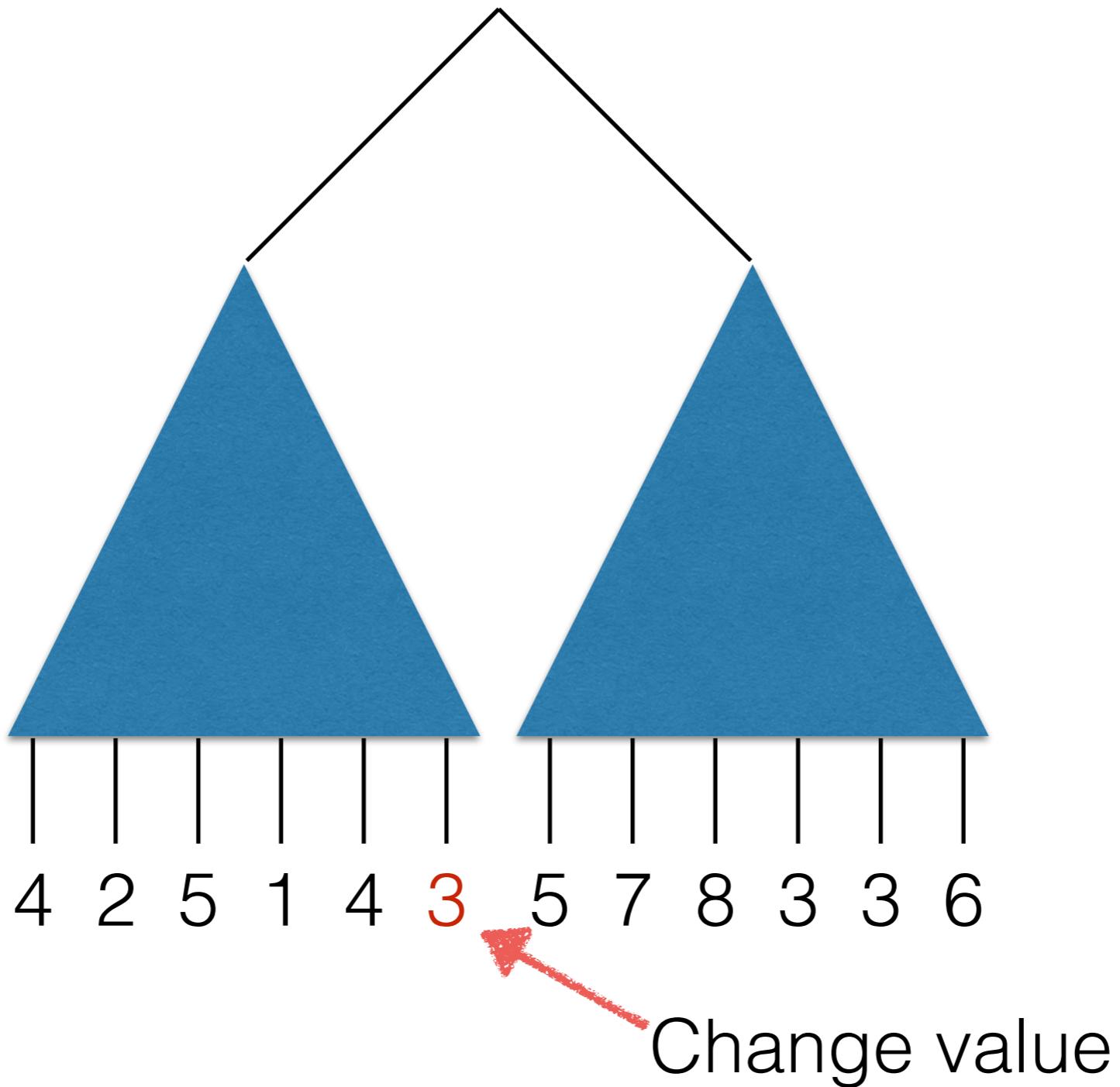


Using Memo Tables



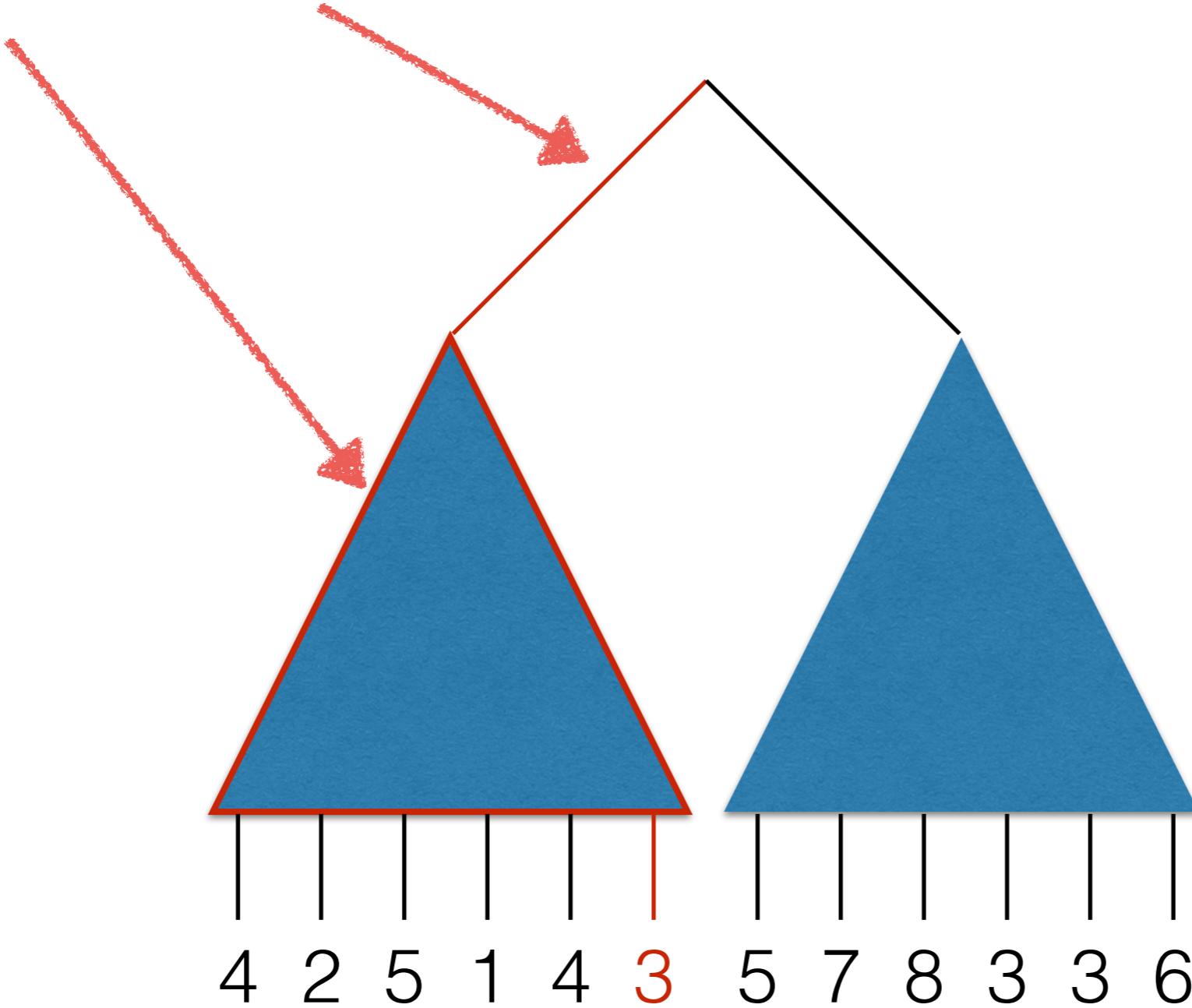
Add results
to table

Using Memo Tables

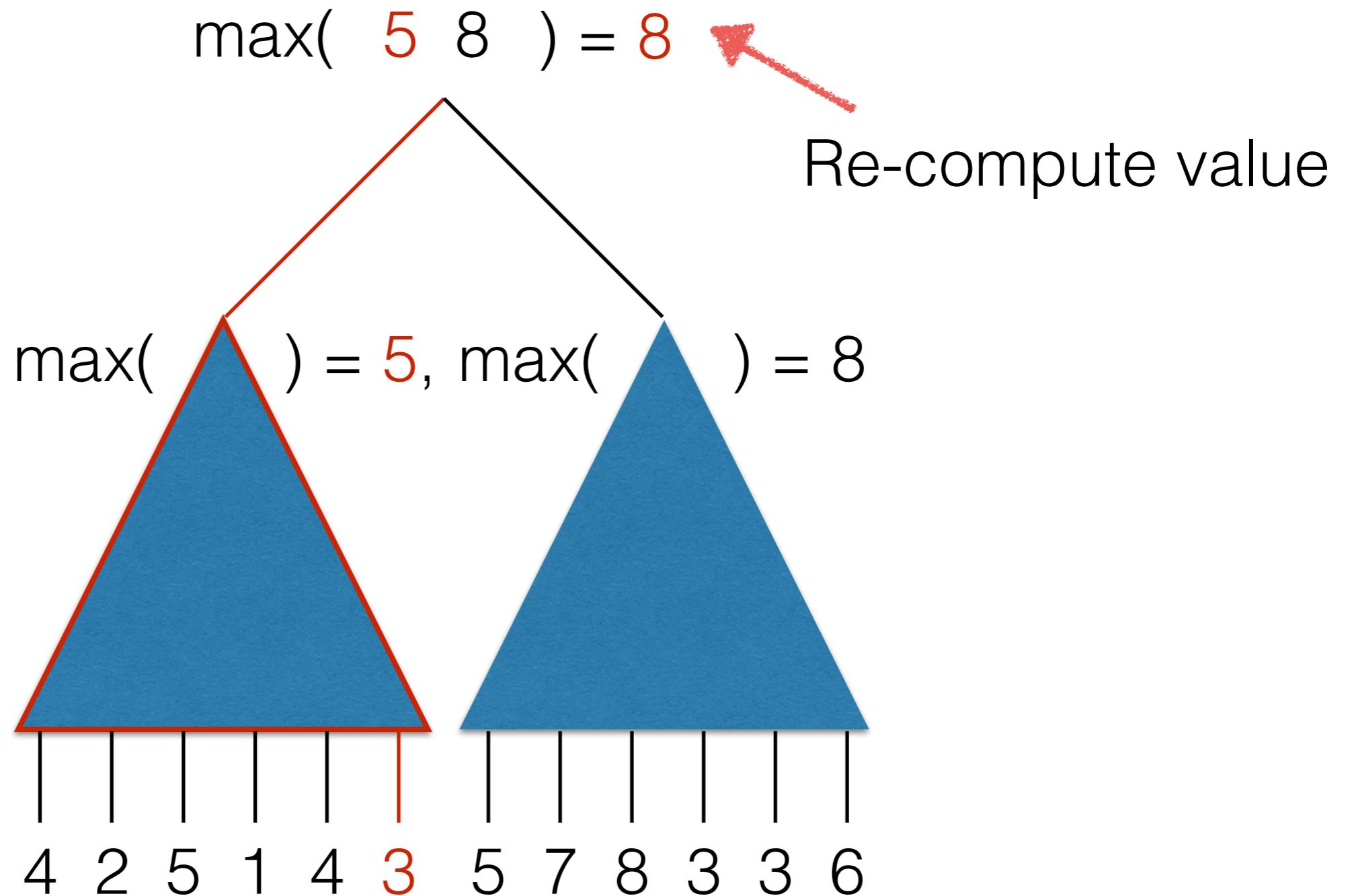


Using Memo Tables

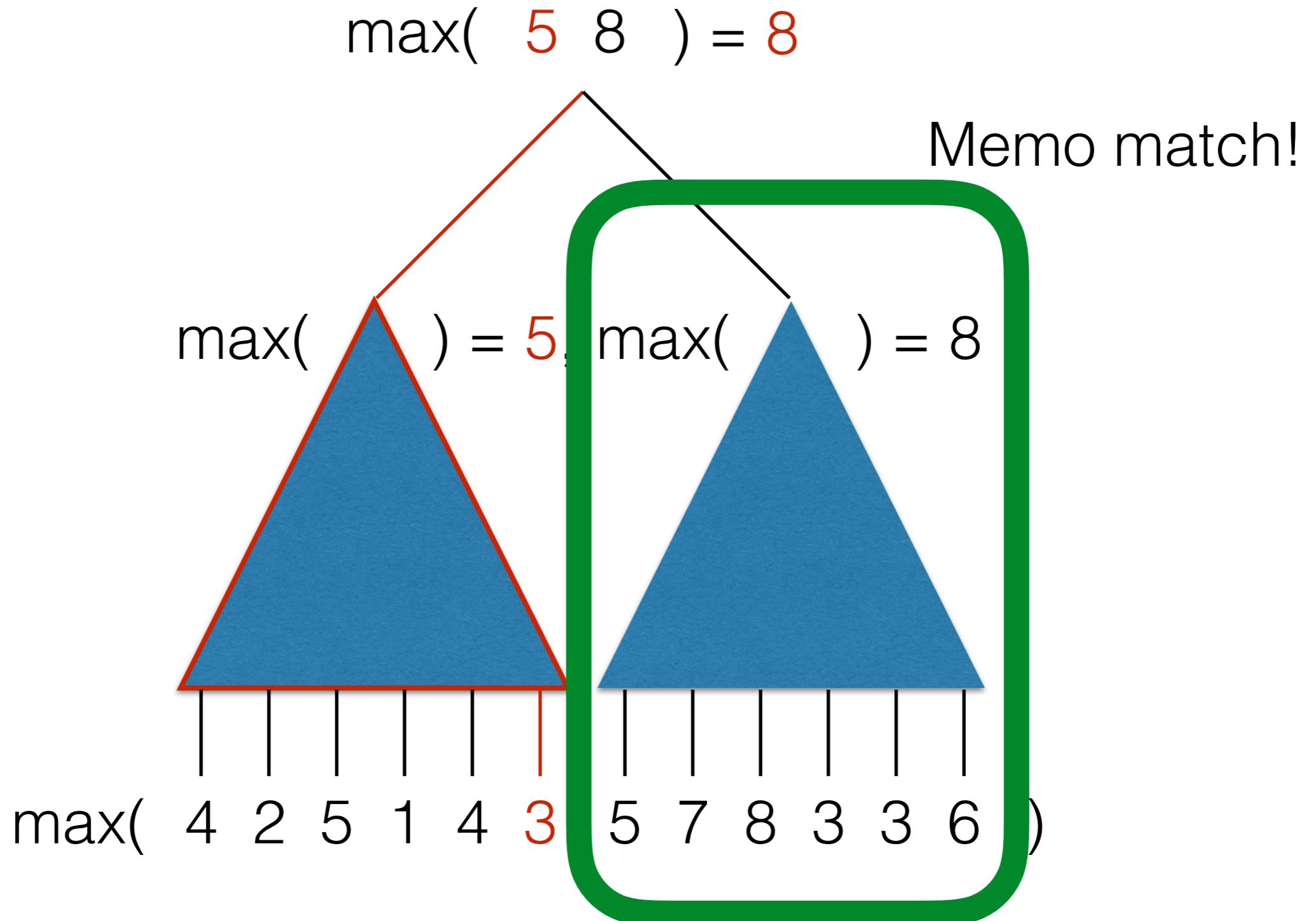
Update persistent tree



Using Memo Tables



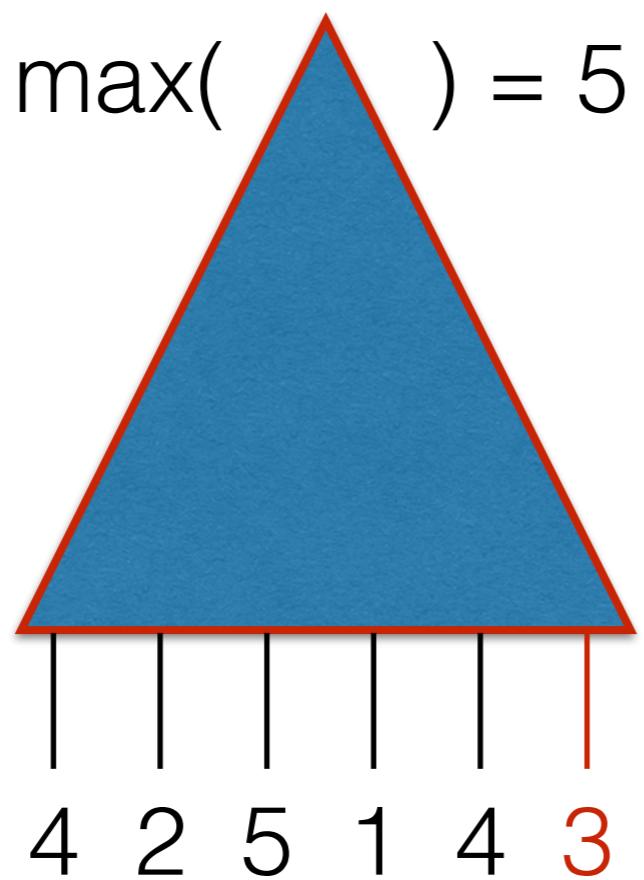
Using Memo Tables



Using Memo Tables

But there's a problem with memo tables

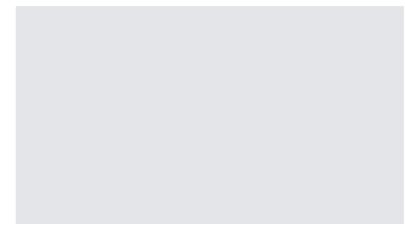
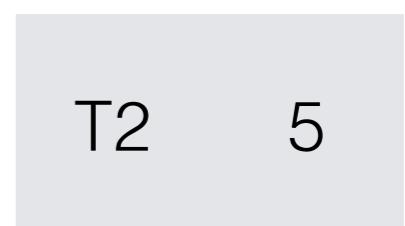
Using Memo Tables



