Missing Data Basics

Utrecht University Winter School: Missing Data in R

Kyle M. Lang

Department of Methodology & Statistics
Utrecht University

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Introduction

1. What’s your name?
2. Where are you from/where do you work?
3. What type of research do you do?
4. What type of missing data problems do you encounter in your research?
5. What statistical software do you use/do you have programming experience?
6. What’s your math background?

Outline

What are Missing Data?

Missing data are empty cells in a dataset where there should be observed values.

- The missing cells correspond to true population values, but we haven't observed those values.

What are Missing Data?

Missing data are empty cells in a dataset where there should be observed values.

- The missing cells correspond to true population values, but we haven't observed those values.

Not every empty cell is a missing datum.

- Quality-of-life ratings for dead patients in a mortality study
- Firm profitability after the company goes out of business
- Self-reported severity of menstrual cramping for men
- Empty blocks of data following “gateway” items

A Little Notation

$$Y := \text{An } N \times P \text{ Matrix of Arbitrary Data}$$

$$Y_{mis} := \text{The \textit{missing} part of } Y$$

$$Y_{obs} := \text{The \textit{observed} part of } Y$$

$$R := \text{An } N \times P \text{ response matrix}$$

$$M := \text{An } N \times P \text{ missingness matrix}$$

The R and M matrices are complementary.

- $r_{np} = 1$ means y_{np} is observed; $m_{np} = 1$ means y_{np} is missing.
- $r_{np} = 0$ means y_{np} is missing; $m_{np} = 0$ means y_{np} is observed.
- M_p is the *missingness* of Y_p.

MISSING DATA DESCRIPTIVES

Missing Data Pattern

Missing data (or response) patterns represent unique combinations of observed and missing items.

- P items $\Rightarrow 2^P$ possible patterns.

	X	Y
1	x	y
2	x	.
3	.	y
4	.	.

Patterns for $P = 2$

	X	Y	Z
1	x	y	z
2	x	y	.
3	x	.	z
4	.	y	z
5	x	.	.
6	.	.	z
7	.	y	.
8	.	.	.

Patterns for $P = 3$

Missing Data Pattern

The concept of a "missing data pattern" can also be used to classify the spatial arrangement of missing cells on a data set.

- Univariate
  - Missing data occur on only one variable

- Monotone
  - The proportion of complete elements, in both rows and columns, decreases when traversing the data set.
  - The observed cells can be arranged into a "staircase" pattern.

- Arbitrary
  - Missing values are "randomly" scattered throughout the data set.

Example Missing Data Patterns

	X	Y	Z
1	x	y	z
2	x	y	z
3	x	y	z
4	x	y	z
5	x	y	z
6	x	.	z
7	x	.	z
8	x	.	z
9	x	.	z
10	x	.	z

Univariate Pattern

	X	Y	Z
1	x	y	z
2	x	y	z
3	x	y	z
4	x	y	.
5	x	y	.
6	x	y	.
7	x	.	.
8	x	.	.
9	x	.	.
10	.	.	.

Monotone Pattern

	X	Y	Z
1	x	.	z
2	x	y	z
3	x	y	z
4	x	.	z
5	x	y	z
6	x	.	z
7	.	y	z
8	x	y	z
9	x	.	.
10	x	y	.

Arbitrary Pattern

Nonresponse Rates

Proportion Missing

- The proportion of cells containing missing data
- Good early screening measure
- Should be computed for each variable, not for the entire dataset

Attrition Rate

- The proportion of participants that drop-out of a study at each measurement occasion

Nonresponse Rates

Proportion of Complete Cases

- The proportion of observations with no missing data
- Often reported but nearly useless quantity

Fraction of Missing Information

- Associated with an estimated parameter, not with an incomplete variable
- Like an R^2 for the missing data
- Most important diagnostic value for missing data problems
- Can only be computed after treating the missing data

Coverage Measures

Covariance Coverage

$$CC_{jk} = N^{-1} \sum_{n=1}^{N} r_{nj} r_{nk}$$

- The proportion of cases available to estimate a given pairwise relationship (e.g., a covariance between two variables)
- Very important to have adequate coverage of the parameters you want to estimate

