Analysis I
MATH 6710/7210-01, Credit Hours: 3
Fall 2022
MWF 9:00 AM – 9:50 AM, Gibson 310

Instructor: Kyle Kun Zhao
Office Hours: By appointment
Email Address: kzhao@tulane.edu

Catalog/Course description
This course is intended for beginning graduate students in mathematics. Prerequisites include calculus, linear algebra, and real analysis.

Course Goals
The primary goal of this course is to introduce basic concepts and results in measure theory, including Lebesgue measure, measurable functions, Lebesgue integral, convergence theorems, functions of bounded variation, Lebesgue differentiation, Lp-space, and abstract measure theory.

Course Learning Objectives
After completing this course, students are expected to be familiar with elementary measure theory and acquire a better understanding of devising rigorous proofs. It is also expected that after completing this course, students will be ready to explore more advanced topics in analysis and related areas, such as functional analysis, harmonic analysis, dynamical systems, partial differential equations, probability, stochastic processes etc.

Detailed Topics
The topics to be covered in this course include, but are not limited to

- **Outer measure**: definition, monotonicity, countable sub-additivity, equivalent definition, countable additivity
- **Measurable set and Lebesgue measure**: definition and examples, countable additivity, limiting sets, invariance properties, sigma-algebra, Borel set, non-measurable set
- **Measurable function**: characteristic function, step function, simple function, definition and properties of measurable functions, approximation by simple functions and step functions
- **Modes of convergence**: convergence almost everywhere, uniform convergence, convergence in measure, Egorov theorem, relation between different modes of convergence, Cauchy sequence in measure, Riesz theorem, Lusin theorem
- **Lebesgue integral**: integral of non-negative measurable simple functions, definition and properties, integral of non-negative measurable functions, definition and properties, monotone convergence theorem (Levi's theorem) and consequences, term-by-term integration, Fatou's lemma, integral of measurable function, definition and properties, absolute continuity, dominated convergence theorem
- **Differentiation**: differentiability of monotone functions, functions of bounded variation, definition and properties, Jordan decomposition, differentiability of BV functions, differentiability of indefinite integral of Lebesgue integrable function, differentiability of absolutely continuous functions, integration by parts
- **Lp-space**: definition and examples, conjugate indices, Holder's inequality, Minkowski inequality, completeness, convergence in Lp space, inner product, orthogonal system, Cauchy-Schwarz inequality, Fourier series, Bessel's inequality, Riesz-Fisher theorem
- **Abstract measure theory**: sigma-algebra, set function, measurable space, measure, measure space, signed measure, Hahn decomposition, mutual singularity, Jordan decomposition, Radon-Nikodym theorem, Radon-Nikodym derivative, Lebesgue decomposition

Required Student Resources
The lectures are based on a collection of notes from the instructor. There is no single textbook for this course. The following classic textbooks are relatively good references:

- *Real Analysis* by G.B. Folland
- *Real Analysis* by H.L. Royden and P.M. Fitzpatrick
- *Measure Theory* by J.K. Hunter [https://www.math.ucdavis.edu/~hunter/measure_theory/measure_notes.pdf]

Homework

There will be bi-weekly homework assignments throughout the semester. Each assignment will be posted on Canvas and sent to each student by email. Students who take this course remotely should send scanned copies of completed homework assignments to the instructor at a timely manner.

Exams

There will be two midterm exams and a final exam. Each midterm exam will have a duration of 50 minutes and the final exam will have a duration of up to 3 hours. In-person exams will take place in the usual classroom. For students taking the exams remotely, exams will be delivered electronically five minutes before the starting time. The students should send scanned copies of their exams to the instructor immediately after completing the exams. The exams at scheduled at the following date and time:

- Midterm 1: **October 3**, 2022, **9:00 AM – 9:50 AM**
- Midterm 2: **November 14**, 2022, **9:00 AM – 9:50 AM**
- Final: **December 13**, 2022, **8:00 AM – 11:00 AM**

Evaluation Procedures and Grading Criteria

Homework assignments are worth 40% of final grade, and each exam is worth 20% of final grade. Final grades are assigned according to the following scheme:

A = 90 to 100%
B = 80 to < 90%
C = 70 to < 80%
D = 60 to < 70%
F = < 60%

Attendance Statement

Faculty and students must comply with University policies on COVID-19 testing and isolation, which are located here [https://tulane.edu/covid-19/health-strategies]. Faculty and students must wear face coverings in all common areas, including classrooms, and follow social distancing rules. Failure to comply is a violation of the Code of Student Conduct and students will be subject to University discipline, which can include suspension or permanent dismissal.

If a student cannot attend class for any reason, the student is responsible for communicating with their instructors to make up any work they may miss. Faculty will provide online options for class participation, outlined in this document, and unless a student is seriously ill, they are expected to use this option. The University Health Center will provide documentation verifying a student is ill, as well as verification that a student may return to class. With the approval of the Newcomb - Tulane College dean, an instructor may have a student who has excessive absences involuntarily withdrawn from a course with a WF grade after written warning at any time during the semester.

