IGME 309-05 E05 – Transformation & Coordinate System

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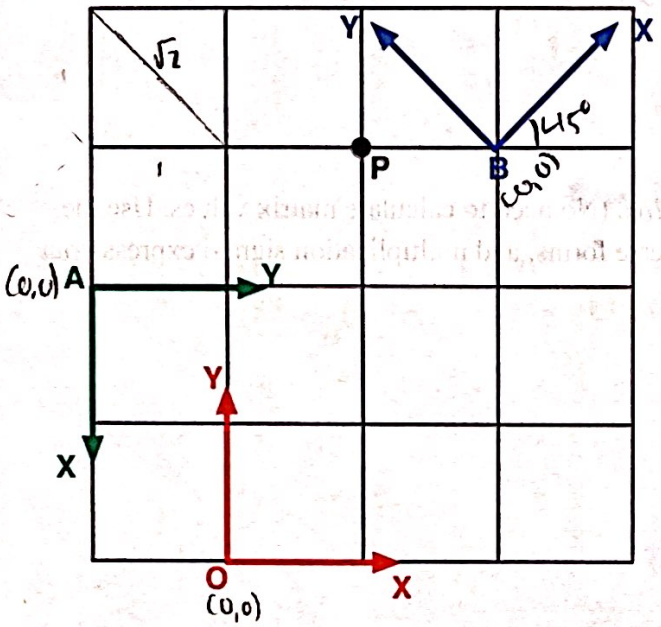
Due Date: 10/29 11:59pm.

Consider the following three coordinate systems *O*, *A* and *B* for Q1 – Q5.



Note:

- Cells of the grid are unit cells, which means the cell edge length is equal to 1.
- M_{AB} is denoted as a math notation to represent the 3x3 matrix that transforms the coordinate system ***A*** to ***B***.
- M^{-1}_{AB} represents the inverse matrix of M_{AB}, transforming the coordinate system **B** to **A**.

Q1: (15pts) What are the coordinates of *P* in the coordinate system *O*?

In coordinate system O, P has coordinates $\boxed{(1, 3).}$

Q2: (15pts) What are the coordinates of *P* in the coordinate system *A*?

In coordinate system A, P has coordinates $\boxed{(-1, 2).}$

Q3: (15pts) What are the coordinates of *P* in the coordinate system *B*?

In coordinate system B, P has coordinates $\boxed{\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)}$

$P = (-1, 0) \quad \theta = 45° = \frac{\pi}{4}$

$P_x' = x\cos\theta + y\sin\theta = -1 \cdot \cos\left(\frac{\pi}{4}\right) + 0\sin\left(\frac{\pi}{4}\right) = -1 \cdot \frac{\sqrt{2}}{2} = -\frac{\sqrt{2}}{2}$

$P_y' = -x\sin\theta + y\cos\theta = -(-1)\cdot\sin\left(\frac{\pi}{4}\right) + 0\cos\left(\frac{\pi}{4}\right) = 1 \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}$

Q4: (15pts) Derive and calculate the values of M_{AB}.

$P = (-1, 3)$ $\theta = 135° = \frac{3\pi}{4}$

A is x, y

B is x', y'

Translation: $\begin{bmatrix} t_x \\ t_y \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$

Rotation: $\begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} \cos(\frac{3\pi}{4}) & \sin(\frac{3\pi}{4}) \\ \sin(\frac{3\pi}{4}) & \cos(\frac{3\pi}{4}) \end{bmatrix} = \begin{bmatrix} \cos(\frac{3\pi}{4}) & -\sin(\frac{3\pi}{4}) & 0 \\ \sin(\frac{3\pi}{4}) & \cos(\frac{3\pi}{4}) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Q5: (15pts) Use M_{AO} and M_{BA} to represent M_{OB}. (No need to calculate matrix values. Use the given math notations (M_{AO} and M_{BA}), their inverse forms, and multiplication sign to express your answer.) $M_{AO} = T_{2,1}\, R_{90°}$ $M_{BA} = R_{-135°}\, T_{1,-3}$ $M_{OB} = T_{2,3},\, R_{45°}$

Q5 answer

$M_{AO}^{-1} \times M_{BA}^{-1} = M_{OB}$

M_{AO}^{-1}: Goes from O to A

M_{BA}^{-1}: Goes from A to B

M_{OB}: Goes from O to B

Goes from O to B

Q4. (cont)

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & -1 \\ -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Translate Rotate Transformation

Q4 answer