Missing Data Basics

Utrecht University Winter School: Missing Data in R

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Outline

Missing Data Descriptives

- Missing Data Patterns
- Nonresponse Rates
- Coverage Measures

Missing Data Mechanisms

- Testing the Missing Data Mechanism

What are Missing Data?

Missing data are empty cells in a dataset where there should be observed values.

- The missing cells correspond to true population values, but we haven't observed those values.

What are Missing Data?

Missing data are empty cells in a dataset where there should be observed values.

- The missing cells correspond to true population values, but we haven't observed those values.

Not every empty cell is a missing datum.

- Quality-of-life ratings for dead patients in a mortality study
- Firm profitability after the company goes out of business
- Self-reported severity of menstrual cramping for men
- Empty blocks of data following "gateway" items

A Little Notation

$$
\begin{aligned}
Y &:= \text{An } N \times P \text{ Matrix of Arbitrary Data} \\
Y_{mis} &:= \text{The \textit{missing} part of } Y \\
Y_{obs} &:= \text{The \textit{observed} part of } Y \\
R &:= \text{An } N \times P \text{ response matrix} \\
M &:= \text{An } N \times P \text{ missingness matrix}
\end{aligned}
$$

The R and M matrices are complementary.

- $r_{np} = 1$ means y_{np} is observed; $m_{np} = 1$ means y_{np} is missing.
- $r_{np} = 0$ means y_{np} is missing; $m_{np} = 0$ means y_{np} is observed.
- M_p is the *missingness* of Y_p.

Missing Data Descriptives

Missing Data Pattern

Missing data (or response) patterns represent unique combinations of observed and missing items.

- P items $\Rightarrow 2^P$ possible patterns.

	X	Y
1	x	y
2	x	.
3	.	y
4	.	.

Patterns for $P = 2$

	X	Y	Z
1	x	y	z
2	x	y	.
3	x	.	z
4	.	y	z
5	x	.	.
6	.	.	z
7	.	y	.
8	.	.	.

Patterns for $P = 3$

Missing Data Pattern

The concept of a “missing data pattern” can also be used to classify the spatial arrangement of missing cells on a data set.

- Univariate
  - Missing data occur on only one variable

- Monotone
  - The proportion of complete elements, in both rows and columns, decreases when traversing the data set.
  - The observed cells can be arranged into a “staircase” pattern.

- Arbitrary
  - Missing values are “randomly” scattered throughout the data set.

Example Missing Data Patterns

	X	Y	Z
1	x	y	z
2	x	y	z
3	x	y	z
4	x	y	z
5	x	y	z
6	x	.	z
7	x	.	z
8	x	.	z
9	x	.	z
10	x	.	z

Univariate Pattern

	X	Y	Z
1	x	y	z
2	x	y	z
3	x	y	z
4	x	y	.
5	x	y	.
6	x	y	.
7	x	.	.
8	x	.	.
9	x	.	.
10	.	.	.

Monotone Pattern

	X	Y	Z
1	x	.	z
2	x	y	z
3	x	y	z
4	x	.	z
5	x	y	z
6	x	.	z
7	.	y	z
8	x	y	z
9	x	.	.
10	x	y	.

Arbitrary Pattern

Nonresponse Rates

Proportion Missing

- The proportion of cells containing missing data
- Good early screening measure
- Should be computed for each variable, not for the entire dataset

Attrition Rate

- The proportion of participants that drop-out of a study at each measurement occasion

Nonresponse Rates

Proportion of Complete Cases

- The proportion of observations with no missing data
- Often reported but nearly useless quantity

Fraction of Missing Information

- Associated with an estimated parameter, not with an incomplete variable
- Like an R^2 for the missing data
- Most important diagnostic value for missing data problems
- Can only be computed after treating the missing data

Coverage Measures

Covariance Coverage

$$CC_{jk} = N^{-1} \sum_{n=1}^{N} r_{nj} r_{nk}$$

- The proportion of cases available to estimate a given pairwise relationship (e.g., a covariance between two variables)
- Very important to have adequate coverage of the parameters you want to estimate

Coverage Measures

INBOUND STATISTIC

$$I_{jk} = \frac{\sum_{n=1}^{N}(1 - r_{nj})r_{nk}}{\sum_{n=1}^{N}(1 - r_{nj})}$$

- The proportion of missing cases in Y_j for which Y_k is observed

OUTBOUND STATISTIC

$$O_{jk} = \frac{\sum_{n=1}^{N} r_{nj}(1 - r_{nk})}{\sum_{n=1}^{N} r_{nj}}$$

- The proportion of observed cases in Y_j for which Y_k is missing

Coverage Measures

Influx Coefficient

$$I_j = \frac{\sum_{k=1}^{P} \sum_{n=1}^{N} (1 - r_{nj}) r_{nk}}{\sum_{k=1}^{P} \sum_{n=1}^{N} r_{nk}}$$