Coverage Measures

Inbound Statistic

$$I_{jk} = \frac{\sum_{n=1}^{N}(1 - r_{nj})r_{nk}}{\sum_{n=1}^{N}(1 - r_{nj})}$$

- The proportion of missing cases in Y_j for which Y_k is observed

Outbound Statistic

$$O_{jk} = \frac{\sum_{n=1}^{N} r_{nj}(1 - r_{nk})}{\sum_{n=1}^{N} r_{nj}}$$

- The proportion of observed cases in Y_j for which Y_k is missing

Coverage Measures

Influx Coefficient

$$I_j = \frac{\sum_{k=1}^{P} \sum_{n=1}^{N} (1 - r_{nj}) r_{nk}}{\sum_{k=1}^{P} \sum_{n=1}^{N} r_{nk}}$$

- The proportion of observed cells in Y that exists in cases for which Y_j is missing
- How well the missing values in Y_j connect to the observed values in Y_{-j}

Coverage Measures

OUTflUX COEFfiCIENT

$$O_j = \frac{\sum_{k=1}^{P} \sum_{n=1}^{N} r_{nj}(1 - r_{nk})}{\sum_{k=1}^{P} \sum_{n=1}^{N} (1 - r_{nk})}$$

- The proportion of missing cells in Y that exists in cases for which Y_j is observed
- How well the observed values in Y_j connect to the missing values in Y_{-j}

Examples

1. What is the coverage for $cov(X, Y)$?
2. What is the coverage for $cov(W, Y)$?
3. What is the coverage for $cov(X, Z)$?
4. What is the outflux coefficient for W?
5. What is the influx coefficient for W?

	W	X	Y	Z
1	w	x	y	.
2	w	x	y	.
3	w	x	y	.
4	w	x	y	.
5	w	x	y	.
6	w	.	y	z
7	w	.	y	z
8	w	.	y	z
9	w	.	y	z
10	w	.	y	z

Examples

1. What is the percent missing at T2?
2. What is the attrition rate at T3?
3. What is the inbound statistic I_{32}?
4. What is the outbound statistic O_{42}?
5. What is the influx coefficient I_3?
6. What is the outflux coefficient O_2?

	T1	T2	T3	T4
1	x1	x2	x3	x4
2	x1	x2	x3	x4
3	x1	x2	x3	x4
4	x1	x2	x3	.
5	x1	x2	x3	.
6	x1	x2	.	.
7	x1	x2	.	.
8	x1	.	.	.
9	x1	.	.	.
10	x1	.	.	.

Missing Data Mechanisms

Missing Data Mechanisms

Missing Completely at Random (MCAR)

- $P(R|Y_{mis}, Y_{obs}) = P(R)$
- Missingness is unrelated to any study variables.

Missing at Random (MAR)

- $P(R|Y_{mis}, Y_{obs}) = P(R|Y_{obs})$
- Missingness is related to only the *observed* parts of study variables.

Missing not at Random (MNAR)

- $P(R|Y_{mis}, Y_{obs}) \neq P(R|Y_{obs})$
- Missingness is related to the *unobserved* parts of study variables.

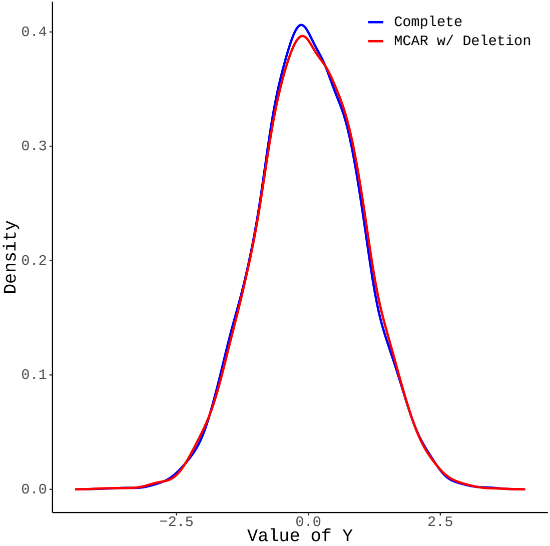
Simulate Some Toy Data

```r
nObs <- 5000 # Sample Size
pm   <- 0.3  # Proportion Missing

sigma <- matrix(c(1.0, 0.5, 0.3,
                  0.5, 1.0, 0.0,
                  0.3, 0.0, 1.0),
                ncol = 3)
tmp <- rmvnorm(nObs, c(0, 0, 0), sigma)

x0 <- tmp[ , 1]
y0 <- tmp[ , 2]
z0 <- tmp[ , 3]

cor(y0, x0) # Check correlation between X and Y

[1] 0.5001822
```