Memo

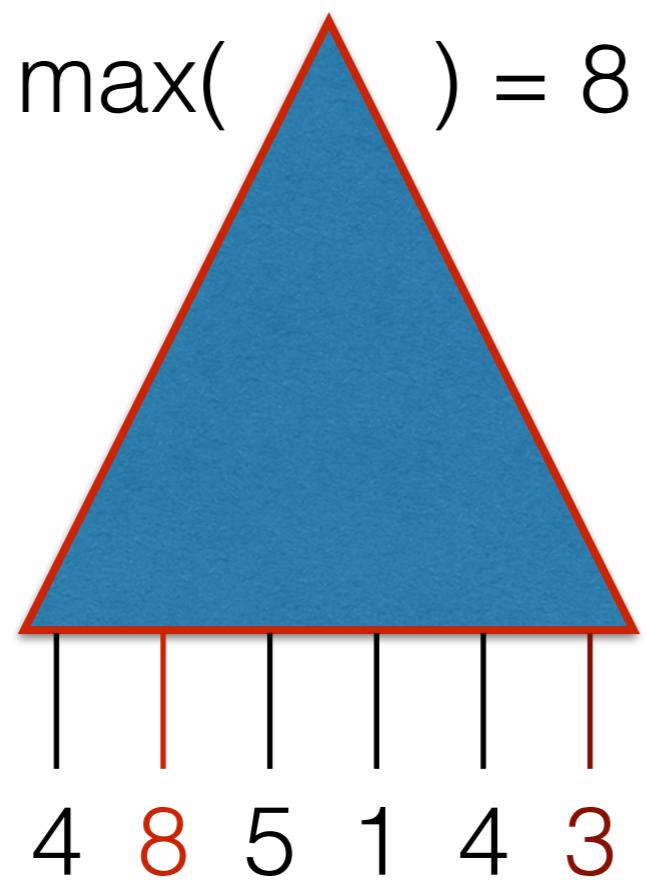
T1 9

T2 5



Editing a persistant data structure

Using Memo Tables



Memo

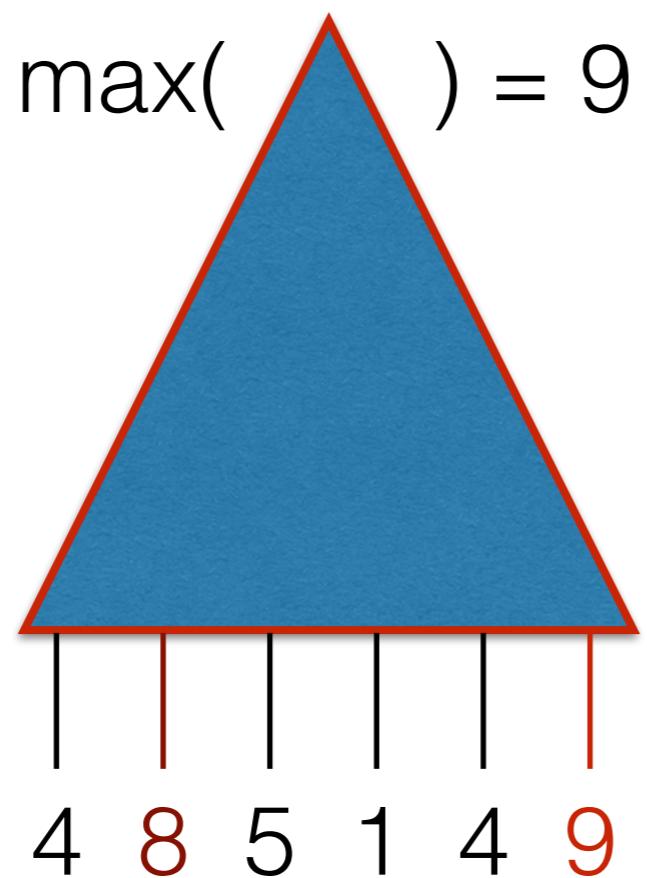
T1 9

T2 5

T3 8

Editing a persistant data structure

Using Memo Tables



Memo

T1 9

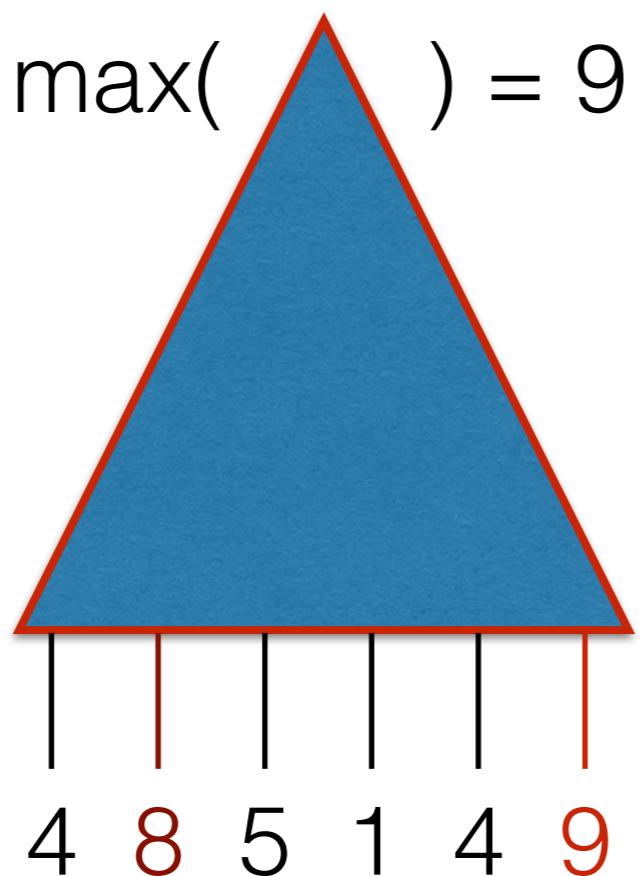
T2 5

T3 8

T4 9

Editing a persistant data structure

Using Memo Tables



Memo

T1 9

T2 5

T3 8

T4 9

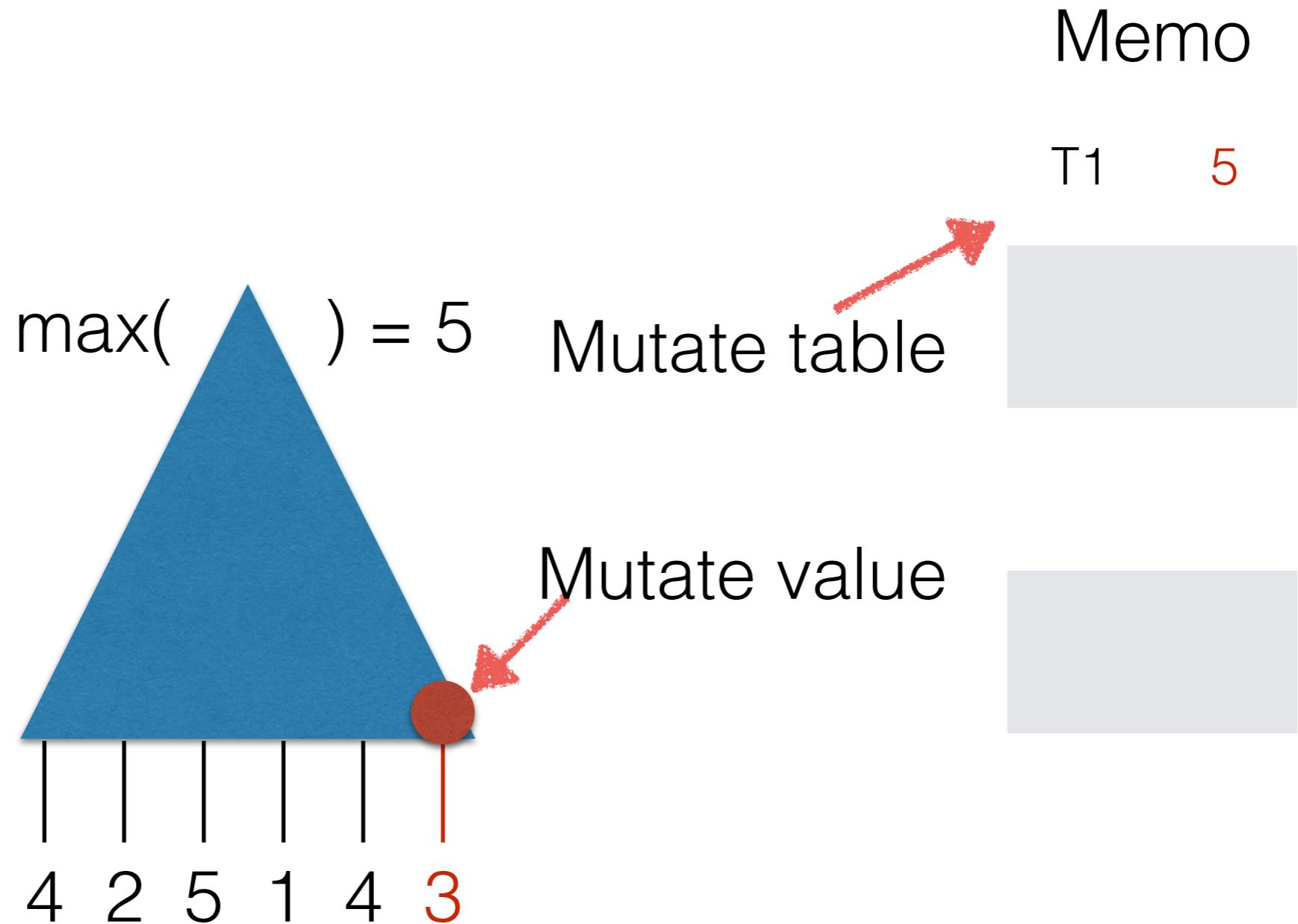
New entry for
each change



Using Memo Tables

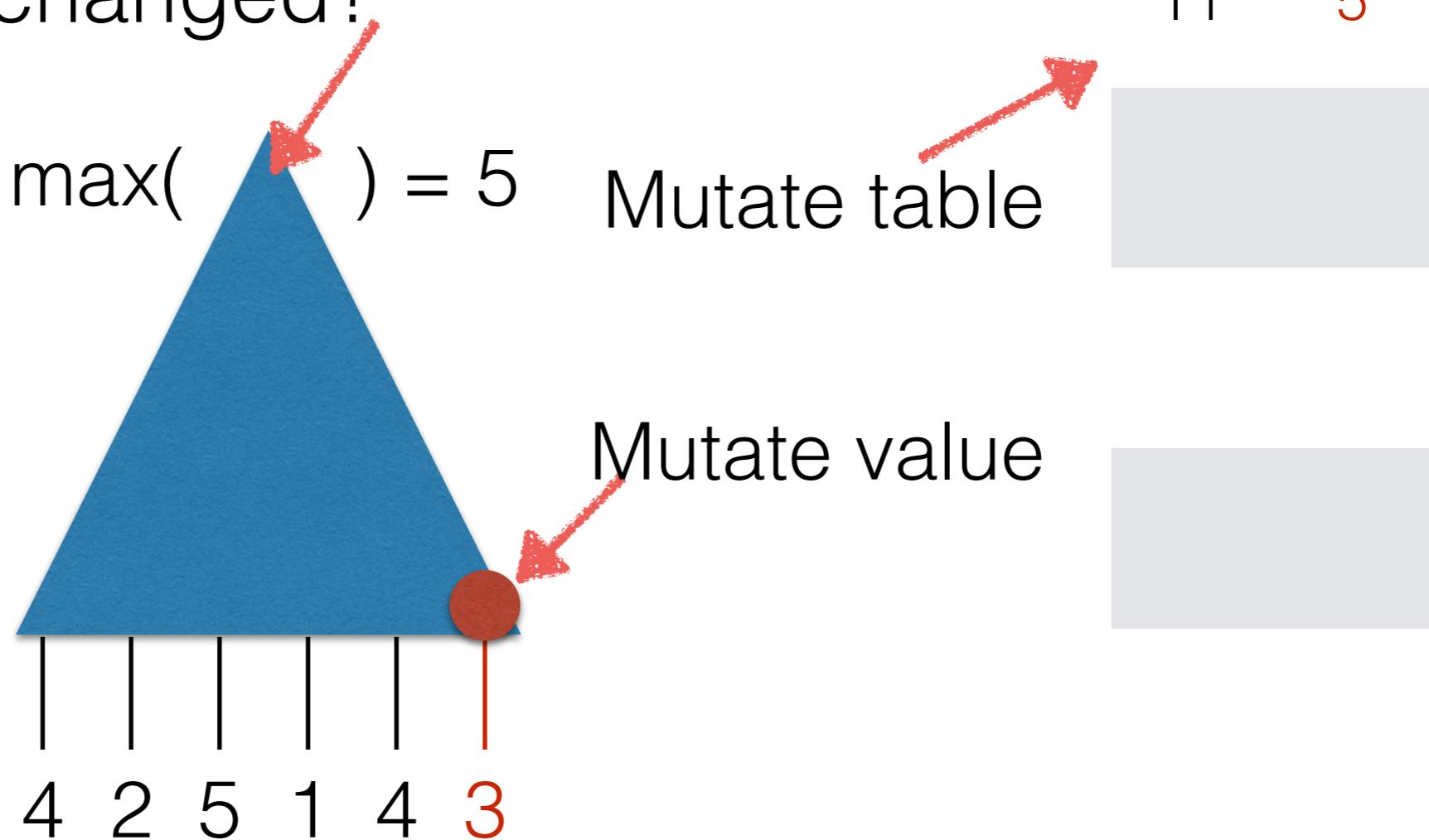
Lets try mutating values

Using Memo Tables



Using Memo Tables

But how do we know that
the computation result at
the root changed?



Language-based Incremental Computation

Reason about the non-incremental computation

Make calls to library functions for data access

Internally, the library makes use of:

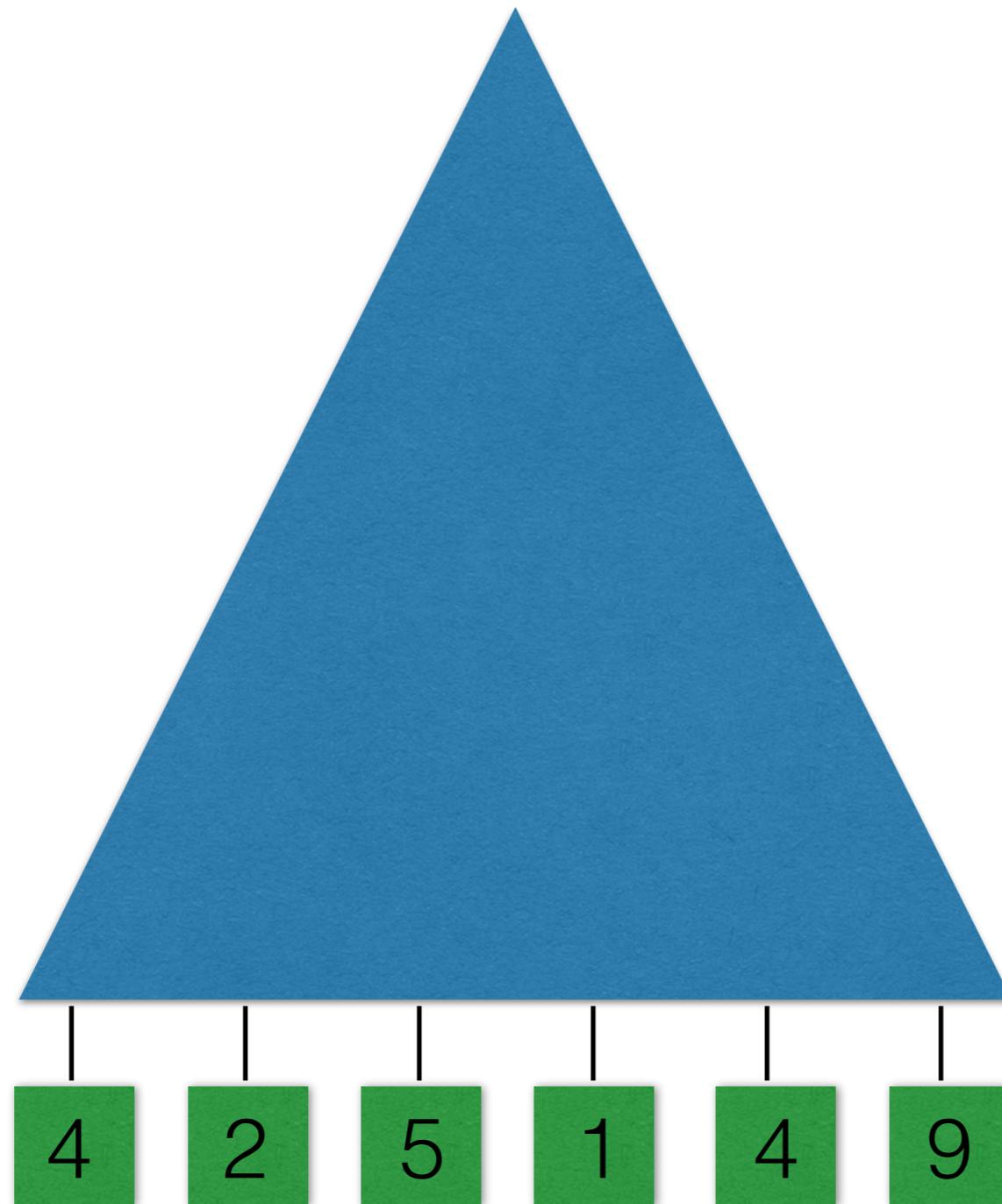
Cached values

Dependency graphs

Dependency Graphs

4

Input Cell



Dependency Graphs

4

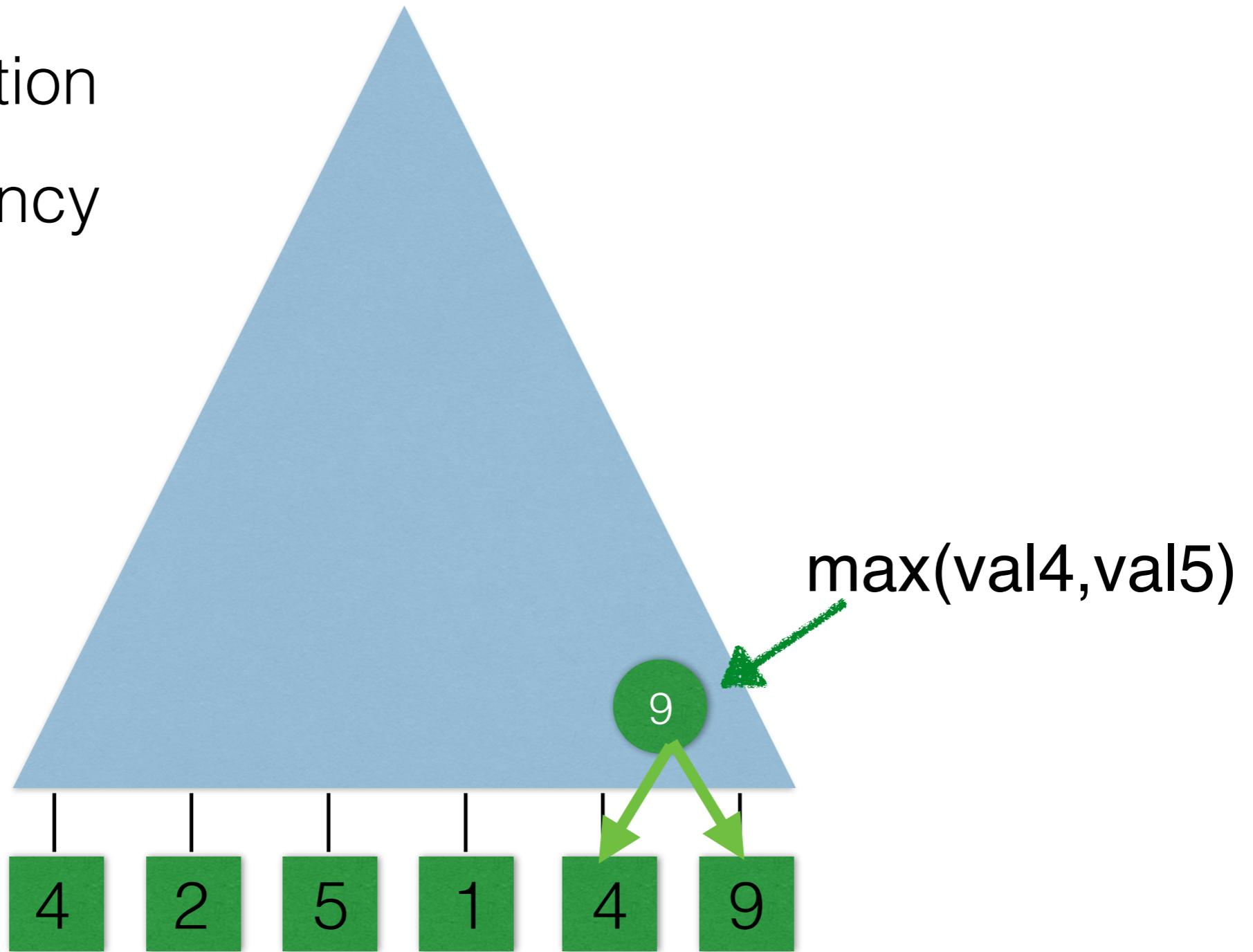
Input Cell

9

Computation



Dependency



Dependency Graphs

4

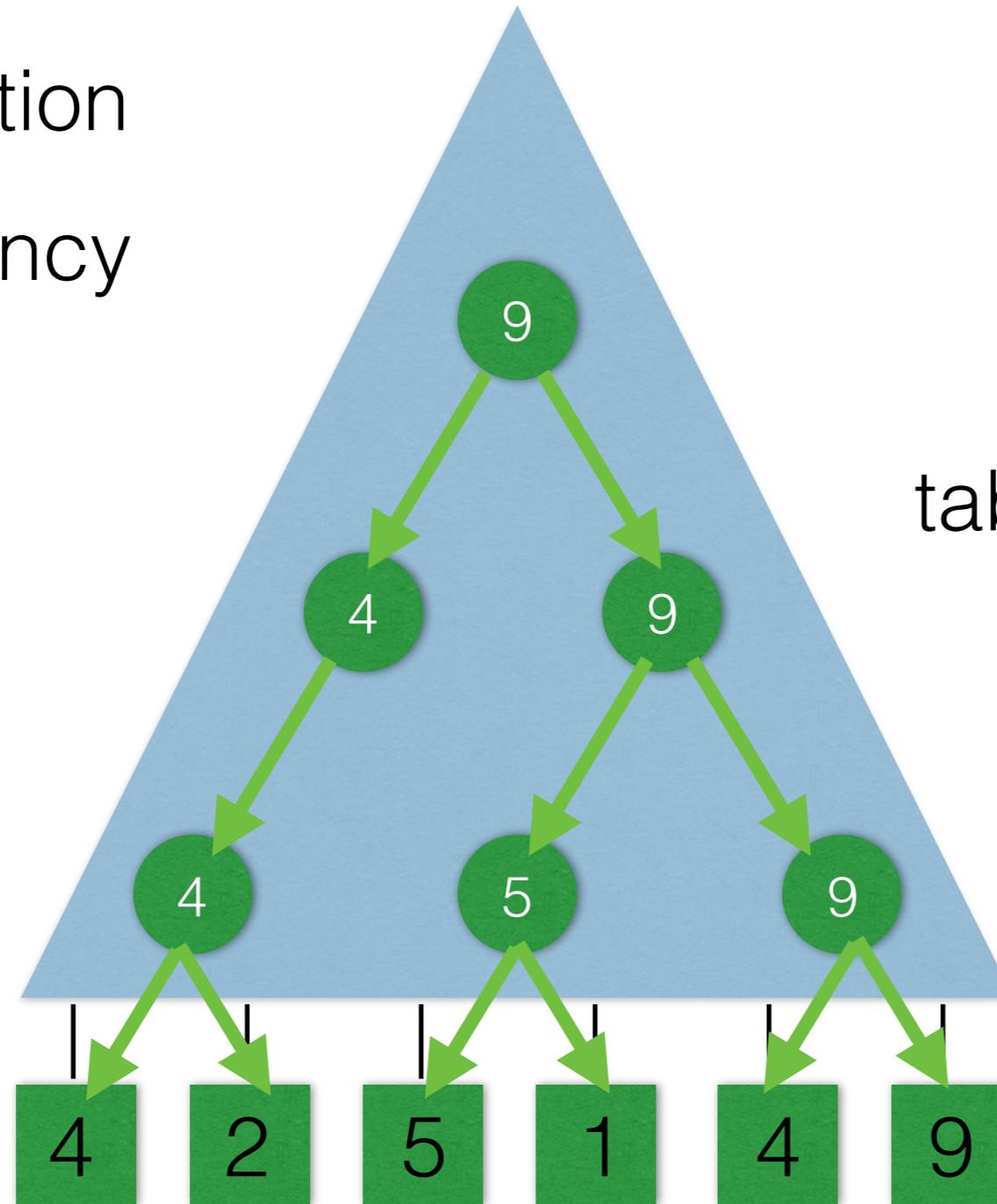
Input Cell

9

Computation



Dependency



With one memo
table entry per thunk

Dependency Graphs

4

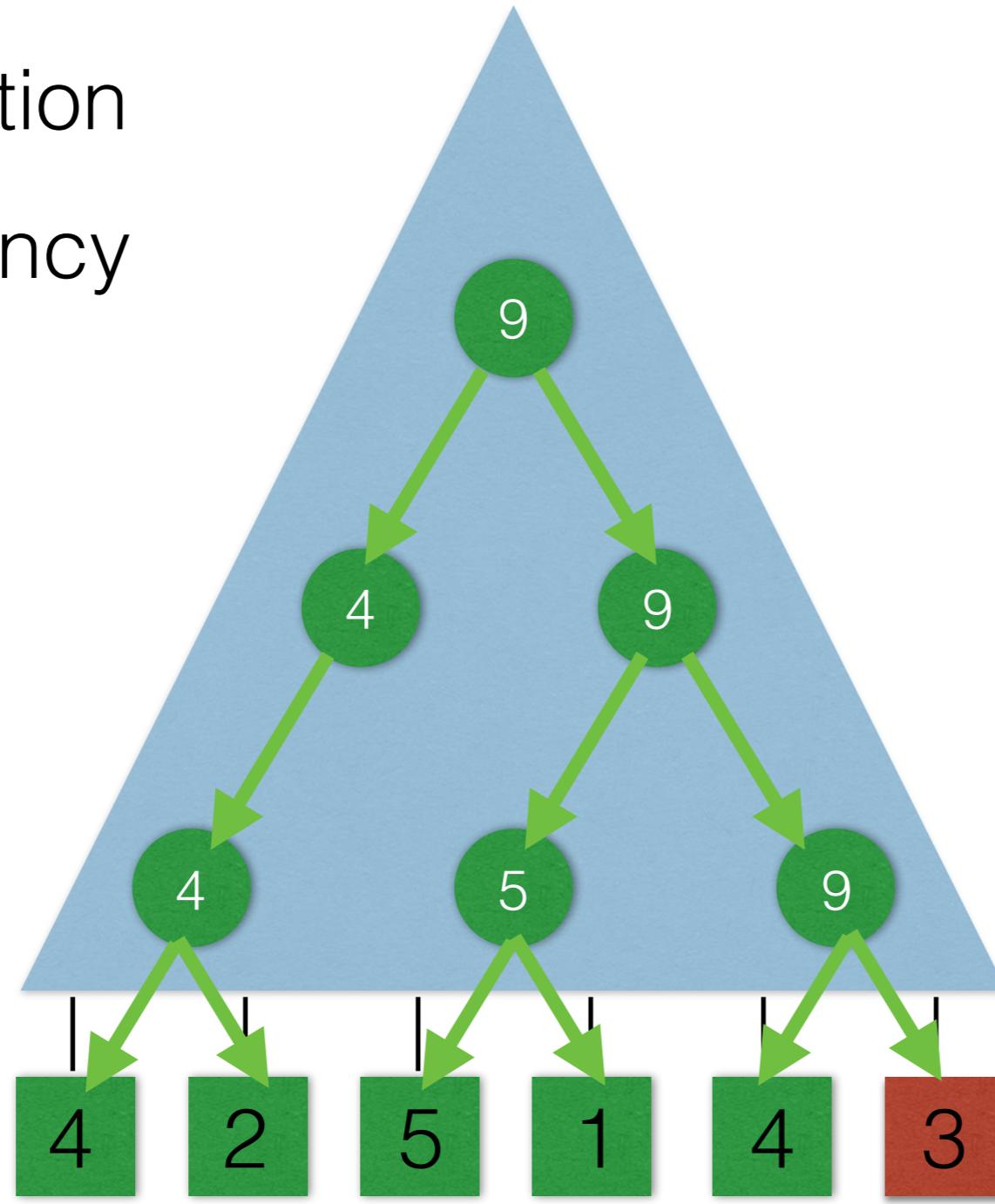
Input Cell

9

Computation



Dependency



Dependency Graphs

4

Input Cell

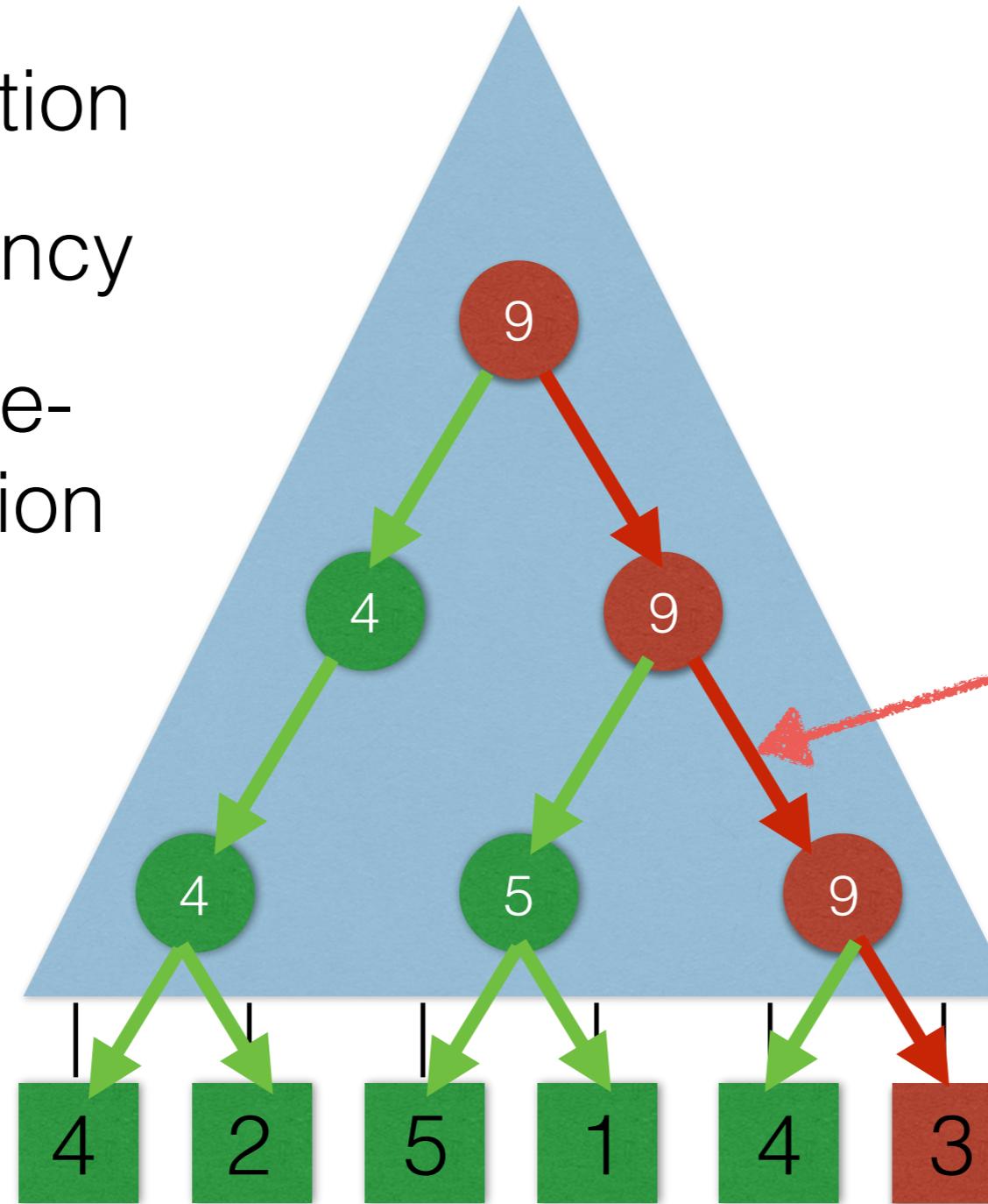
9

Computation

→ Dependency

9

Needs Re-computation



Transitively
mark graph as
“dirty”

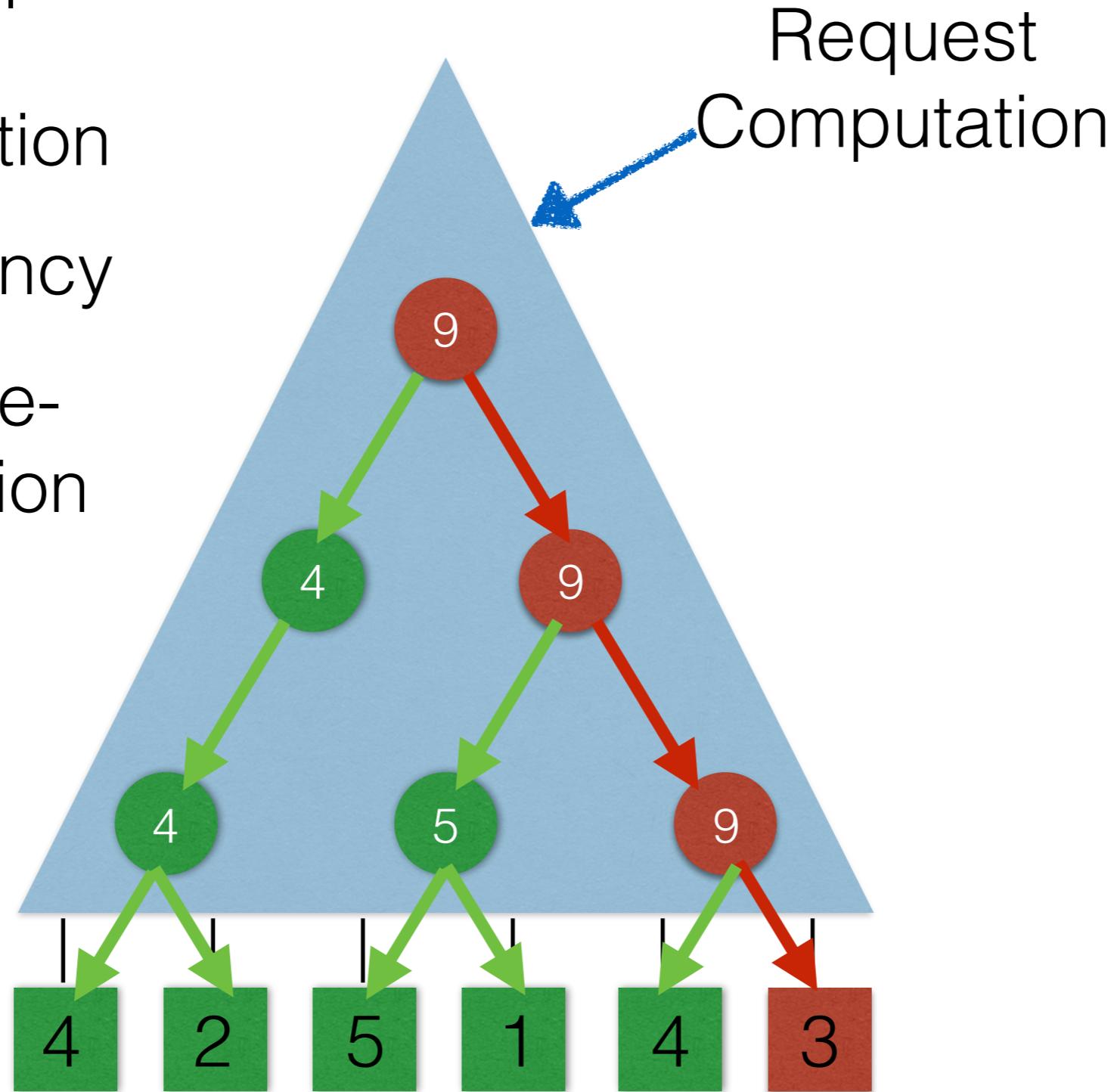
Dependency Graphs

4 Input Cell

9 Computation

→ Dependency

→ Needs Re-computation



Dependency Graphs

4

Input Cell

9

Computation

→ Dependency

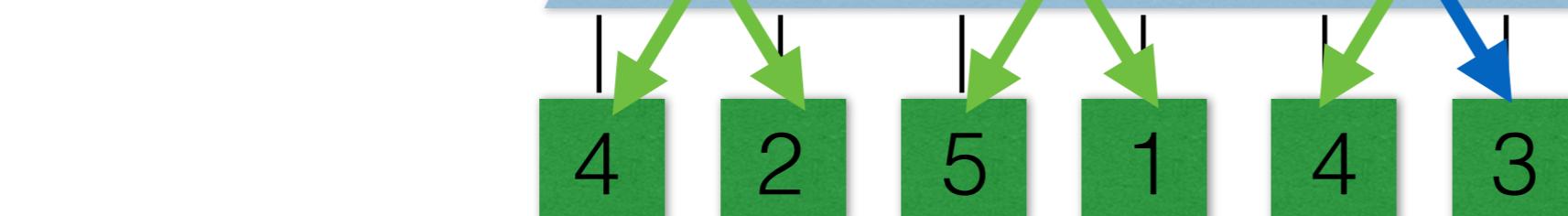
→ Needs Re-

9

computation

9

Transitive Re-



Graph is
“cleaned”

Dependency Graphs

4

Input Cell

9

Computation

→ Dependency

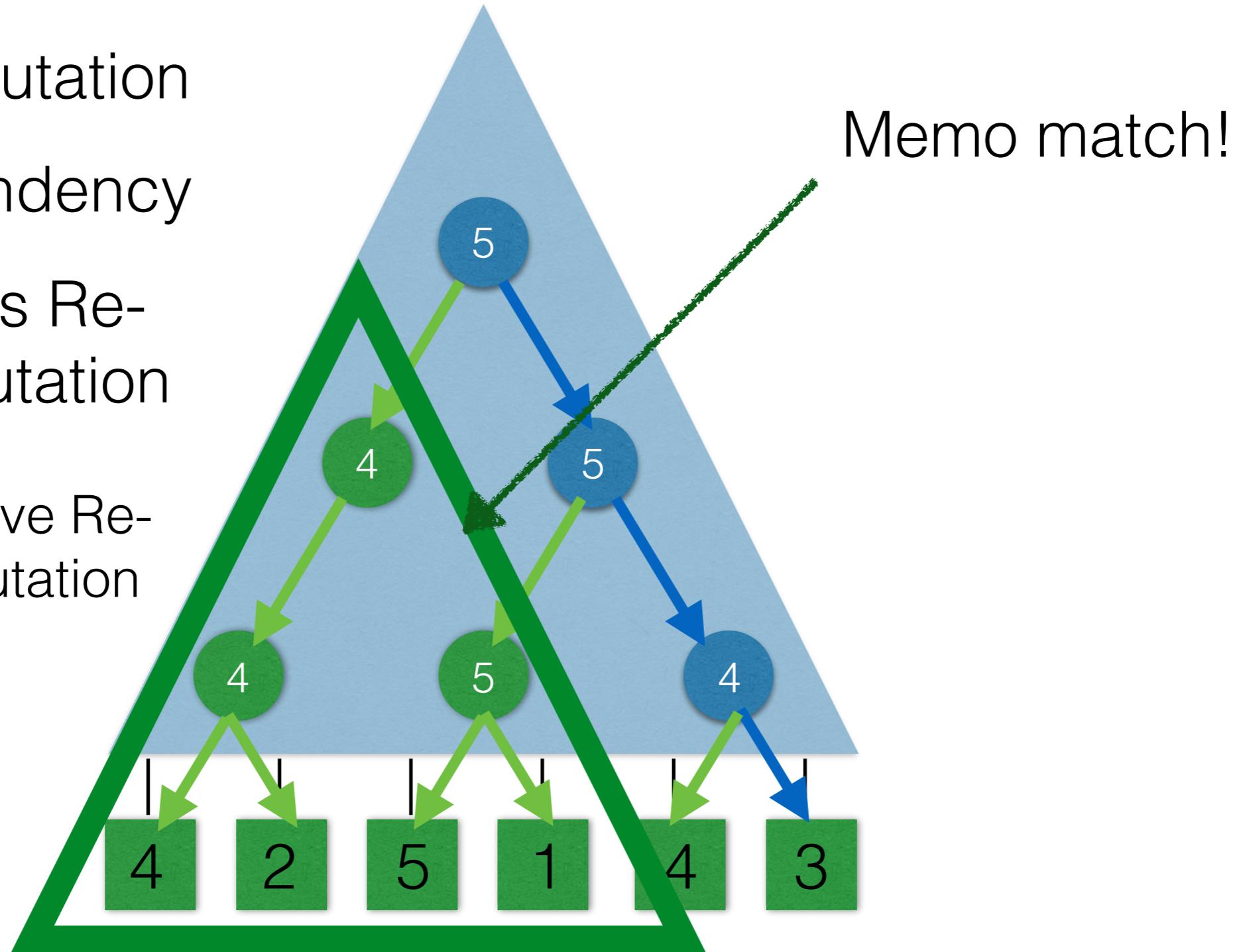
→ Needs Re-computation

9

computation

9

Transitive Re-computation



Research challenge

How do we advance the use of incremental computation, further simplifying code creation and providing speedups over realistic code?

Speed Problem

How do we advance the use of incremental computation, further simplifying code creation and providing speedups over realistic code?

www.rust-lang.org



[Documentation](#)

[Install](#)

[Community](#)

[Cont](#)

Rust is a systems programming language that runs blazingly fast, prevents segfaults, and guarantees thread safety.
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Inst

Speed Problem

Incremental performance improved

Non-Incremental performance improved a lot more

Speed Problem

Computers are better at adding and subtracting numbers than walking through memory

Speed Problem

Computers are better at adding and subtracting numbers than walking through memory

Incremental computation libraries manage a lot of memory

Simplicity Problem

How do we advance the use of incremental computation, further simplifying code creation and providing speedups over realistic code?