- The proportion of observed cells in Y that exists in cases for which Y_j is missing
- How well the missing values in Y_j connect to the observed values in Y_{-j}

Coverage Measures

Outflux Coefficient

$$O_j = \frac{\sum_{k=1}^{P} \sum_{n=1}^{N} r_{nj}(1 - r_{nk})}{\sum_{k=1}^{P} \sum_{n=1}^{N} (1 - r_{nk})}$$

- The proportion of missing cells in Y that exists in cases for which Y_j is observed
- How well the observed values in Y_j connect to the missing values in Y_{-j}

Missing Data Mechanisms

Missing Data Mechanisms

Missing Completely at Random (MCAR)

- $P(R|Y_{mis}, Y_{obs}) = P(R)$
- Missingness is unrelated to any study variables.

Missing at Random (MAR)

- $P(R|Y_{mis}, Y_{obs}) = P(R|Y_{obs})$
- Missingness is related to only the *observed* parts of study variables.

Missing not at Random (MNAR)

- $P(R|Y_{mis}, Y_{obs}) \neq P(R|Y_{obs})$
- Missingness is related to the *unobserved* parts of study variables.

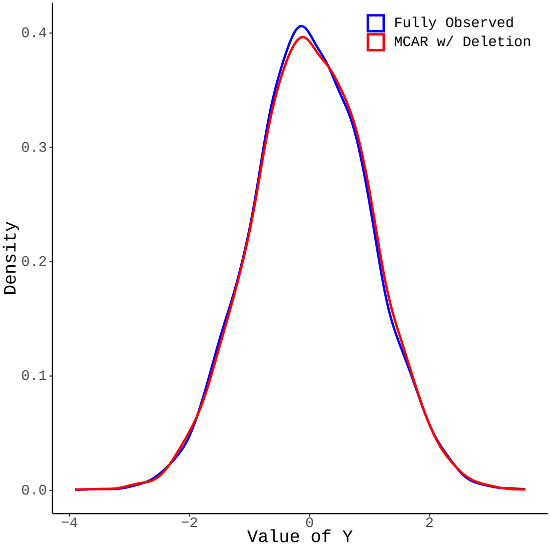
Simulate Some Toy Data

```
library(mvtnorm)
library(dplyr)
library(magrittr)
library(MASS)

nObs <- 5000 # Sample Size
pm   <- 0.3  # Proportion Missing

sigma <- matrix(c(1.0, 0.5, 0.3,
                  0.5, 1.0, 0.0,
                  0.3, 0.0, 1.0),
                ncol = 3)
dat0 <- rmvnorm(nObs, c(0, 0, 0), sigma) %>% data.frame()
colnames(dat0) <- c("x", "y", "z")

dat0 %$% cor(y, x)

[1] 0.5001822
```

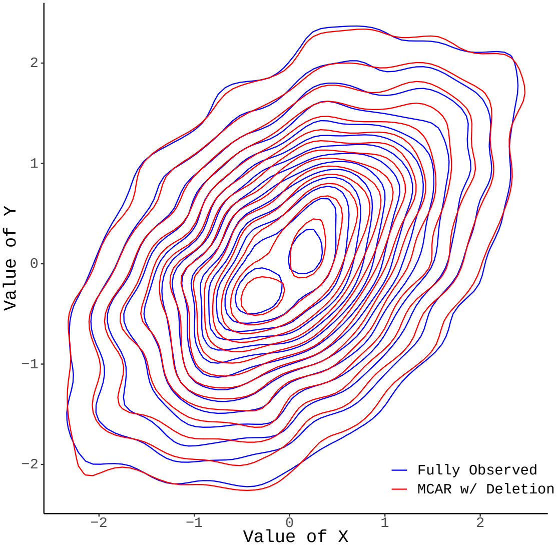
MCAR Example

```r
## Simulate MCAR Missingness:
m <- sample(1:nObs, size = pm * nObs)

## Impose MCAR missing on Y:
mcarData          <- dat0
mcarData[m, "y"] <- NA

## Check the correlation between X & Y:
mcarData %$% cor(y, x, use = "pairwise")

[1] 0.5197437
```