MCAR Example

```r
## Simulate MCAR Missingness:
mVec <- sample(1 : length(y0), size = pm * length(y0))

yMcar       <- y0
yMcar[mVec] <- NA

cor(yMcar, x0, use = "pairwise") # Look at correlation

[1] 0.5197437
```

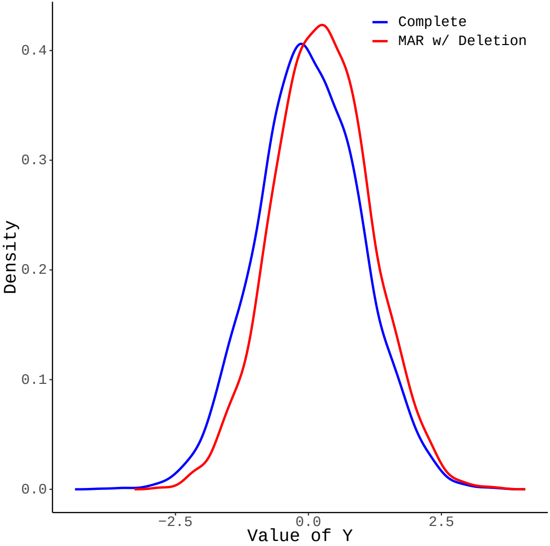
MCAR Example



MAR Example

```
## Simulate MAR Missingness:
mVec <- x0 < quantile(x0, probs = pm)
mean(mVec)

[1] 0.3

yMar       <- y0
yMar[mVec] <- NA

cor(yMar, x0, use = "pairwise") # Not looking so good :(

[1] 0.3825876
```

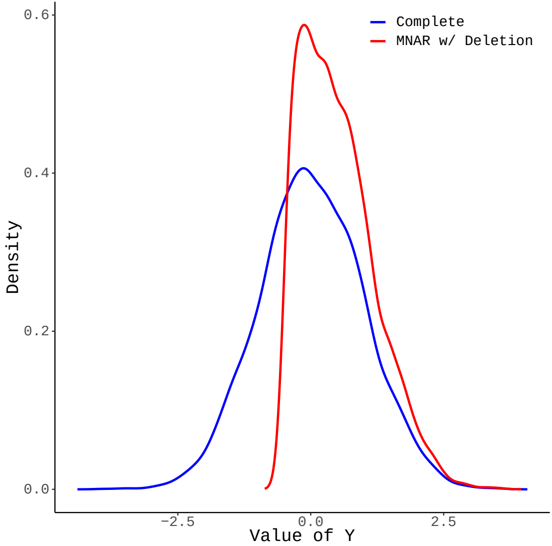
MAR Example


Complete
MAR w/ Deletion
Density
0.0
0.1
0.2
0.3
0.4
−2.5
0.0
2.5
Value of Y

MNAR Example

```r
## Simulate MNAR Missingness:
mVec <- y0 < quantile(y0, probs = pm)
mean(mVec)

[1] 0.3

yMnar        <- y0
yMnar[mVec] <- NA

cor(yMnar, x0, use = "pairwise") # Hmm...looks pretty bad.

[1] 0.3901487
```

MNAR Example



Crucial Nuance

In our previous MAR example, ignoring the predictor of missingness actually produces *Indirect MNAR*.

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QUESTION: What happens if we ignore the predictor of missingness, but that predictor is independent of our study variables?