Simplicity Problem

Memo tables reduce computations

Simplicity Problem

Memo tables reduce computations

Dependency graphs hide the memo tables

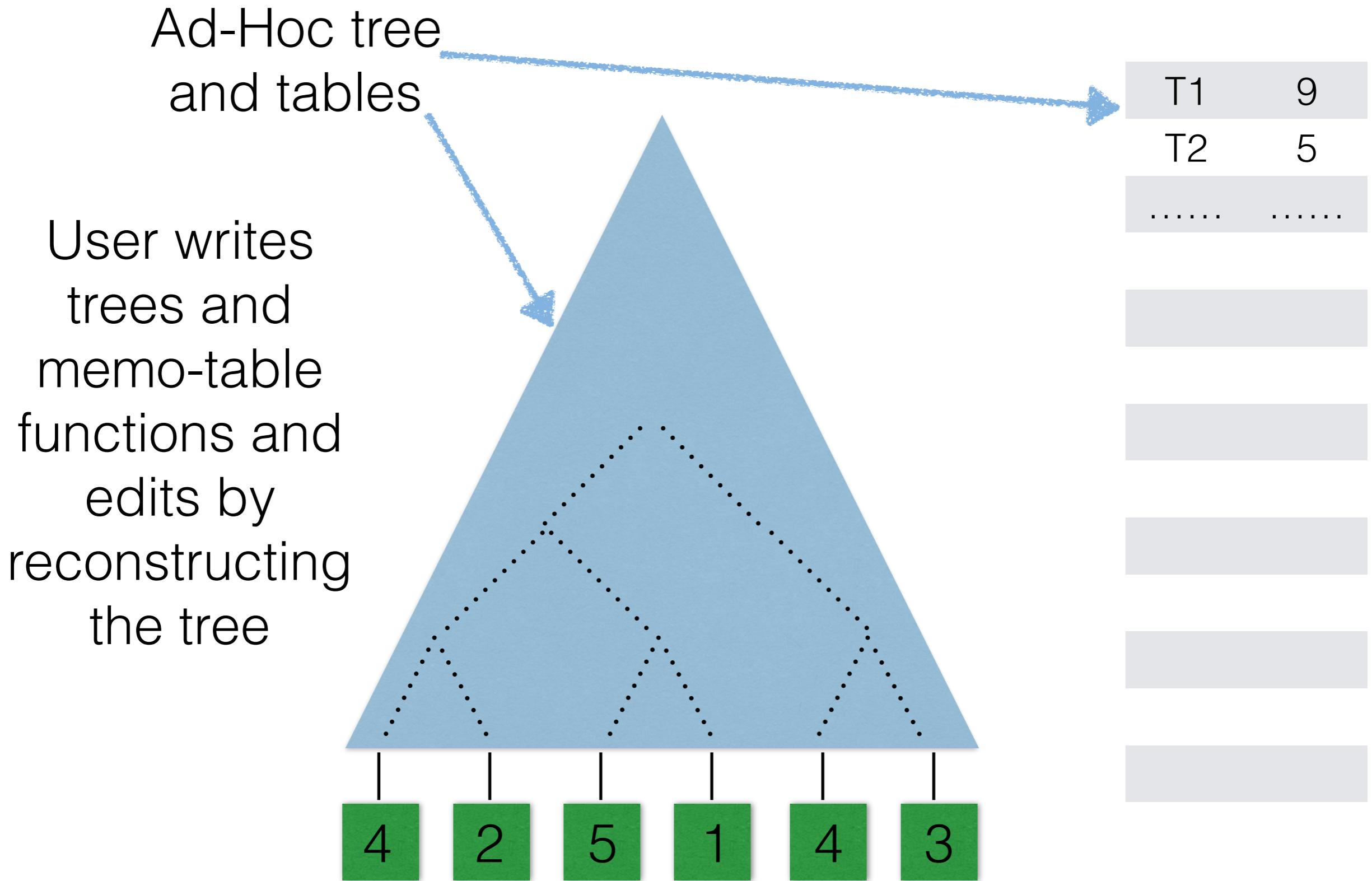
Simplicity Problem

Memo tables reduce computations

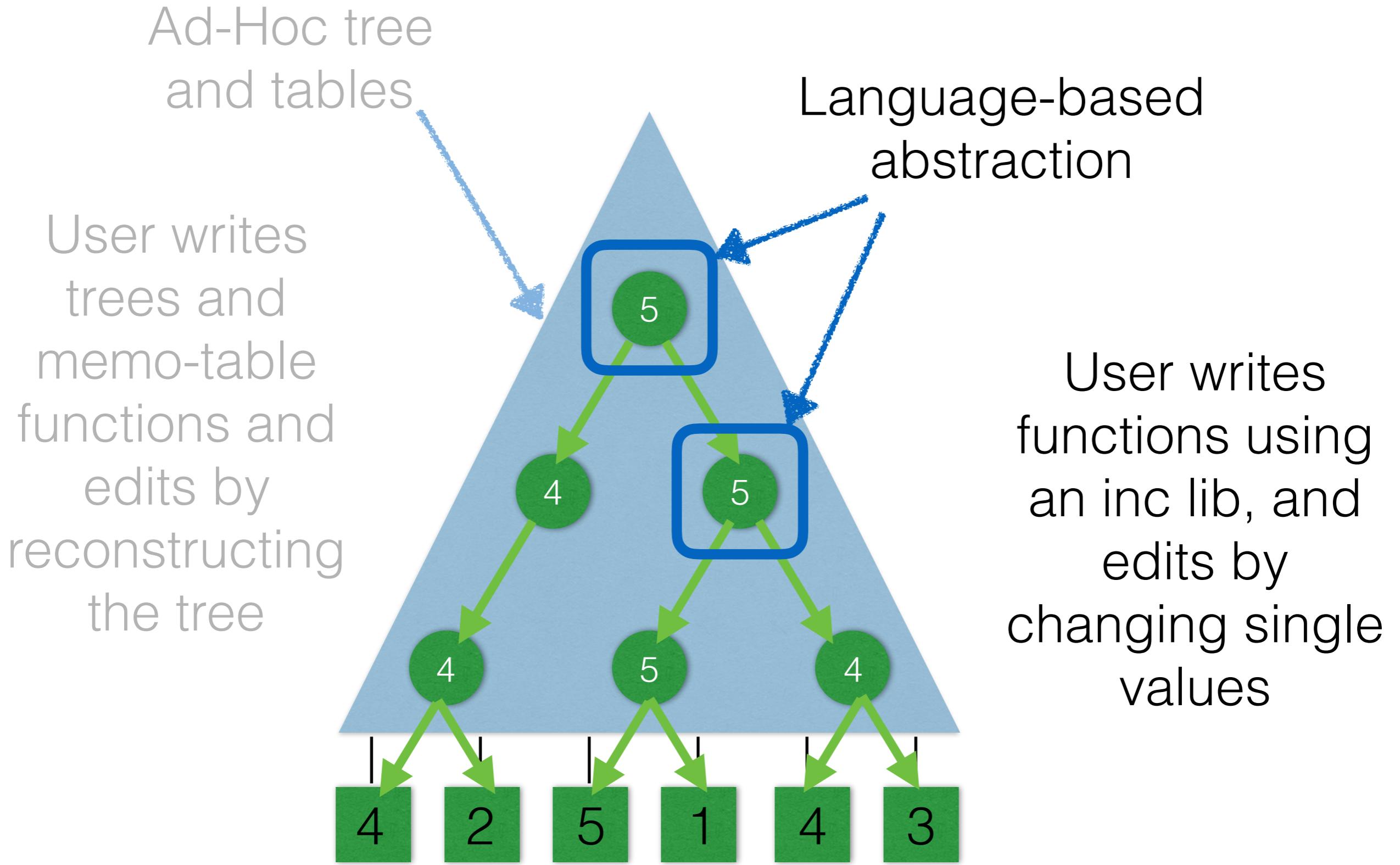
Dependency graphs hide the memo tables

We want a way to generate graphs of similar computations

Simplicity Problem



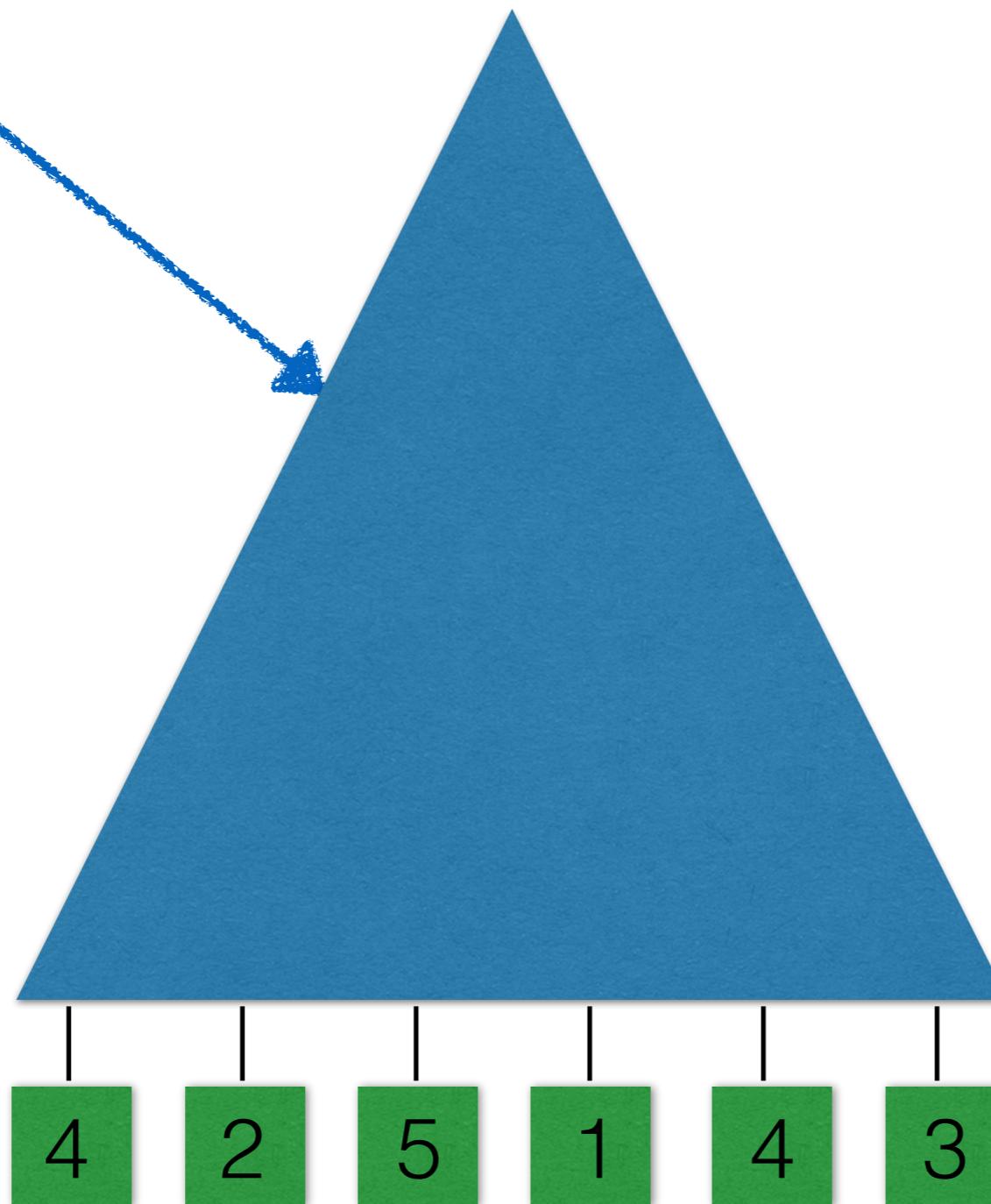
Simplicity Problem



Simplicity Problem

Data collections abstraction

`seq.fold_up(max) = 5`



iodyn

Incremental Collections Library
Based on Adapton

github.com/cuplv/iodyn.rust

rust crate: iodyn

IRaz - Incremental sequences
(Giraz?, Sigraz?)

In progress: Tries, Graphs

Giraz

Incremental Sequence data structure
Based on Adapton

Based on RAZ data structure

Includes incremental functions

Insert

fold_up -

tree fold, compute at leaf and
binary nodes

Delete

fold_lr -

list fold, compute at each element

Move cursor

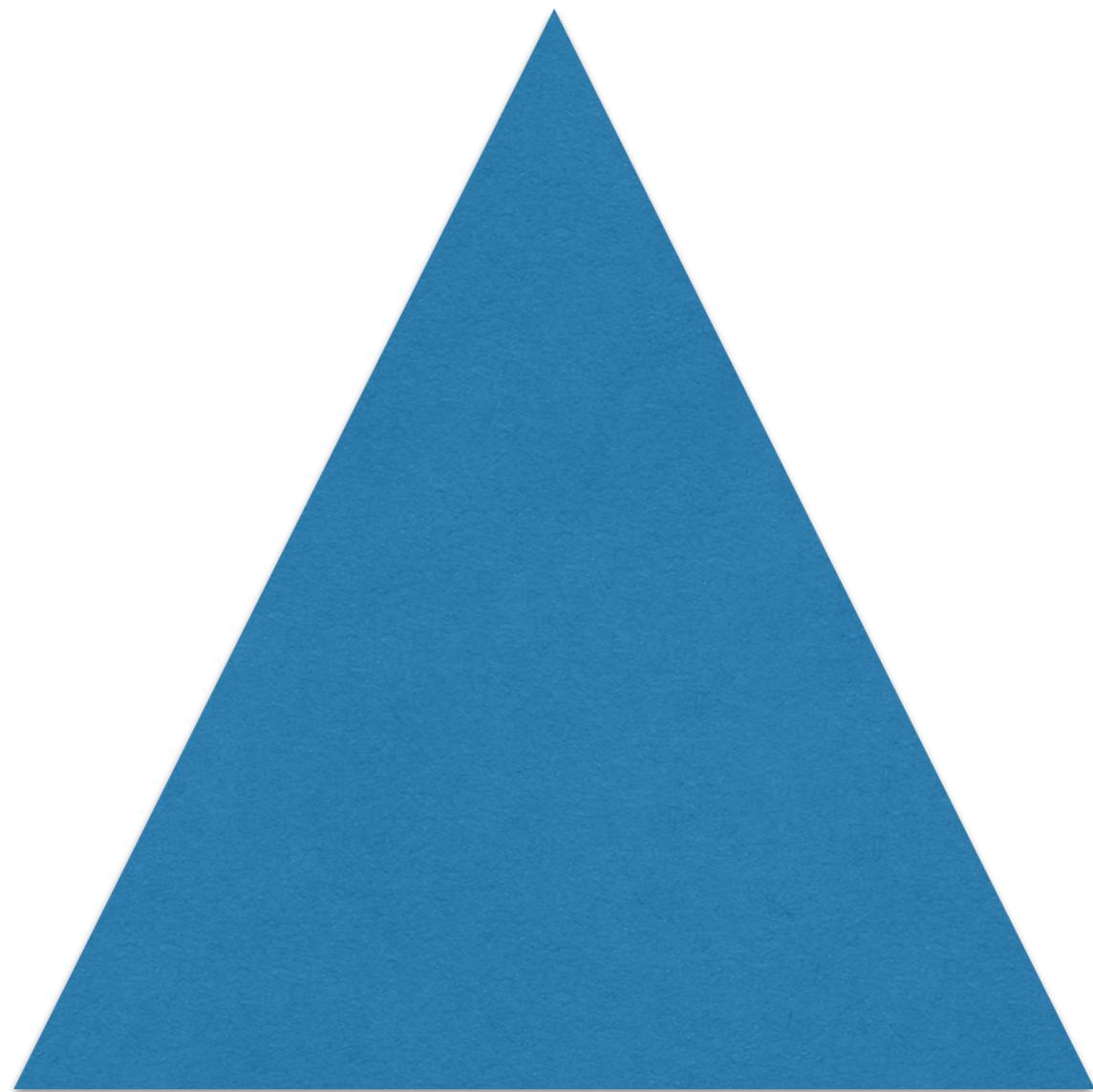
map -

maintain structure and transform each
element

What is a Giraz?

Sequence: 

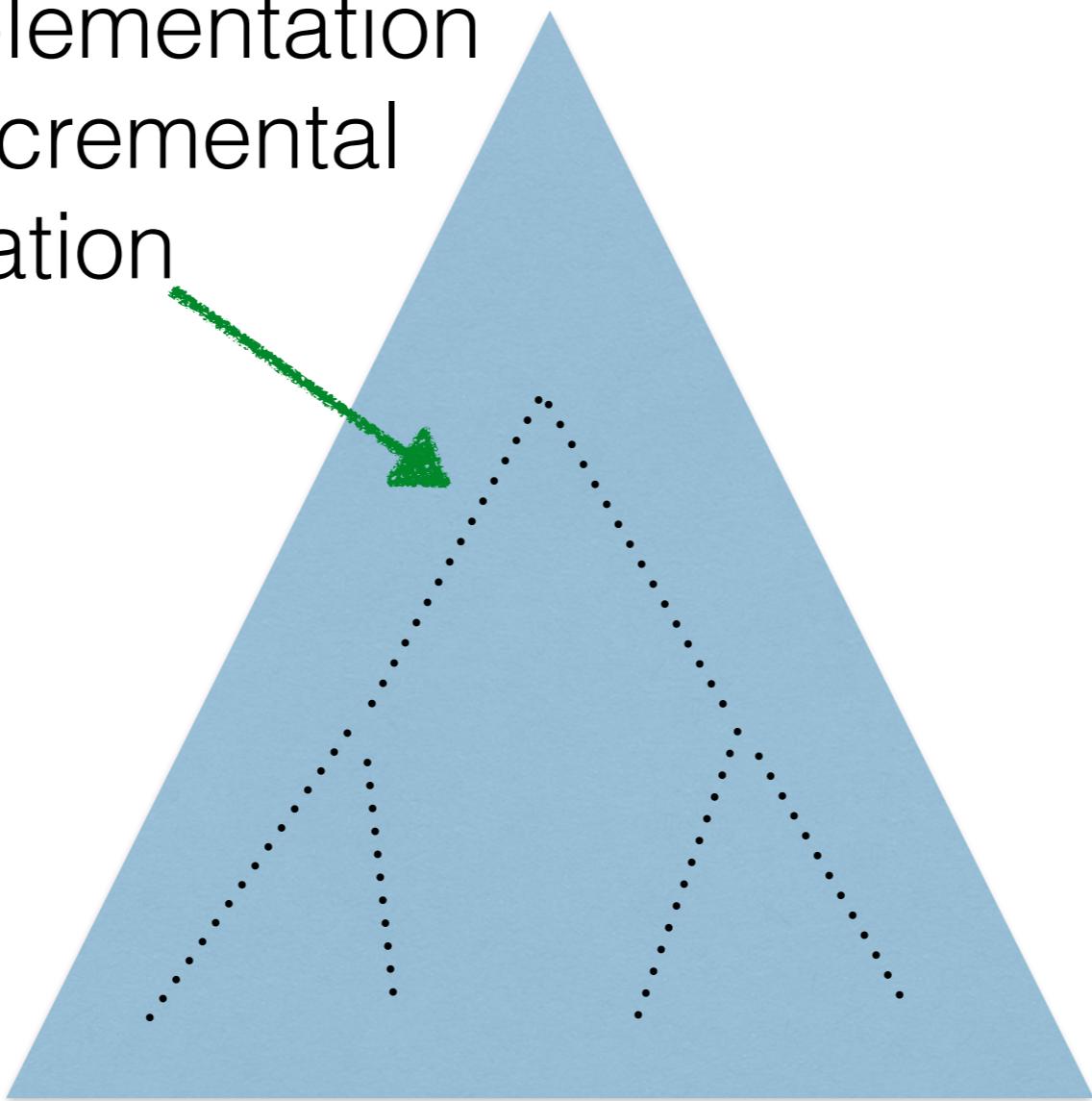
What is a Giraz?

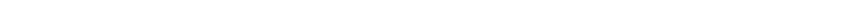


Sequence: ■■■ ■■■■ ■■ ■■■■■

What is a Giraz?

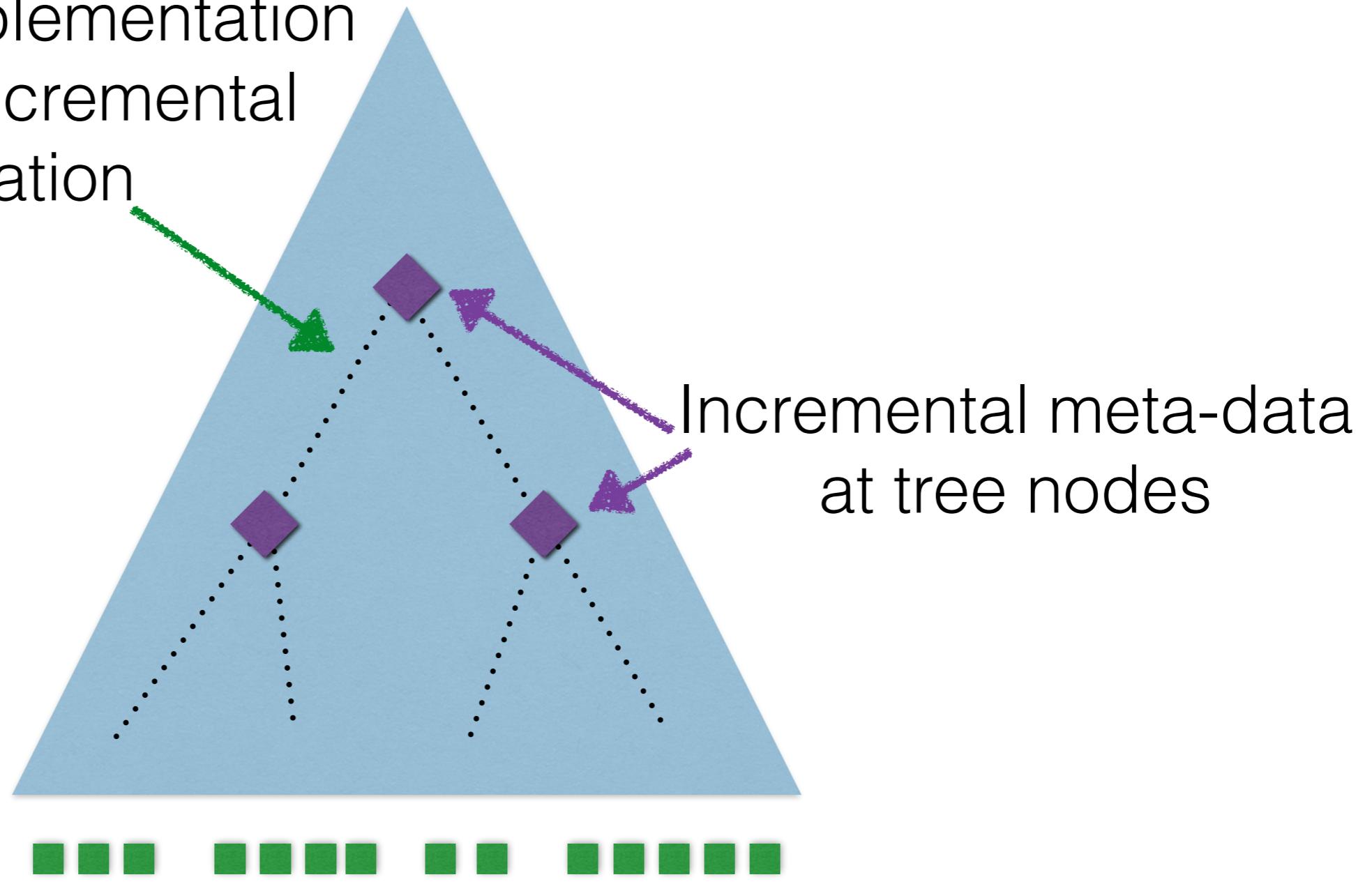
Tree-based implementation for efficient incremental computation



Sequence: 

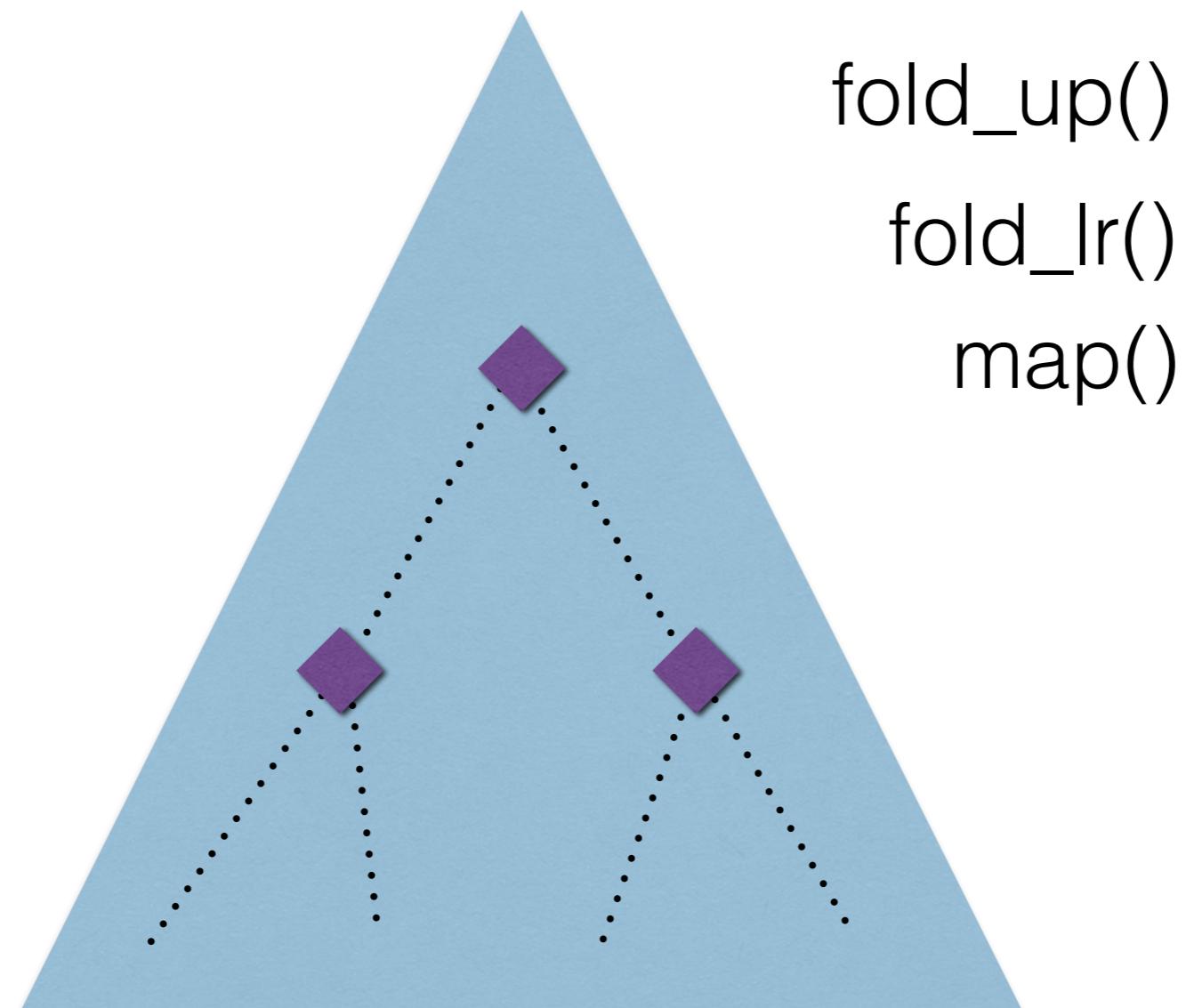
What is a Giraz?

Tree-based implementation
for efficient incremental
computation



What is a Giraz?