MCAR Example



MCAR Example

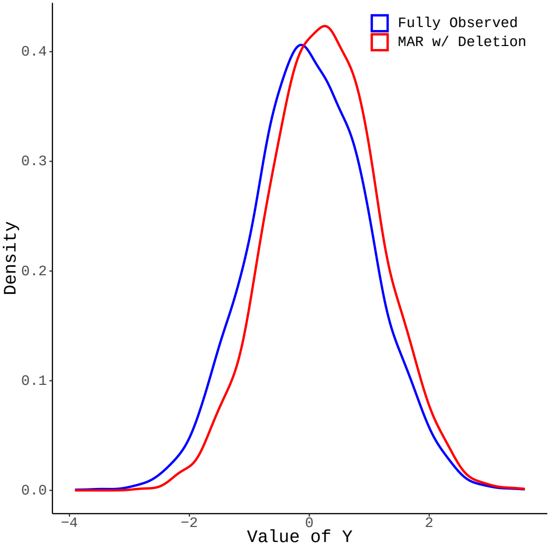


MAR Example

```r
## Simulate MAR Missingness:
m <- with(dat0, x < quantile(x, probs = pm))

## Impose MAR missing on Y:
marData             <- dat0
marData[m, "y"] <- NA

## Check the correlation between X & Y:
marData %$% cor(y, x, use = "pairwise")
```

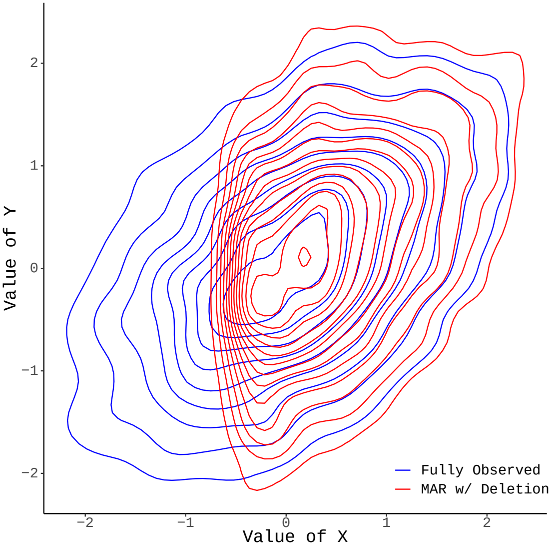
```
[1] 0.3825876
```

MAR Example



Fully Observed
MAR w/ Deletion
Density
0.0
0.1
0.2
0.3
0.4
−4
−2
0
2
Value of Y

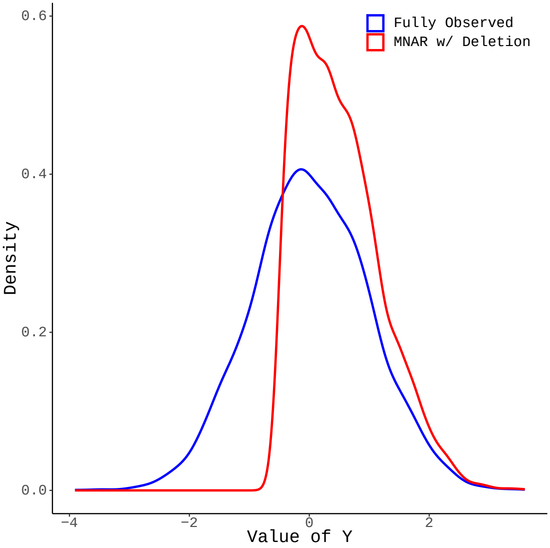
MAR Example


Value of Y
Value of X
2
1
0
−1
−2
−2
−1
0
1
2
Fully Observed
MAR w/ Deletion

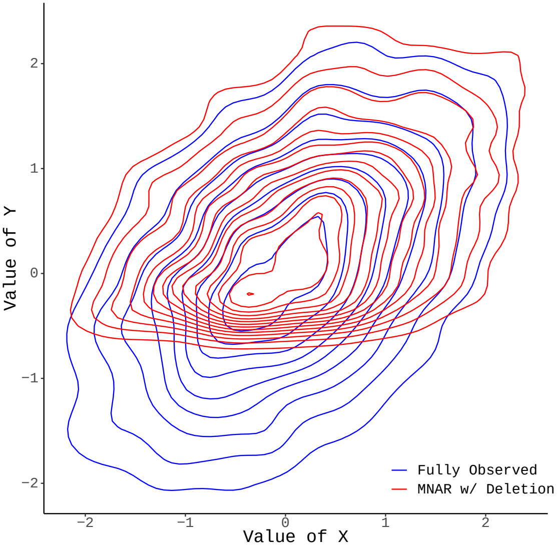
MNAR Example

```r
## Simulate MNAR Missingness:
m <- with(dat0, y < quantile(y, probs = pm))

## Impose MNAR missing on Y:
mnarData            <- dat0
mnarData[m, "y"] <- NA

## Check the correlation between X & Y:
mnarData %$% cor(y, x, use = "pairwise")

[1] 0.3901487
```

MNAR Example



MNAR Example



Crucial Nuance

In our previous MAR example, ignoring the predictor of missingness actually produces *Indirect MNAR*.