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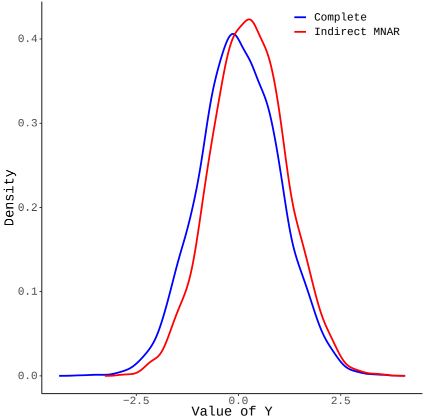
QUESTION: What happens if we ignore the predictor of missingness, but that predictor is independent of our study variables?

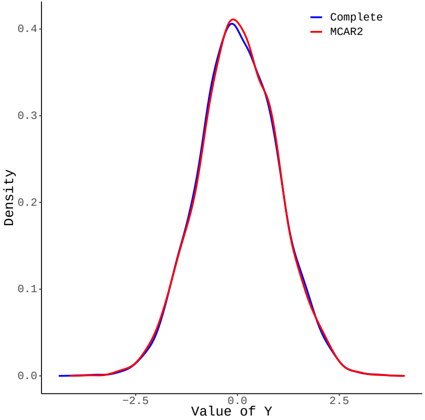
```
mVec <- z0 < quantile(z0, probs = pm)

y        <- y0
y[mVec] <- NA

cor(y, x0, use = "pairwise")

[1] 0.5119953
```

ANSWER: We get back to MCAR :)

Crucial Nuance

The missing data mechanisms are not simply characteristics of an incomplete dataset; we also need to account for the analysis.





Testing the Missing Data Mechanism

We cannot fully test the MAR or MNAR assumptions.

- To do so would require knowing the values of the missing data.
- We can find observed predictors of missingness, but we can never know that we have them all.
- In practice, MAR and MNAR live on the ends of a continuum.
  - Our missing data problem exists at some unknown point along this continuum.
  - We can do a lot to nudge our problem towards the MAR side.

Testing the Missing Data Mechanism

We can test the MCAR assumption.

- With MCAR, the missing data and the observed data should have the same distribution.
- We can test for MCAR by testing the distributions of *auxiliary variables*, $\mathbf{Z}$.
  - Use a t-test to compare the subset of Z_p that corresponds to Y_{mis} to the subset corresponding to Y_{obs}.
  - The Little (1988) MCAR test is a multivariate version of this.

Example

Create some toy datasets from the variables we generated above.

```
mcarData <- data.frame(y = yMcar, x = x0, z = z0,
                       m = as.numeric(is.na(yMcar))
                       )
marData  <- data.frame(y = yMar, x = x0, z = z0,
                       m = as.numeric(is.na(yMar))
                       )
mnarData <- data.frame(y = yMnar, x = x0, z = z0,
                       m = as.numeric(is.na(yMnar))
                       )
```

T-Test Example

Test for dependence between X and M_Y in MCAR data.

```
mcarData %$% t.test(x ~ m) %>% wrap()


Welch Two Sample t-test

data:  x by m
t = 0.68563, df = 2852.8, p-value = 0.493
alternative hypothesis: true difference in means between
group 0 and group 1 is not equal to 0
95 percent confidence interval:
 -0.03921499  0.08138543
sample estimates:
mean in group 0 mean in group 1
    0.013908816    -0.007176408
```

T-Test Example

Test for dependence between Z and M_Y in MCAR data.

```
mcarData %$% t.test(z ~ m) %>% wrap()


Welch Two Sample t-test

data:  z by m
t = 0.38865, df = 2841.9, p-value = 0.6976
alternative hypothesis: true difference in means between
group 0 and group 1 is not equal to 0
95 percent confidence interval:
 -0.04848298  0.07245421
sample estimates:
mean in group 0 mean in group 1
    0.009151786    -0.002833825
```

T-Test Example

Test for dependence between X and M_Y in MAR data.

```
marData %$% t.test(x ~ m) %>% wrap()


Welch Two Sample t-test

data:  x by m
t = 92.56, df = 3832.8, p-value < 2.2e-16
alternative hypothesis: true difference in means between
group 0 and group 1 is not equal to 0
95 percent confidence interval:
 1.614203 1.684066
sample estimates:
mean in group 0 mean in group 1
      0.5023237      -1.1468112
```