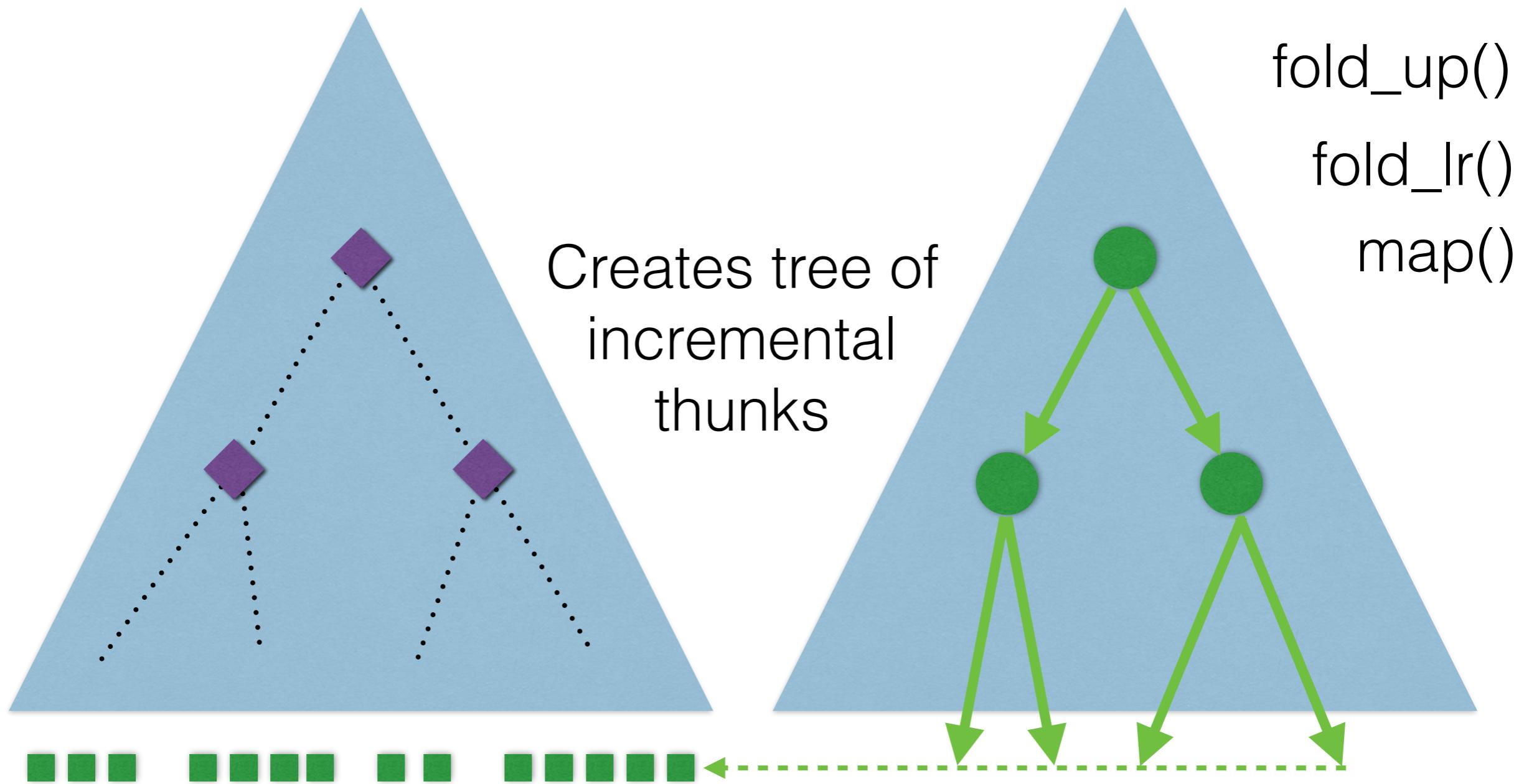
Higher-order collections combinator



Sequence: ■■■ ■■■■ ■■ ■■■■■■

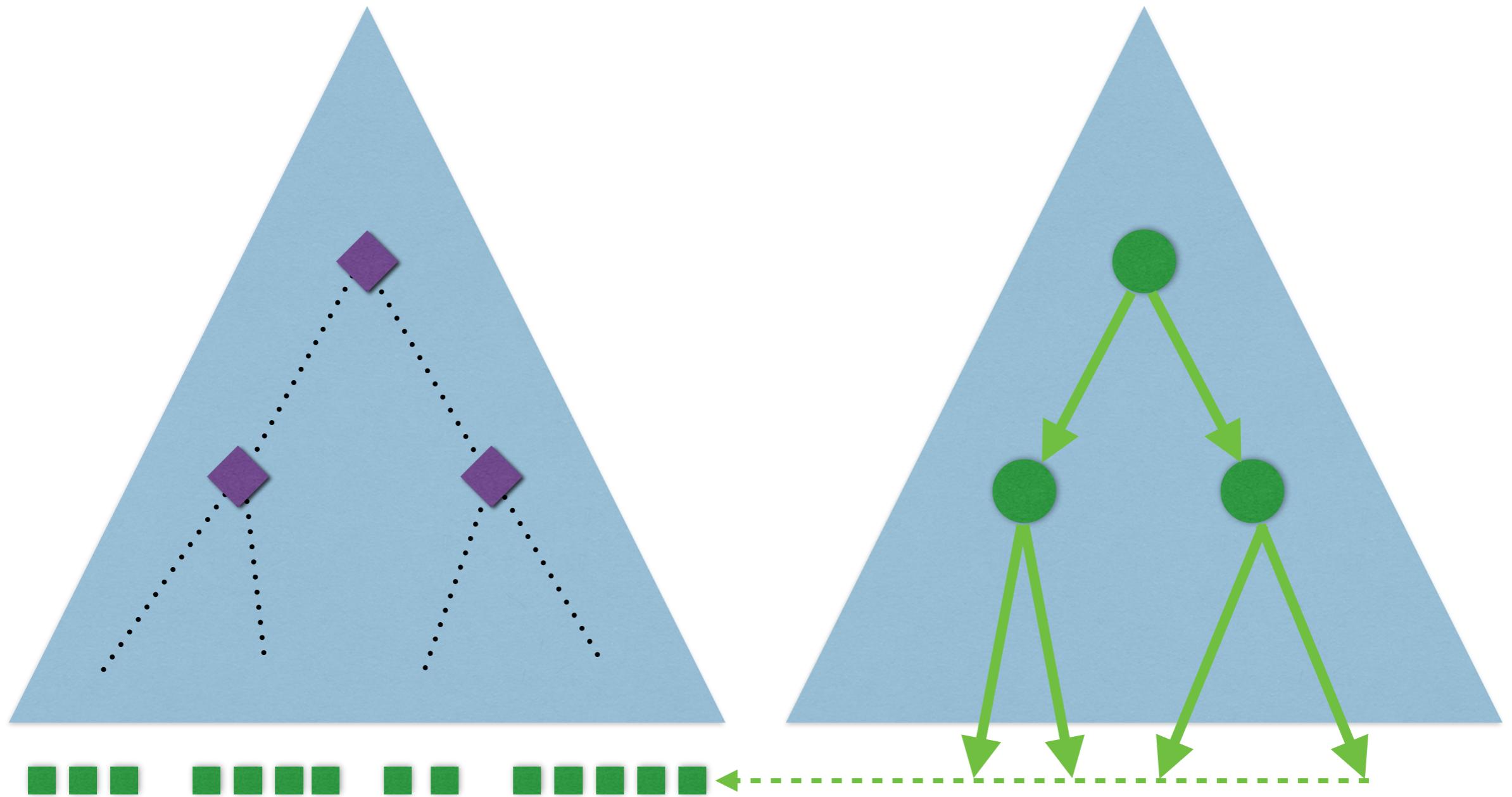
What is a Giraz?

Higher-order collections combinator



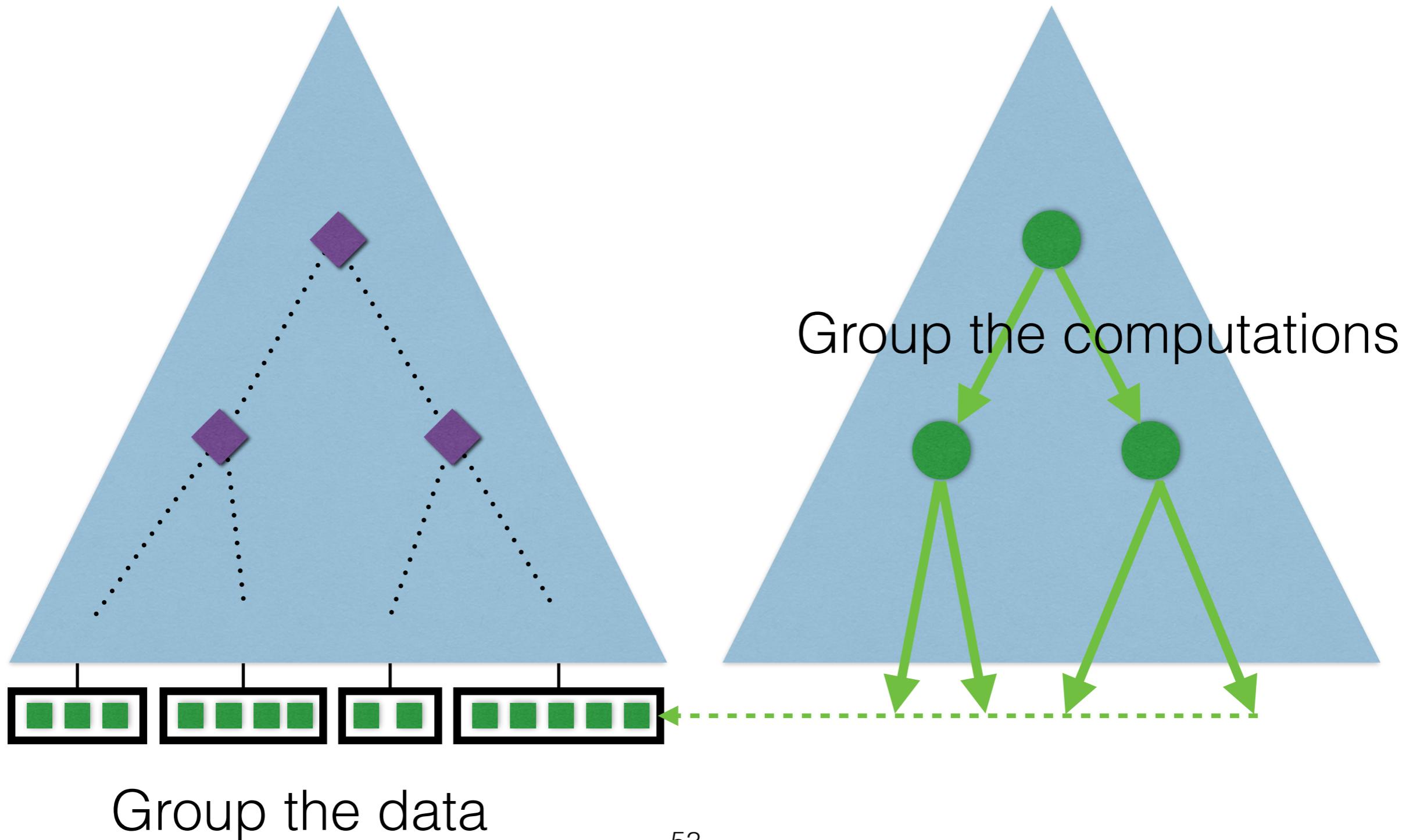
What is a Giraz?

Allows user control over subproblem size



What is a Giraz?

Allows user control over subproblem size



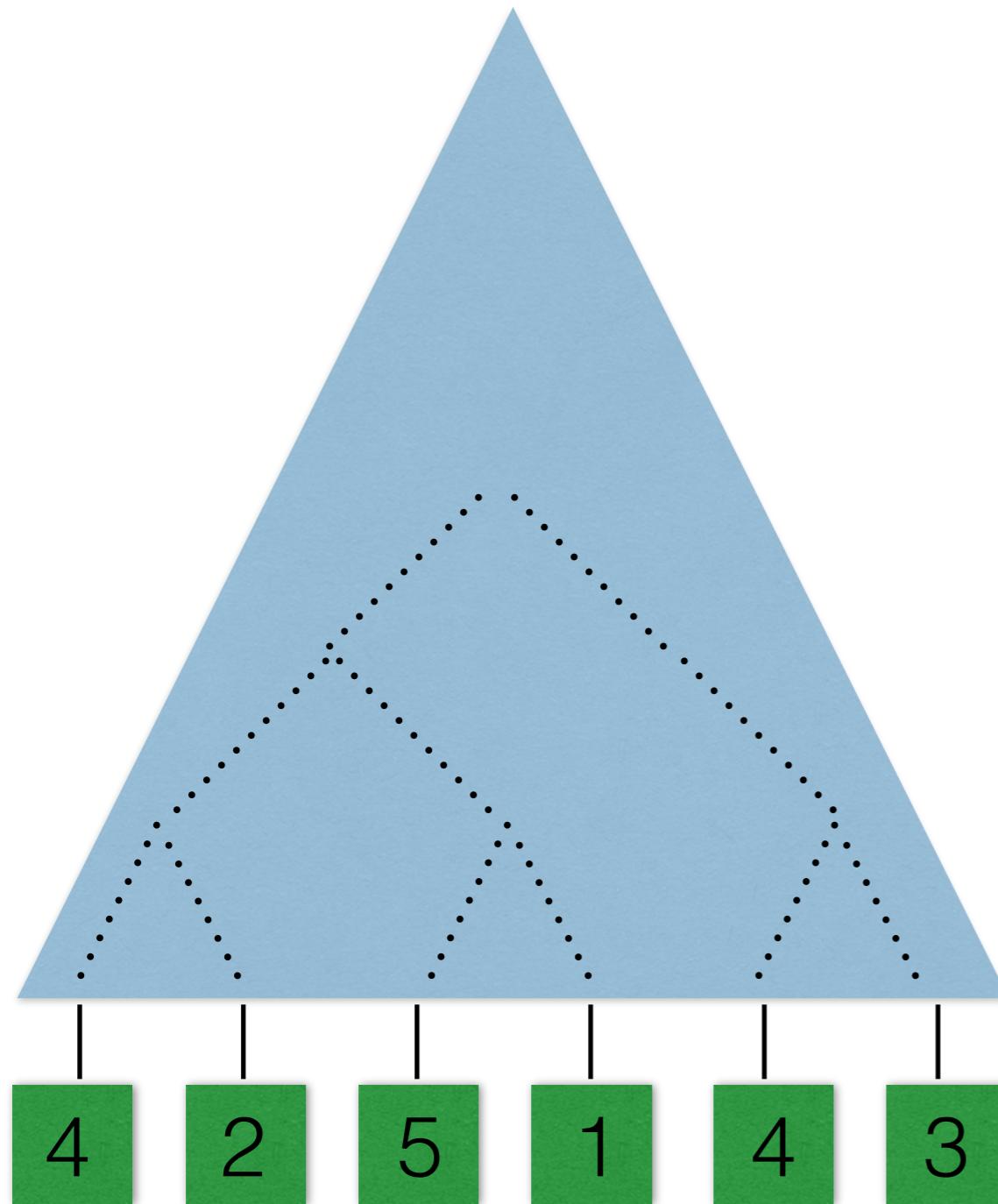
Overhead of incremental computation

Computers are better at adding and subtracting numbers than walking through memory

With optimized code, it's faster to re-compute subproblems than to manage them through our dependency graphs