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QUESTION: What happens if we ignore the predictor of missingness, but that predictor is independent of our study variables?

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In our previous MAR example, ignoring the predictor of missingness actually produces *Indirect MNAR*.

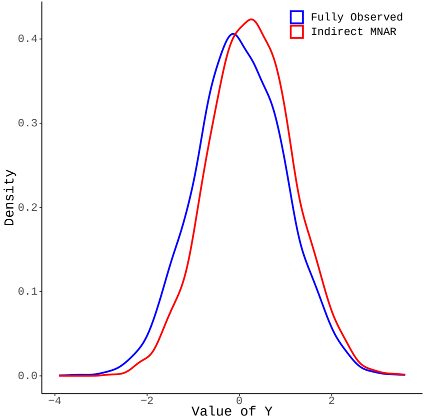
QUESTION: What happens if we ignore the predictor of missingness, but that predictor is independent of our study variables?

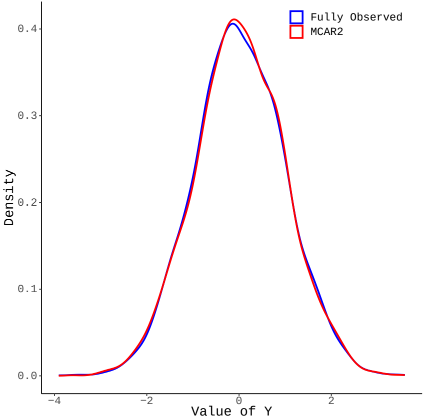
```
m <- with(dat0, z < quantile(z, probs = pm))

mcarData2          <- dat0
mcarData2[m, "y"] <- NA

mcarData2 %$% cor(y, x, use = "pairwise")

[1] 0.5119953
```

ANSWER: We get back to MCAR :)

Crucial Nuance

The missing data mechanisms are not simply characteristics of an incomplete dataset; we also need to account for the analysis.





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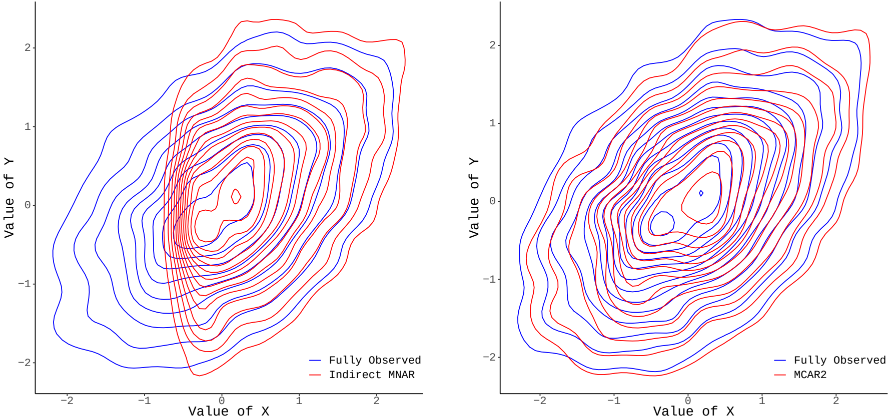

Testing the Missing Data Mechanism

We cannot fully test the MAR or MNAR assumptions.

- To do so would require knowing the values of the missing data.
- We can find observed predictors of missingness.
    - Use classification algorithms to predict missingness from Y_{obs}.
    - We can never know that we have discovered all MAR predictors.
- In practice, MAR and MNAR live on the ends of a continuum.
    - Our missing data problem exists at some unknown point along this continuum.
    - We can do a lot to nudge our problem towards the MAR side.

Testing the Missing Data Mechanism

We can (partially) test the MCAR assumption.

- With MCAR, the missing data and the observed data should have the same distribution.
- We can test for MCAR by testing the distributions of *auxiliary variables*, $\mathbf{Z}$.
    - Use a t-test to compare the subset of Z_p that corresponds to Y_{mis} to the subset corresponding to Y_{obs}.
    - The Little (1988) MCAR test is a multivariate version of this.

These procedures actually test if the data are *observed* completely at random.

References

Little, R. J. A. (1988). A test of missing completely at random for multivariate data with missing values. *Journal of the American Statistical Association*, *83*(404), 1198–1202. doi: 10.1080/01621459.1988.10478722