ADA/Accessibility Statement

Tulane University strives to make all learning experiences as accessible as possible. If you anticipate or experience academic barriers based on your disability, please let me know immediately so that we can privately discuss options. I will never ask for medical documentation from you to support potential accommodation needs. Instead, to establish reasonable accommodations, I may request that you register with the Goldman Center for Student Accessibility. After registration, make arrangements with me as soon as possible to discuss your accommodations so that they may be implemented in a timely fashion. **Goldman Center contact information:** goldman@tulane.edu; (504) 862-8433; accessibility.tulane.edu.

Code of Academic Conduct

The Code of Academic Conduct applies to all undergraduate students, full-time and part-time, in Tulane University. Tulane University expects and requires behavior compatible with its high standards of scholarship. By accepting admission to the university, a student accepts its regulations (i.e., Code of Academic Conduct and Code of Student Conduct) and acknowledges the right of the university to take disciplinary action, including suspension or expulsion, for conduct judged unsatisfactory or disruptive.

Religious accommodation policy

Per Tulane's religious accommodation policy, I will make every reasonable effort to ensure that students are able to observe religious holidays without jeopardizing their ability to fulfill their academic obligations. Excused absences do not relieve the student from the responsibility for any course work required during the period of absence. Students should notify me within the first two weeks of the semester about their intent to observe any holidays that fall on a class day or on the day of the final exam.

Recordings of class sessions

Classes will be recorded and the recordings will be posted to Canvas. Students may not post a class recording elsewhere, either wholly or in part. Instructors may not use a class recording in another course or in a subsequent semester.

Title IX:

Tulane University recognizes the inherent dignity of all individuals and promotes respect for all people. As such, Tulane is committed to providing an environment free of all forms of discrimination including sexual and gender-based discrimination, harassment, and violence like sexual assault, intimate partner violence, and stalking. If you (or someone you know) has experienced or is experiencing these types of behaviors, know that you are not alone. Resources and support are available: you can learn more at allin.tulane.edu. Any and all of your communications on these matters will be treated as either "Confidential" or "Private" as explained in the chart below. Please know that if you choose to confide in me I am mandated by the university to report to the Title IX Coordinator, as Tulane and I want to be sure you are connected with all the support the university can offer. You *do not* need to respond to outreach from the university if you do not want. You can also make a report yourself, including an anonymous report, through the form at tulane.edu/concerns.

Confidential	Private
Except in extreme circumstances, involving imminent danger to one's self or others, nothing will be shared without your explicit permission.	*Conversations are kept as confidential as possible, but information is shared with key staff members so the University can offer resources and accommodations and take action if necessary for safety reasons.*
Counseling & Psychological Services (CAPS) | (504) 314-2277 or The Line (24/7) | (504) 264-6074	**Case Management & Victim Support Services | (504) 314-2160 orsrss@tulane.edu**
Student Health Center | (504) 865-5255	**Tulane University Police (TUPD) | Uptown - (504) 865-5911. Downtown – (504) 988-5531**
Sexual Aggression Peer Hotline and Education (SAPHE) | (504) 654-9543	**Title IX Coordinator | (504) 314-2160 or** msmith76@tulane.edu

Emergency Preparedness & Response:

EMERGENCY NOTIFICATION SYSTEM: TU ALERT

In the event of a campus emergency, Tulane University will notify students, faculty, and staff by email, text, and phone call. You were automatically enrolled in this system when you enrolled at the university.

Check your contact information annually in Gibson Online to confirm its accuracy.

RAVE GUARDIAN

- Download the RAVE Guardian app from the App Store
- Communicate with dispatchers silently by selecting "Submit Tip" feature in the app
- Use the Safety Timer feature to alert your "guardian" (TUPD, family, friend) when travelling alone at night

For more information, visit publicsafety.tulane.edu/rave-guardian

Tulane University
EMERGENCY PREPAREDNESS & RESPONSE

ACTIVE SHOOTER / VIOLENT ATTACKER

- **RUN** – run away from or avoid the affected area, if possible
- **HIDE** – go into the nearest room that can be locked, turn out the lights, silence cell phones, and remain hidden until all-clear message is given through TU ALERT
- **FIGHT** – do not attempt this option, except as a last resort

For more information on Active Shooter emergency procedures or to schedule a training, visit emergencyprep.tulane.edu

SEVERE WEATHER

- Follow all TU Alerts and outdoor warning sirens
- Seek shelter indoors until the severe weather threat has passed and an all-clear message is given
- Do not attempt to travel outside if weather is severe
- Monitor the Tulane Emergency website (tulane.edu/emergency/) for university-wide closures during a severe weather event