Overhead of incremental computation

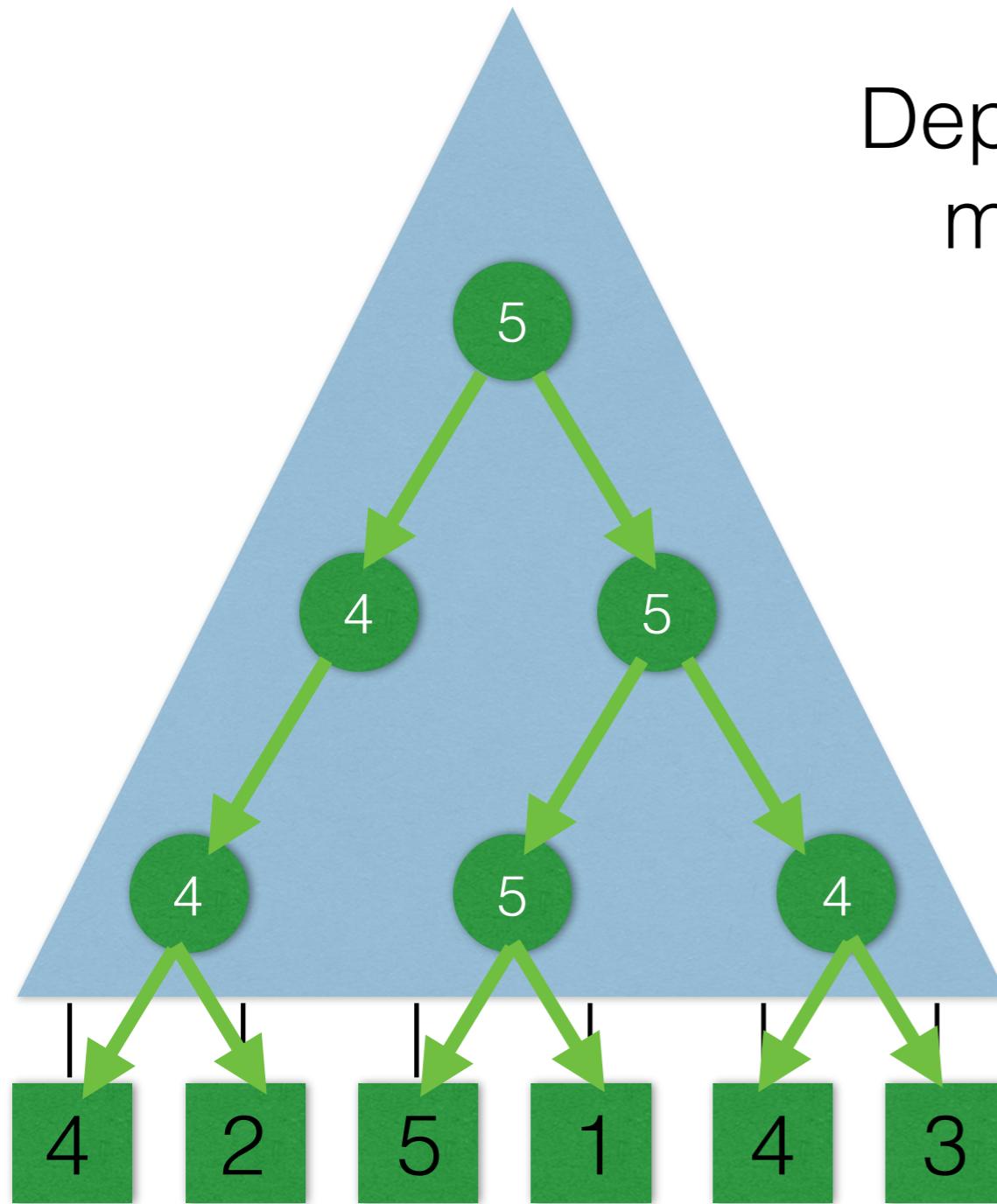
Ad-hoc: many memo-entries per change



Overhead of incremental computation

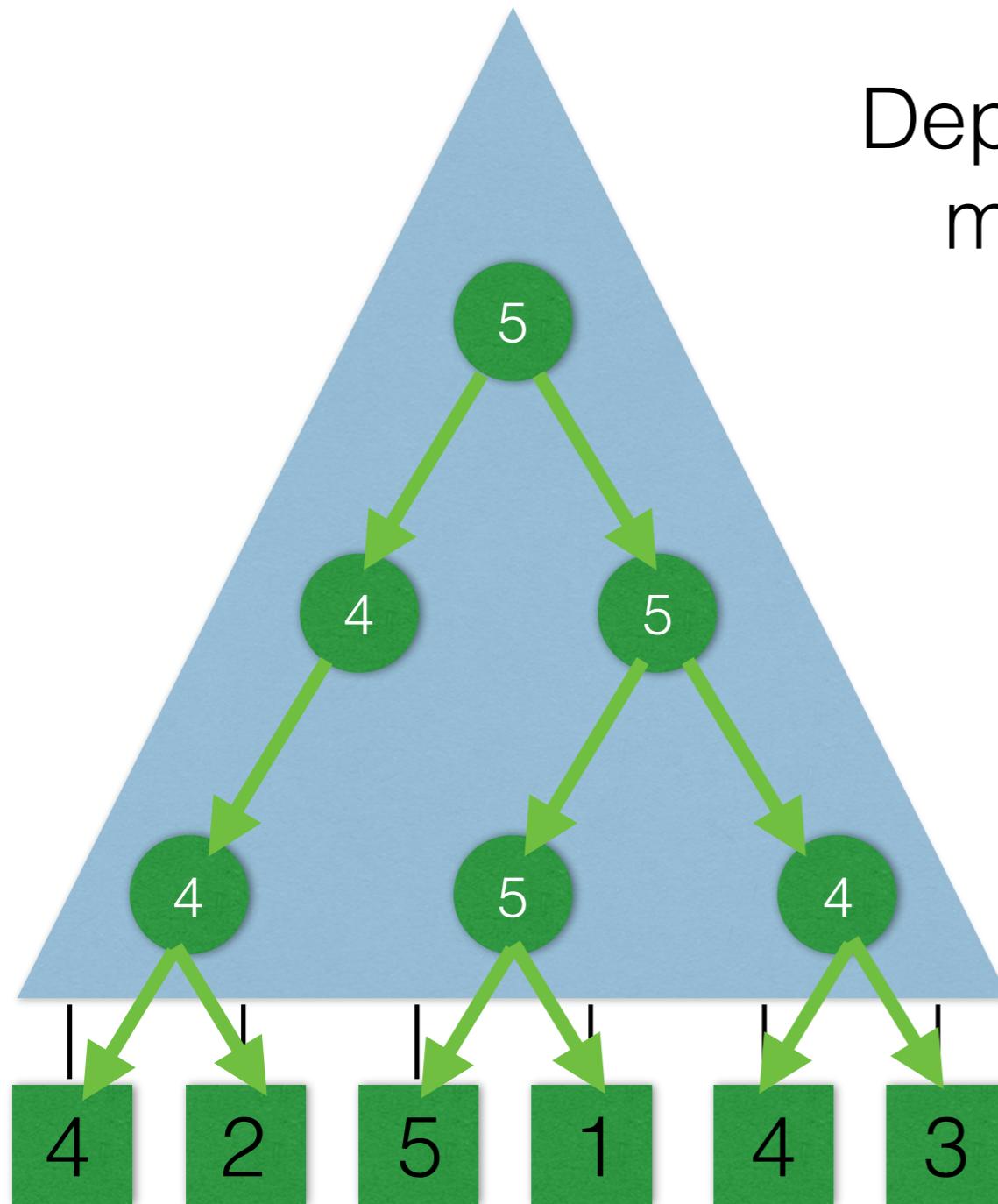
Ad-hoc: many memo-entries per change

Dependency graphs:
memo-entry per computation



Overhead of incremental computation

Ad-hoc: many memo-entries per change

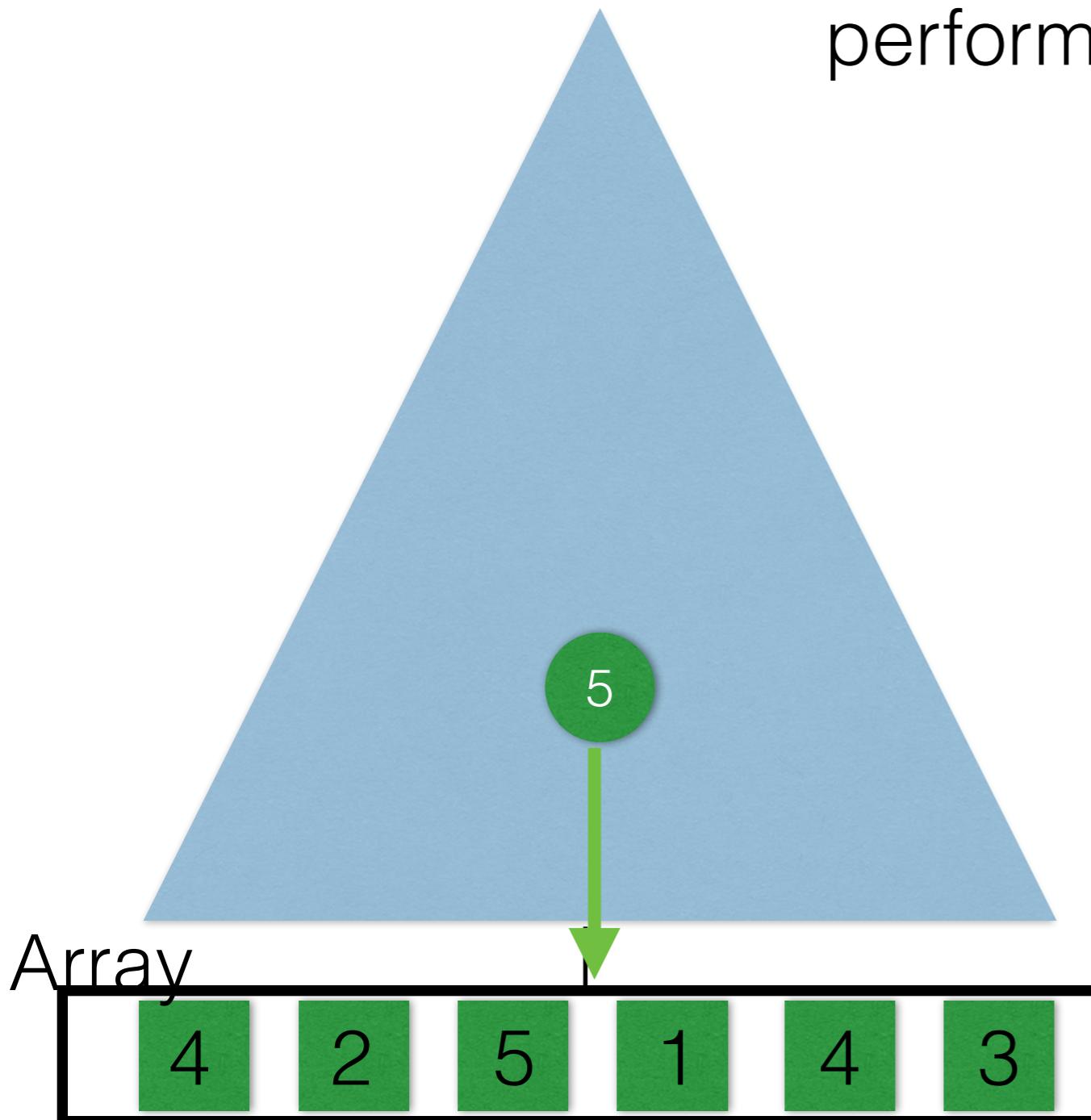


Dependency graphs:
memo-entry per computation

Need to reduce further

Reducing the overhead of incremental computation

One memorized computation is performed over an array of data



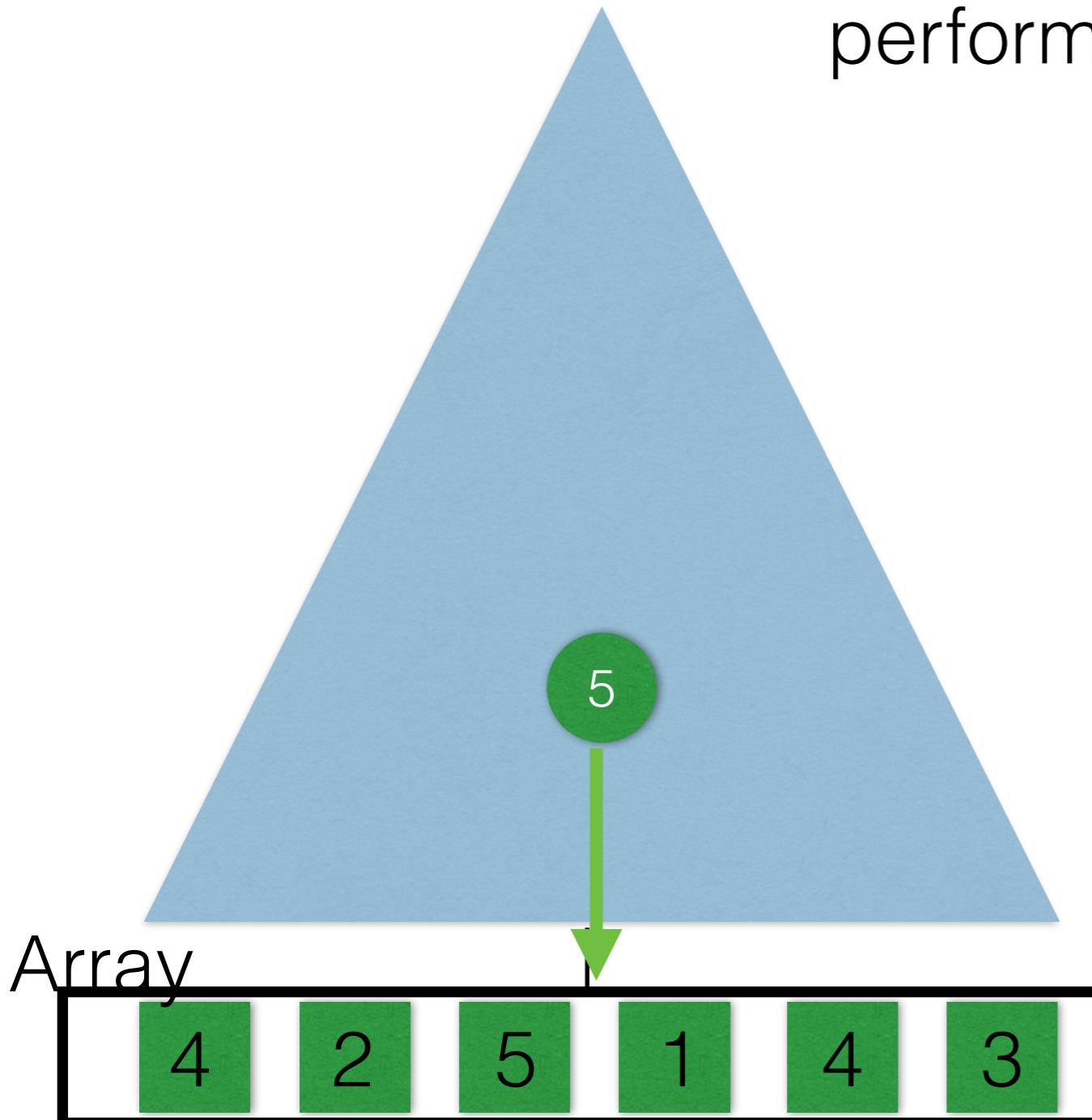
Reducing the overhead of incremental computation

One memorized computation is performed over an array of data

Improve cache coherence

Reduce incremental overhead

Requires management



Reducing the overhead of incremental computation

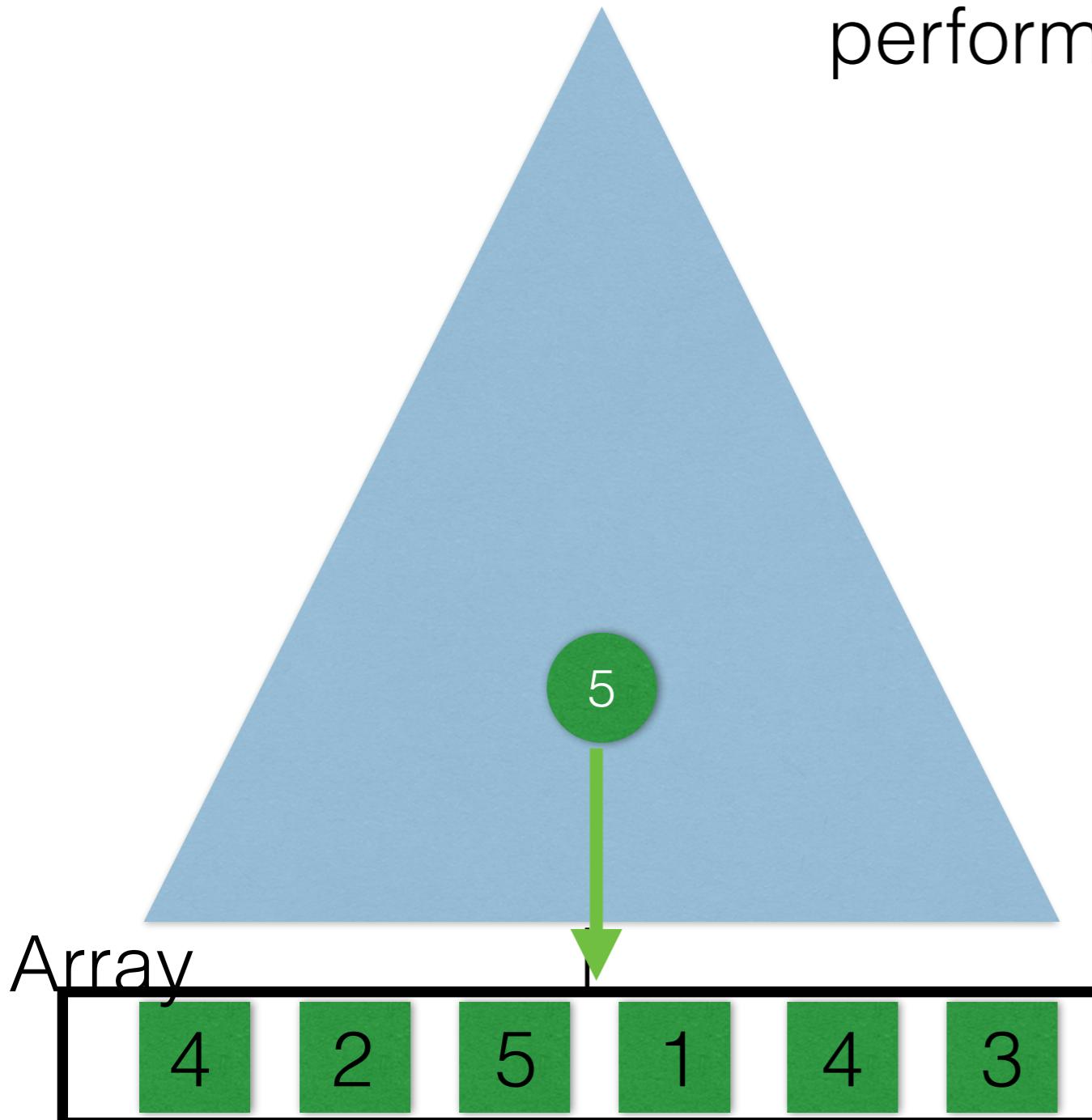
One memorized computation is performed over an array of data

Improve cache coherence

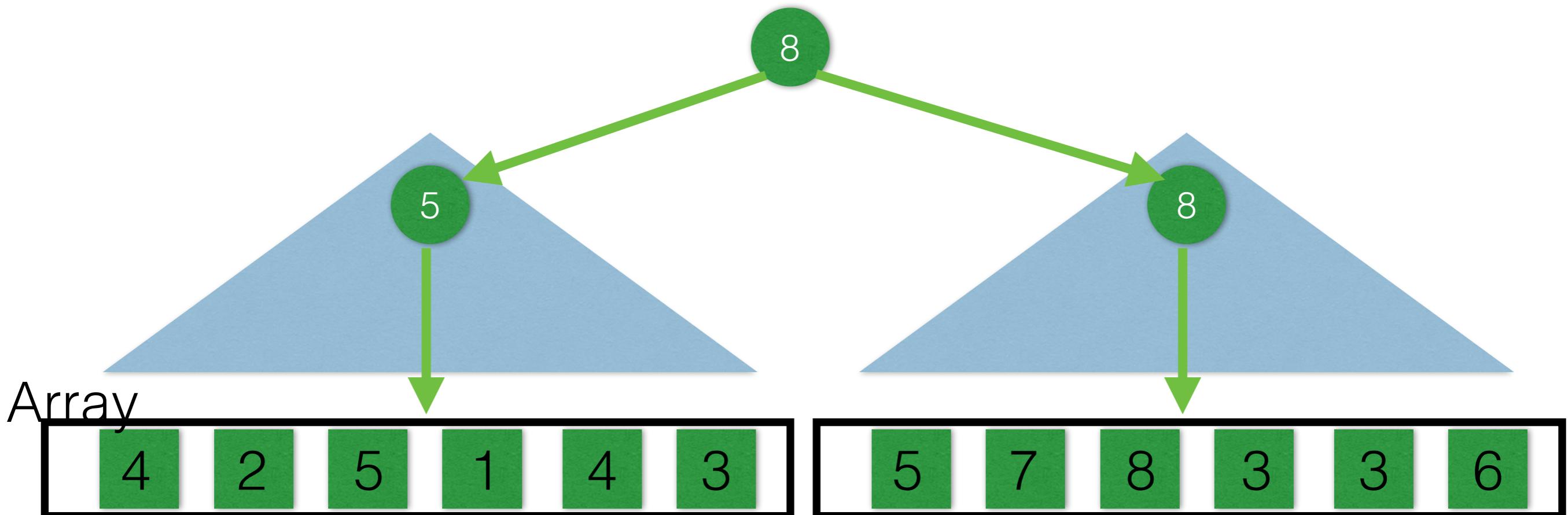
Reduce incremental overhead

Requires management

We can still do more!

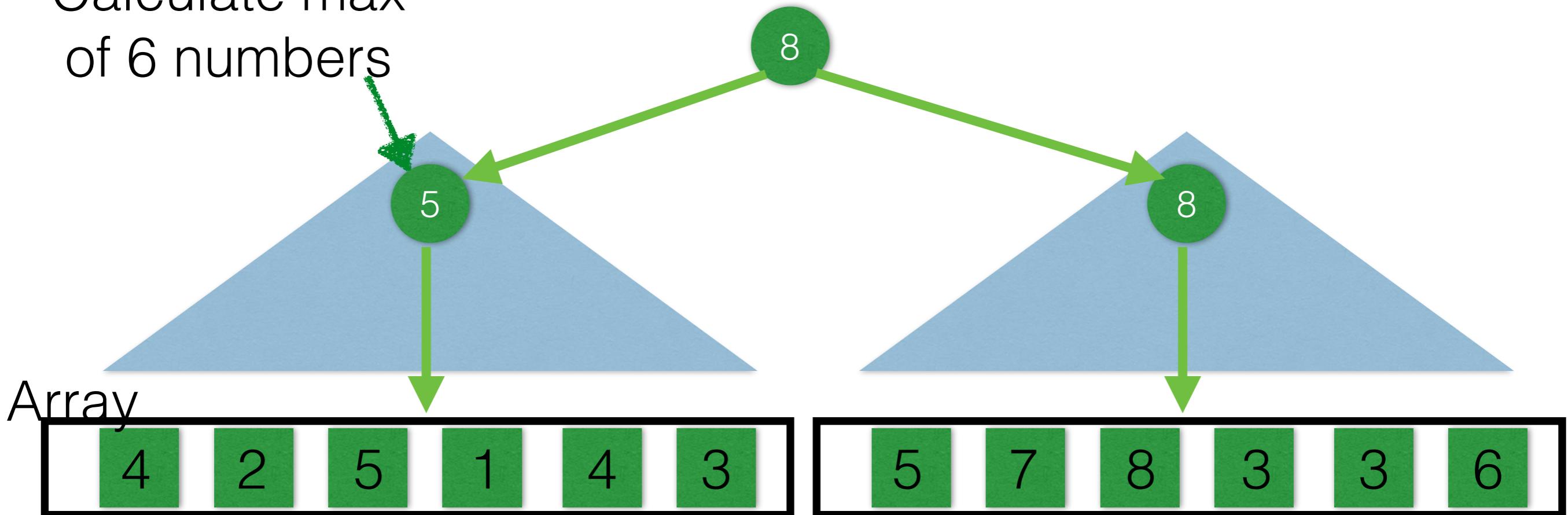


Reducing the overhead of incremental computation

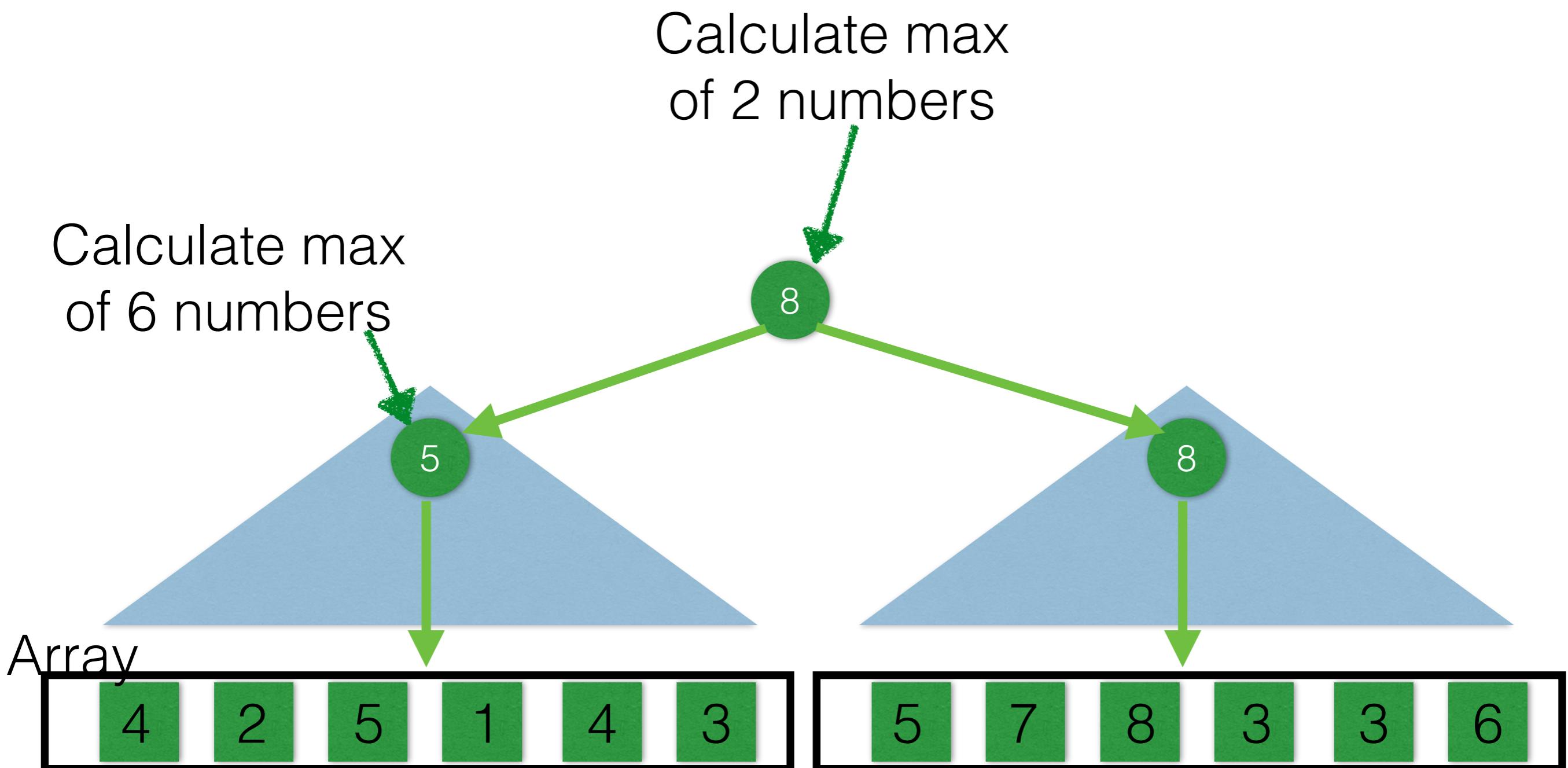


Reducing the overhead of incremental computation

Calculate max
of 6 numbers



Reducing the overhead of incremental computation



Reducing the overhead of incremental computation

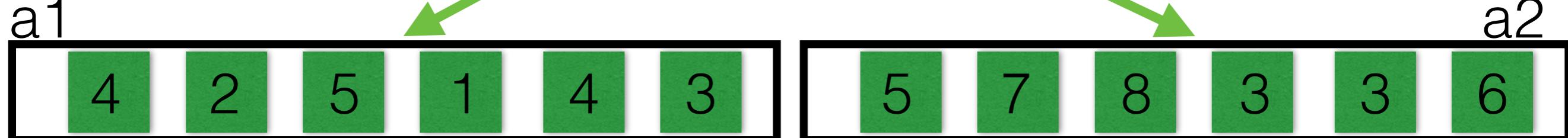
One memorized thunk tracks
multiple user function calls

Improve cache coherence

Reduce incremental overhead

Requires management

Works best with
large data sets



Experimental Evaluation

Experimental Evaluation

Crossover plot

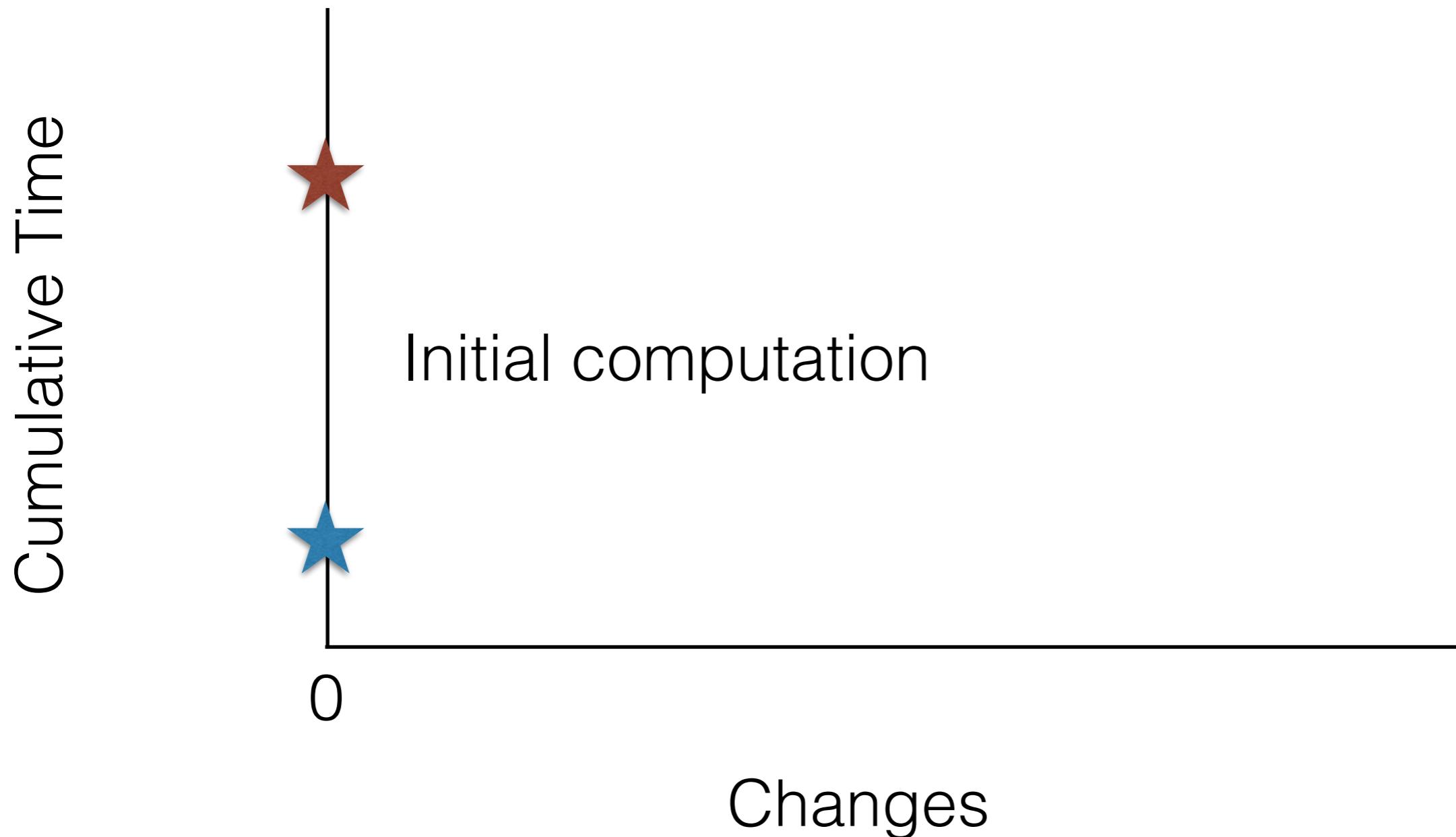
Cumulative Time



Changes

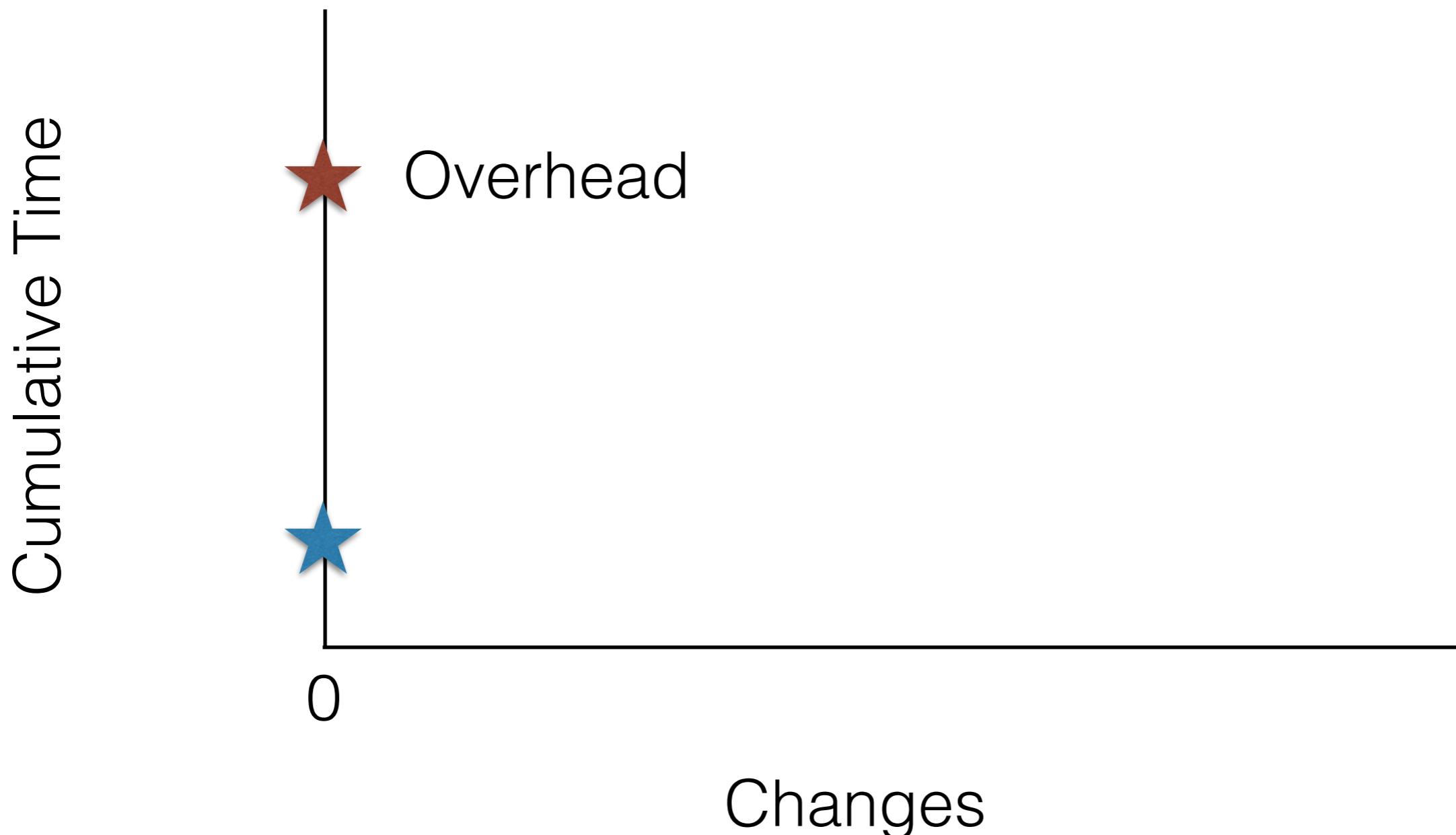
Experimental Evaluation

Crossover plot



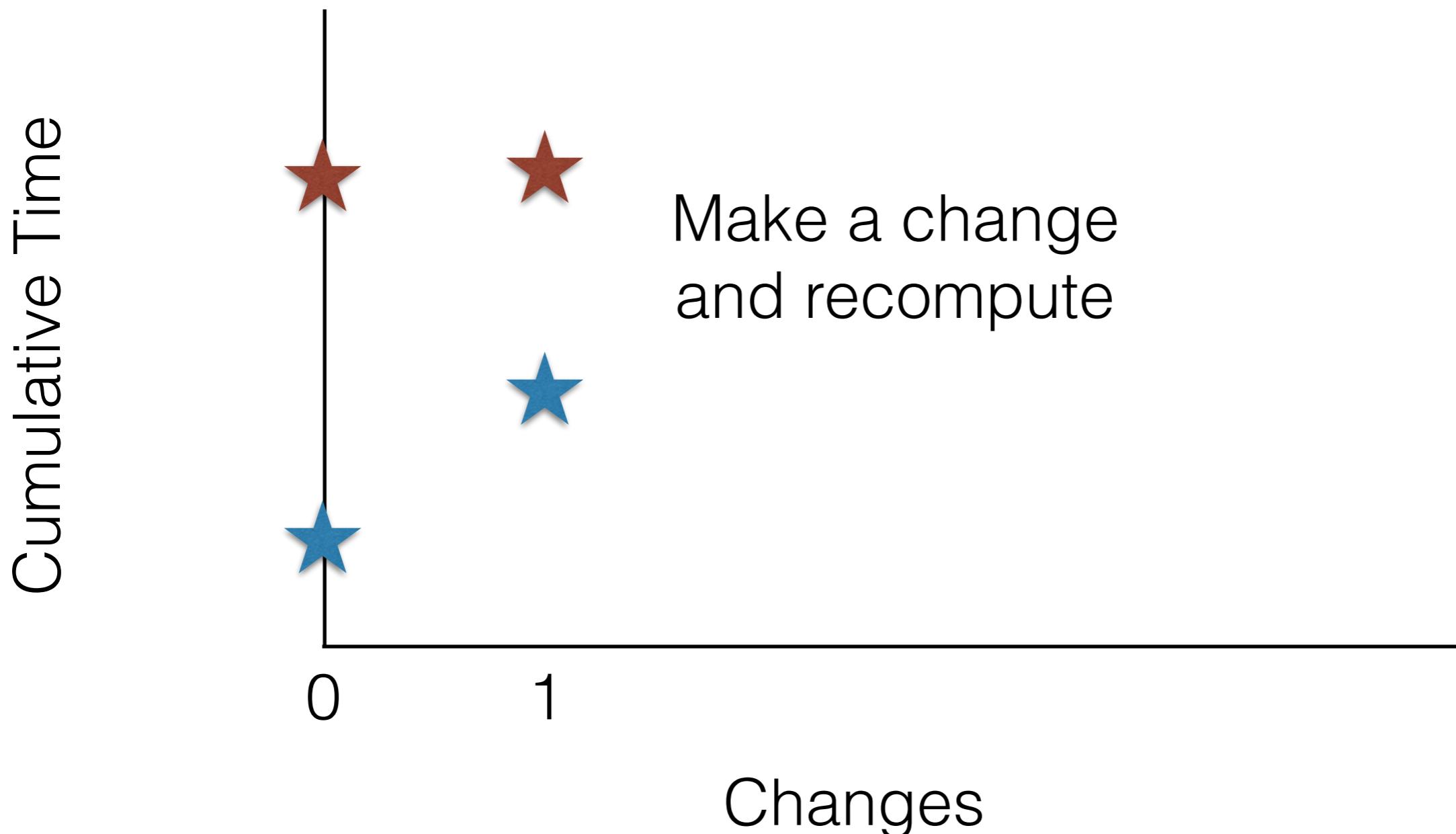
Experimental Evaluation

Crossover plot



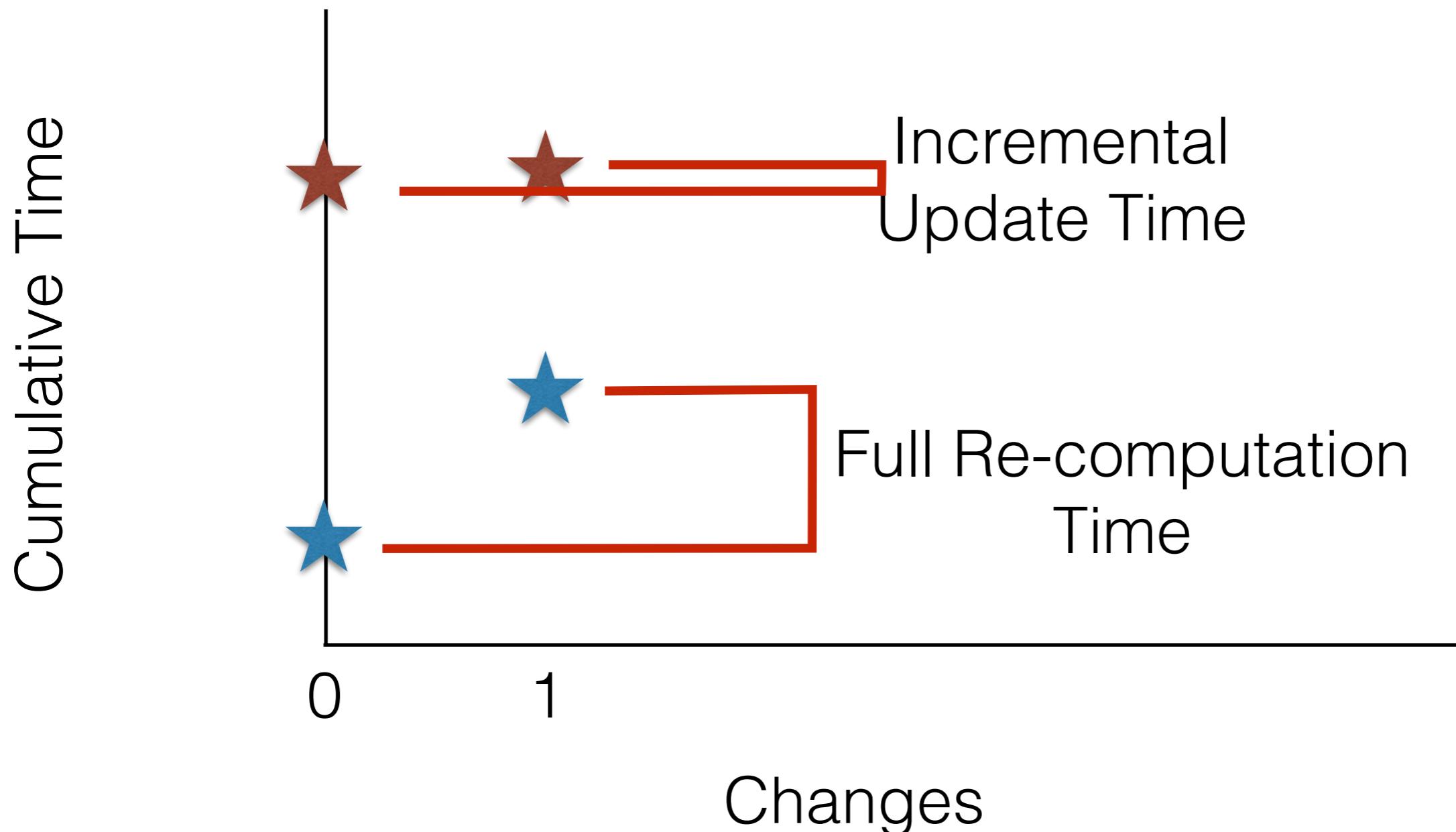
Experimental Evaluation

Crossover plot



Experimental Evaluation

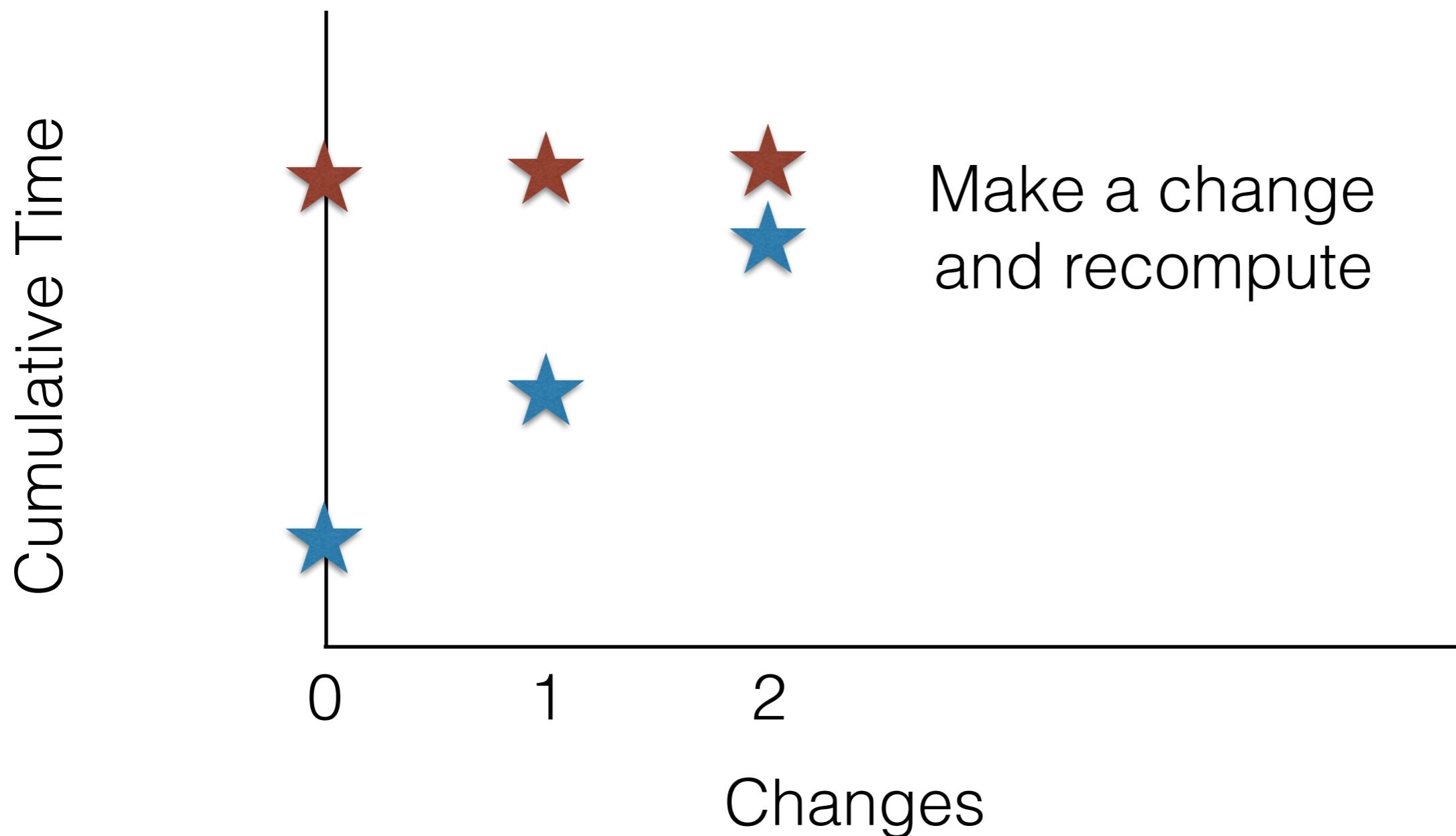
Crossover plot



Experimental Evaluation

Crossover plot

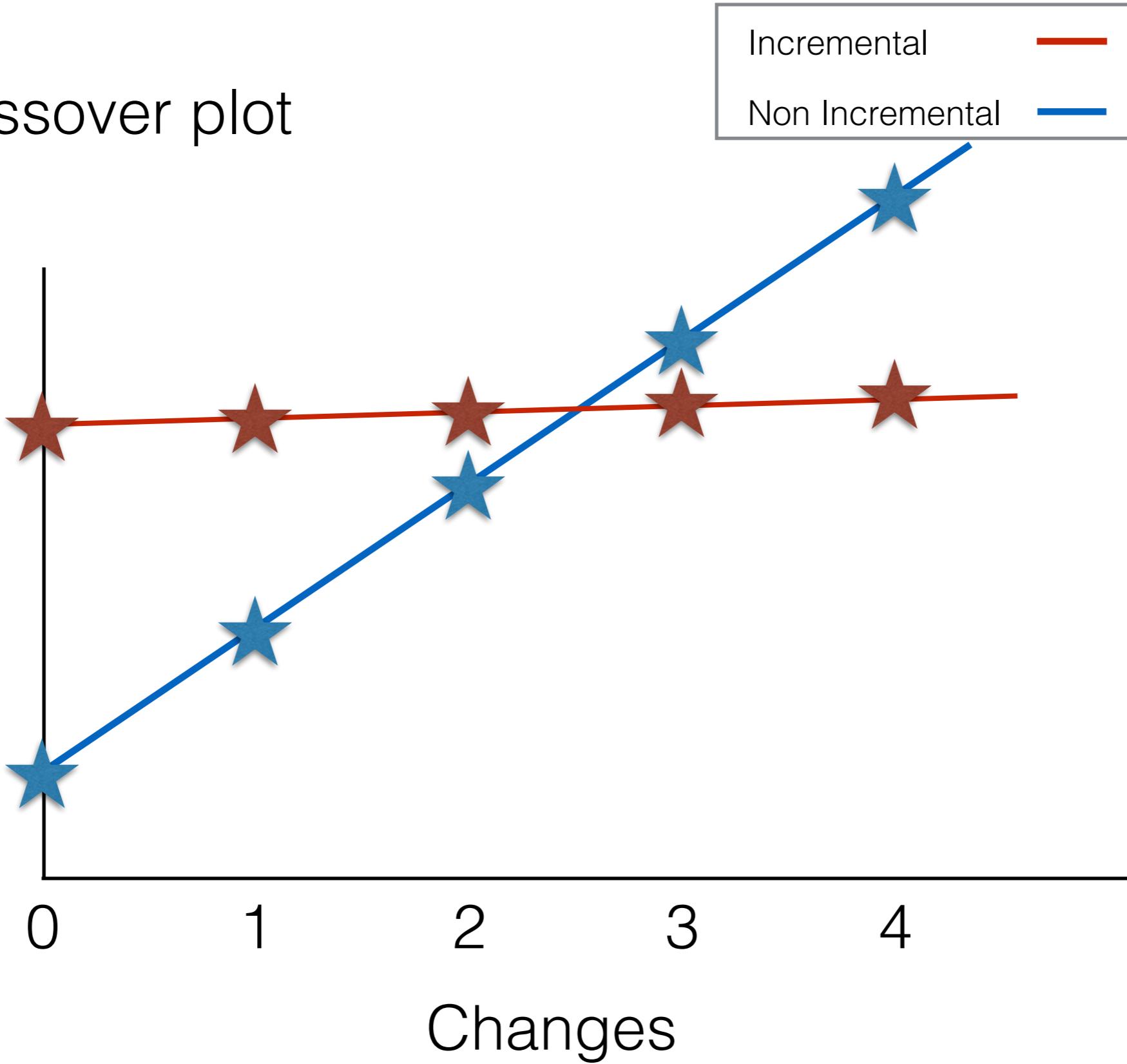
Incremental —
Non Incremental —



Experimental Evaluation

Crossover plot

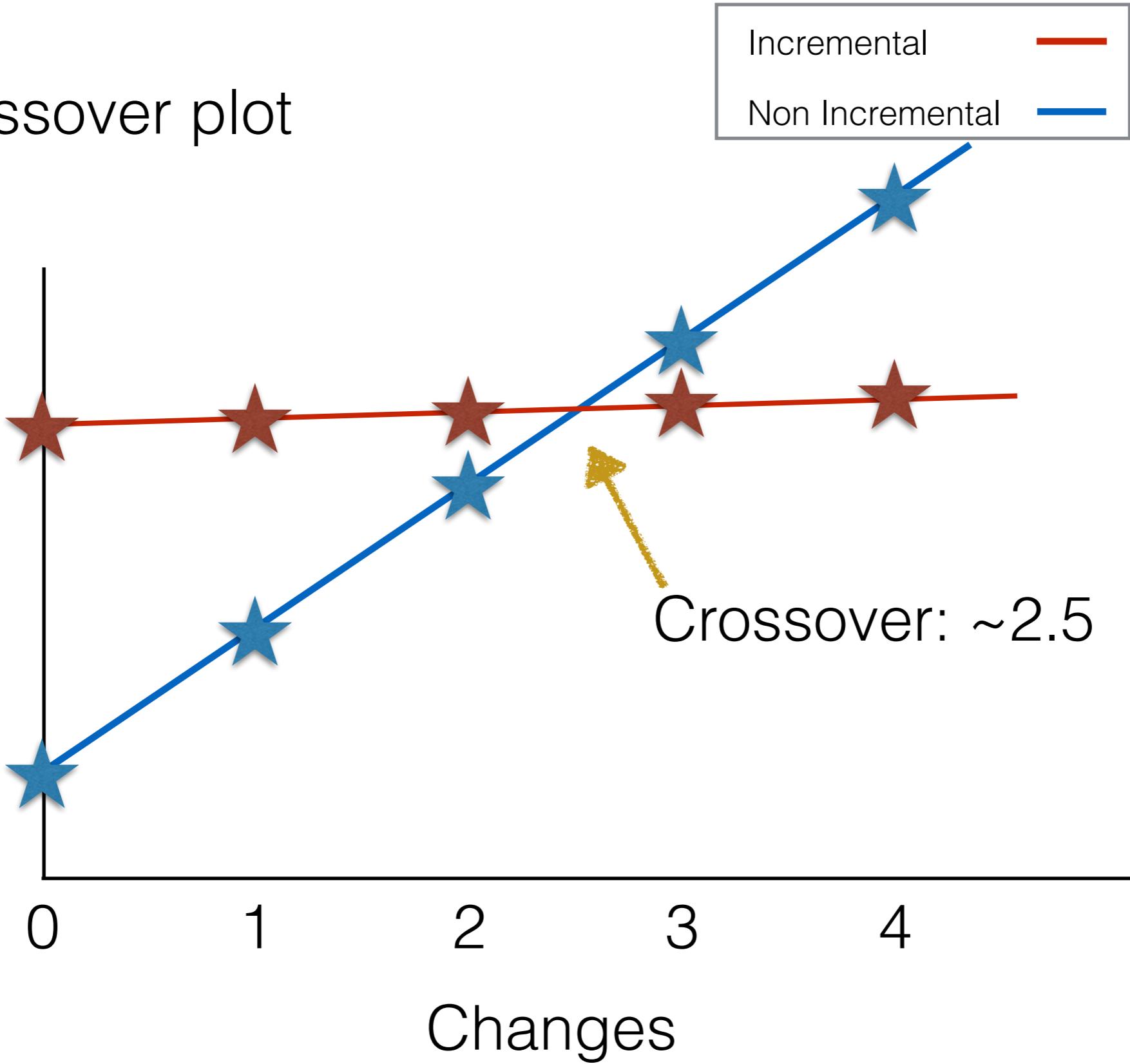
Cumulative Time



Experimental Evaluation

Crossover plot

Cumulative Time



Experimental Evaluation

Compute max of a collection

Non-Incremental

```
inputvec.iter().max()
```

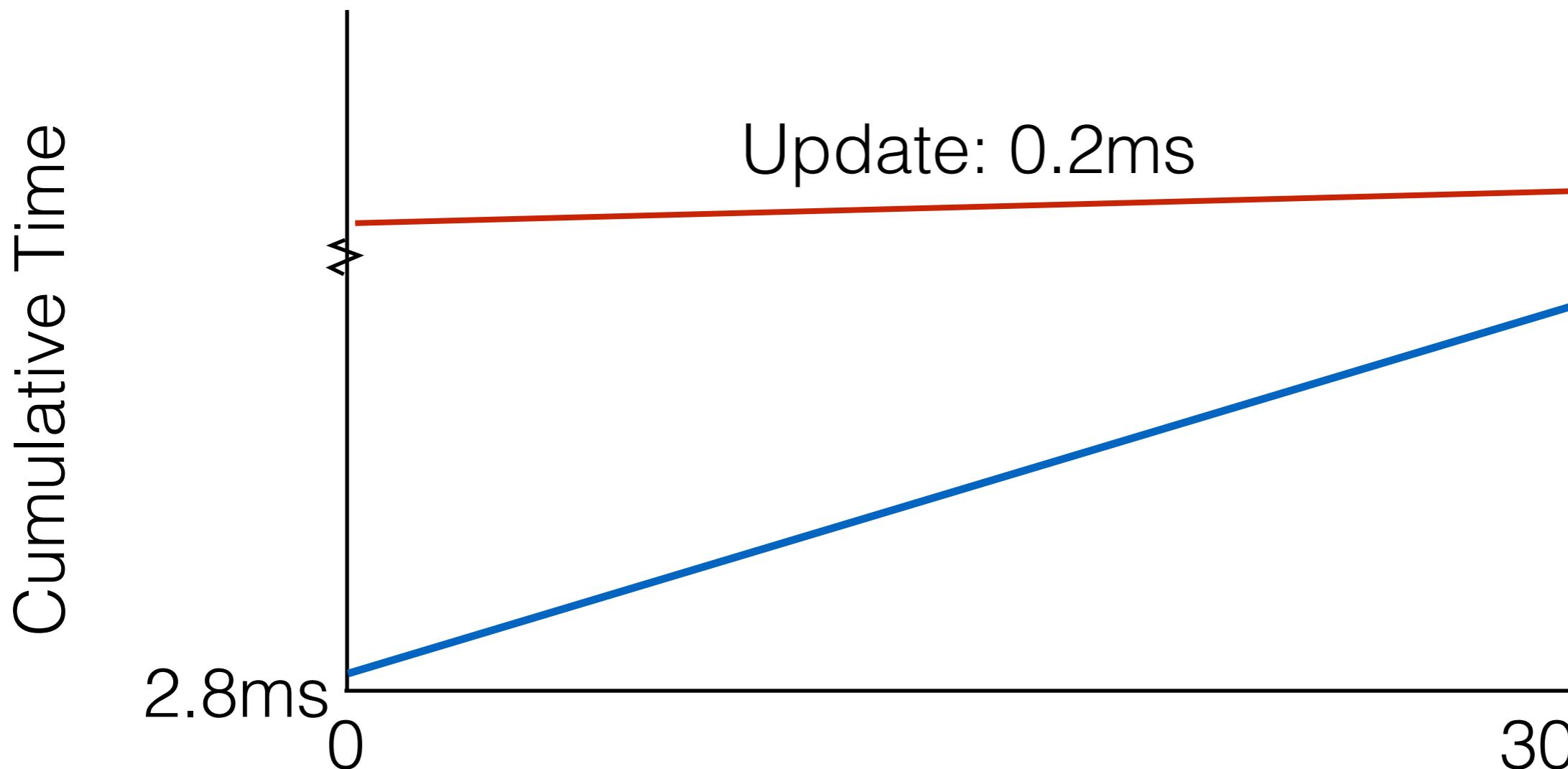
Incremental

```
inputgiraz.fold_up(λx.match x {  
    Leaf(vec) => vec.iter().max(),  
    Bin(m1,m2) => max(m1,m2)  
})
```

Experimental Evaluation

Max of 1M elements,
arrays of 1 element

Incremental
Non Incremental

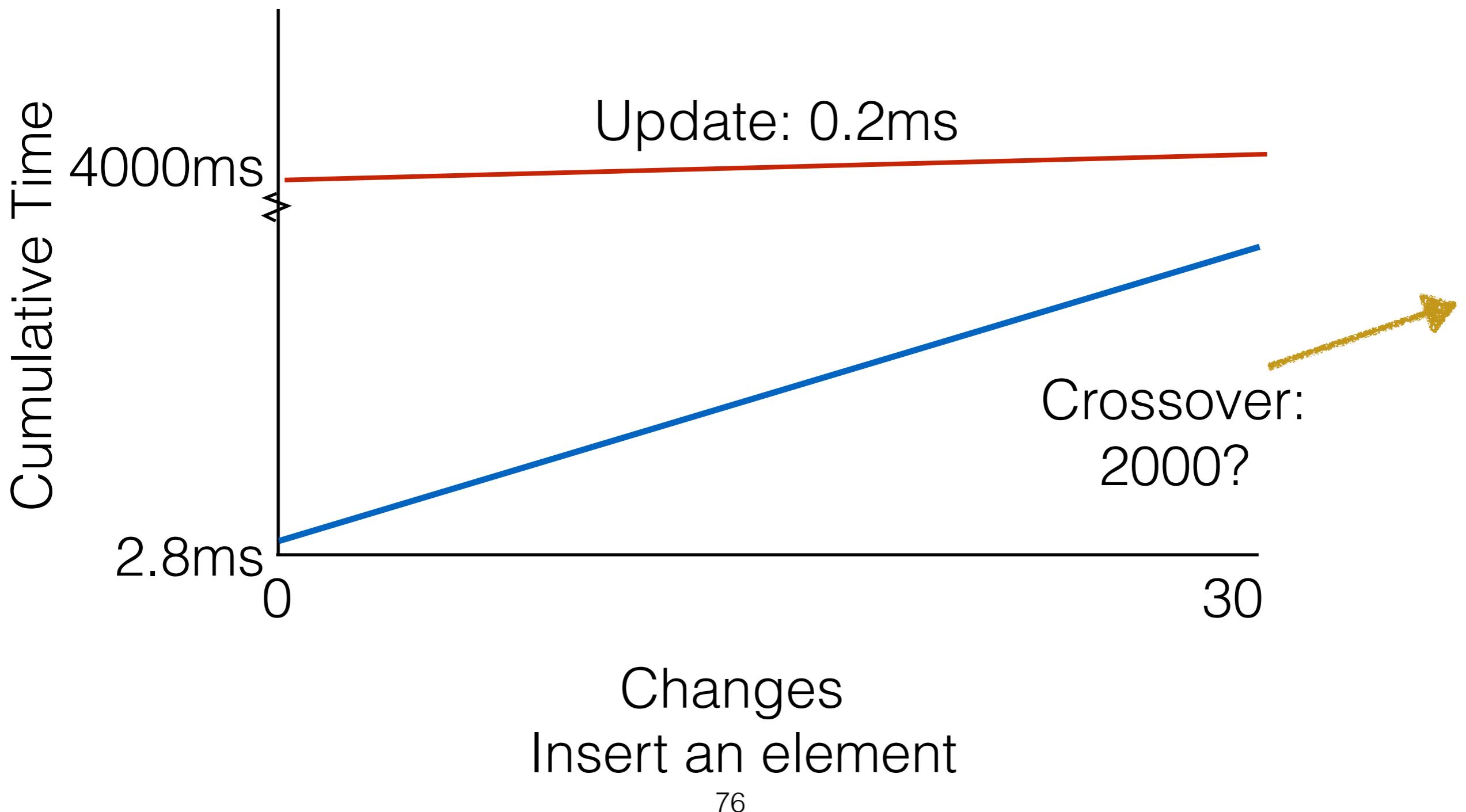


Changes
Insert an element

Experimental Evaluation

Max of 1M elements,
arrays of 1 element

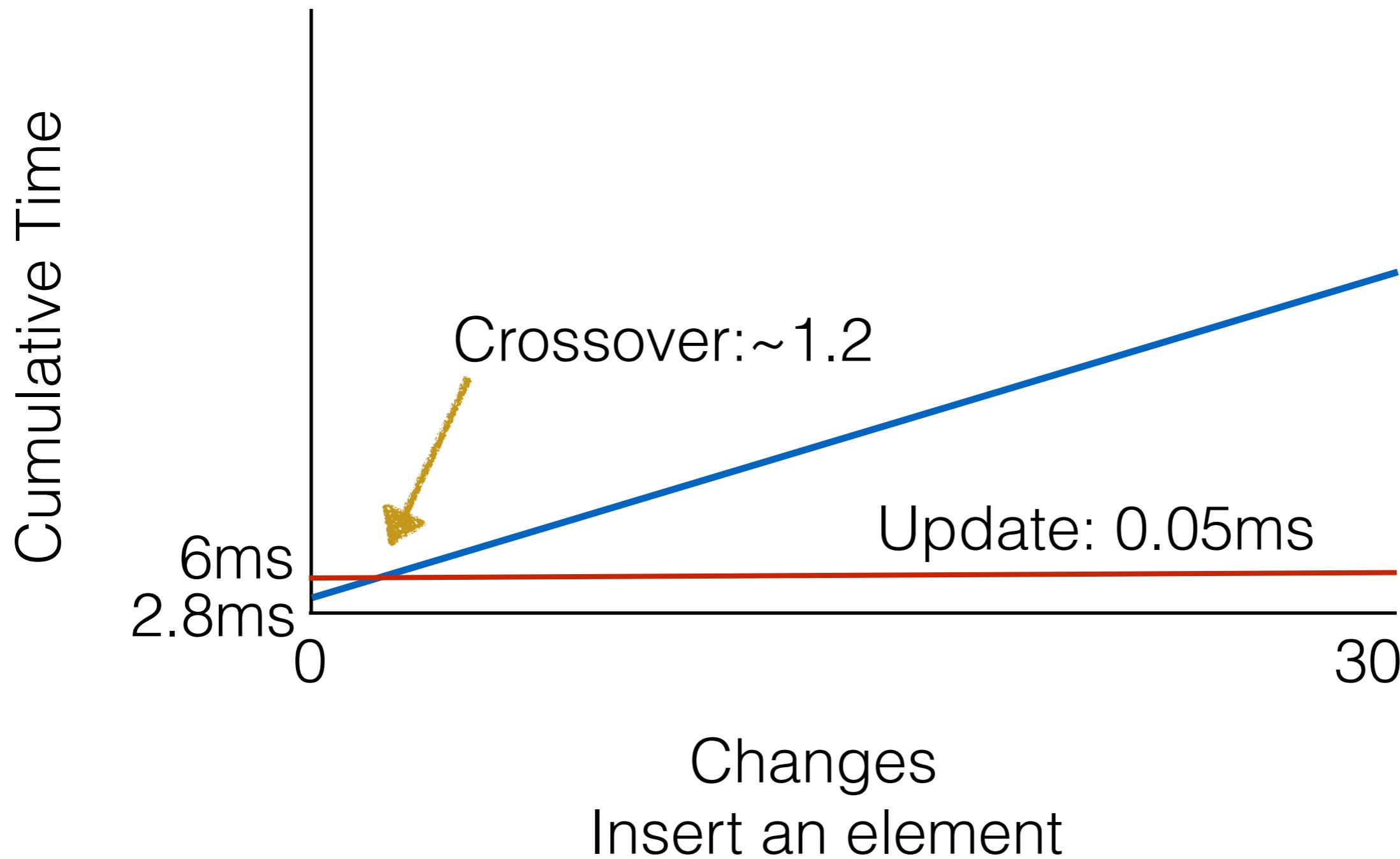
Incremental
Non Incremental



Experimental Evaluation

Max of 1M elements,
arrays of 1000 elements

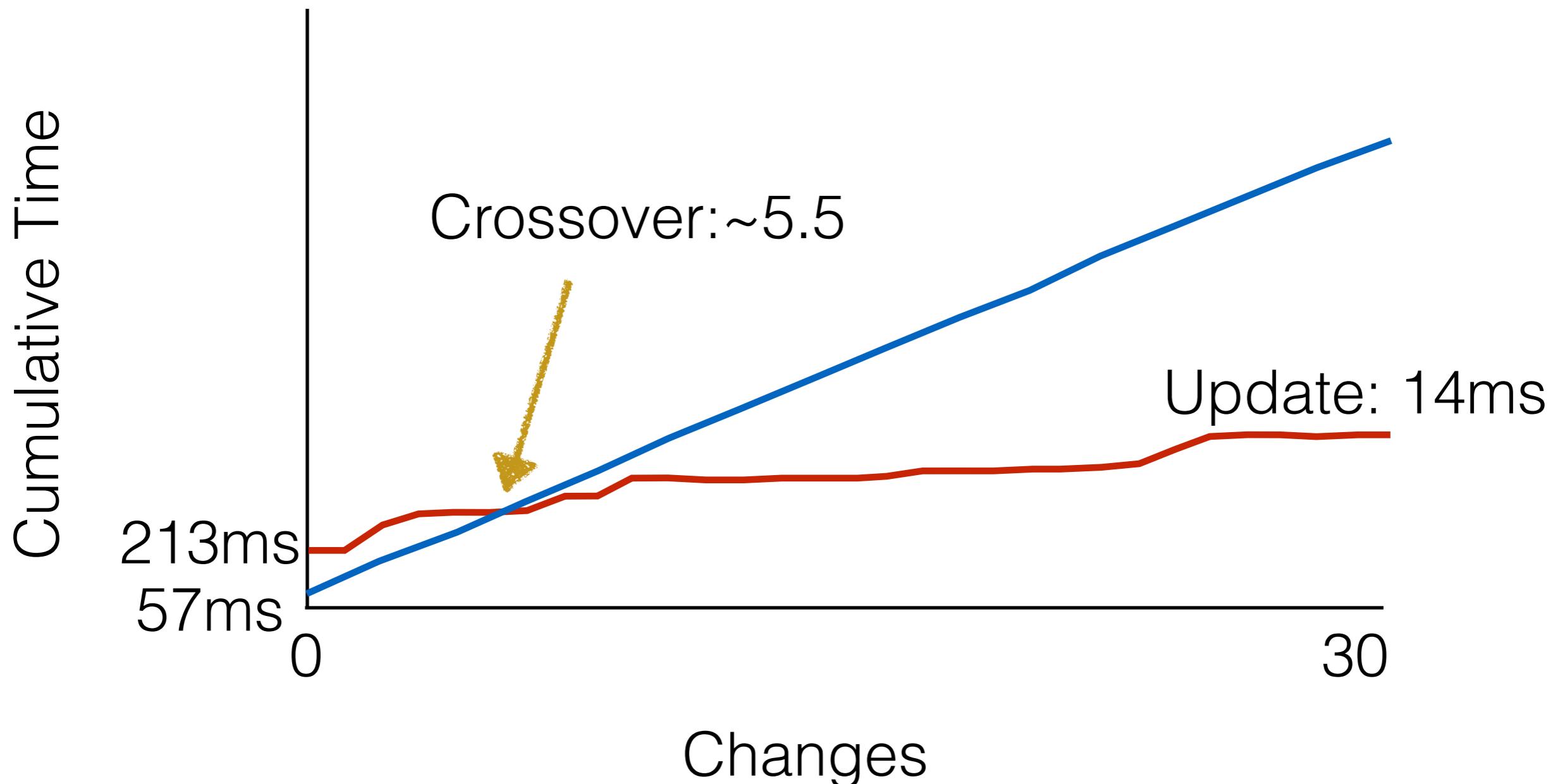
Incremental
Non Incremental



Experimental Evaluation

Quickhull of 1M points,
arrays of 1000 elements

Incremental
Non Incremental



All inputs: 1M, gauges 1k, times in ms

	Native initial	inc initial	inc update	crossover	speedup
max	2.84	5.99	0.05	2	57.5
quickhull	56.6	213	13.5	6	4.20
adder	10.3	91.1	0.43	10	23.9
to_string	93.8	95.5	0.21	1	449
reverse	2.01	7.85	0.09	4	22.2

github.com/cuplv/iodyn.rust

cd eval, cargo run --release --example [name] -- [options]

Summary

Development of an incremental computation library where the user writes non-incremental code

The api allows the user to specify subsequences, which can tune performance to a particular application

This library is competitive with native rust code

Code is available on Github, and can be imported into rust projects through the standard package manager

www.github.com/cuplv/iodyn.rust

kyleheadley.github.io