T-Test Example

Test for dependence between Z and M_Y in MAR data.

```
marData %$% t.test(z ~ m) %>% wrap()


Welch Two Sample t-test

data:  z by m
t = 16.913, df = 2832.1, p-value < 2.2e-16
alternative hypothesis: true difference in means between
group 0 and group 1 is not equal to 0
95 percent confidence interval:
 0.4491108 0.5669049
sample estimates:
mean in group 0 mean in group 1
      0.1579585      -0.3500494
```

T-Test Example

Test for dependence between X and M_Y in MNAR data.

```
mnarData %$% t.test(x ~ m) %>% wrap()


Welch Two Sample t-test

data:  x by m
t = 28.251, df = 2926.7, p-value < 2.2e-16
alternative hypothesis: true difference in means between
group 0 and group 1 is not equal to 0
95 percent confidence interval:
 0.7439001 0.8548632
sample estimates:
mean in group 0 mean in group 1
      0.2473977      -0.5519839
```

T-Test Example

Test for dependence between Z and M_Y in MNAR data.

```
mnarData %$% t.test(z ~ m) %>% wrap()


Welch Two Sample t-test

data:  z by m
t = -0.33313, df = 2778.5, p-value = 0.7391
alternative hypothesis: true difference in means between
group 0 and group 1 is not equal to 0
95 percent confidence interval:
 -0.07145430  0.05070098
sample estimates:
mean in group 0 mean in group 1
    0.002443105     0.012819764
```

Little (1988) MCAR Test Example

Use the Little (1988) MCAR test on MCAR data.

```
mcarData %>% select(-m) %>% mcar_test()

# A tibble: 1 x 4
  statistic    df p.value missing.patterns
      <dbl> <dbl>   <dbl>            <int>
1     0.504     2   0.777                2
```

Little (1988) MCAR Test Example

Use the Little (1988) MCAR test on MAR data.

```
marData %>% select(-m) %>% mcar_test()

# A tibble: 1 x 4
  statistic    df p.value missing.patterns
      <dbl> <dbl>   <dbl>            <int>
1     2862.     2       0                2
```

Little (1988) MCAR Test Example

Use the Little (1988) MCAR test on MNAR data.

```
mnarData %>% select(-m) %>% mcar_test()

# A tibble: 1 x 4
  statistic    df p.value missing.patterns
      <dbl> <dbl>   <dbl>            <int>
1      746.     2       0                2
```

Logistic Regression Example

```r
## Read in some data:
diabetes <- readRDS(paste0(dataDir, "diabetes.rds"))

## Generate MAR missingness:
diabetes$m <- simLogisticMissingness0(data    = diabetes,
                                      pm      = 0.25,
                                      preds   = c("bmi", "tc"),
                                      type    = "high",
                                      stdData = TRUE)$r

## Predict the missingness using logistic regression:
fit <- diabetes %>%
    select(-glu) %>%
    glm(m ~ ., data = ., family = "binomial")
```

Logistic Regression Example

```
partSummary(fit, 3)

Coefficients:
              Estimate Std. Error z value Pr(>|z|)
(Intercept) -1.459e+01  4.031e+00  -3.619 0.000296
age          1.205e-02  1.141e-02   1.056 0.290782
bmi          2.269e-01  4.054e-02   5.596 2.19e-08
bp          -1.213e-02  1.147e-02  -1.057 0.290292
tc           2.949e-02  2.897e-02   1.018 0.308696
ldl          2.703e-03  2.625e-02   0.103 0.917986
hdl         -5.961e-05  3.990e-02  -0.001 0.998808
tch         -3.160e-01  2.889e-01  -1.094 0.274049
ltg          5.588e-01  8.952e-01   0.624 0.532537
progress     2.501e-03  2.380e-03   1.051 0.293237
sexmale      4.336e-02  2.978e-01   0.146 0.884234
```

MISSING DATA TREATMENTS

Bad Methods (These almost never work)

Listwise Deletion (Complete Case Analysis)

- Use only complete observations for the analysis
  - Very wasteful (can throw out lots of useful data)
  - Loss of statistical power

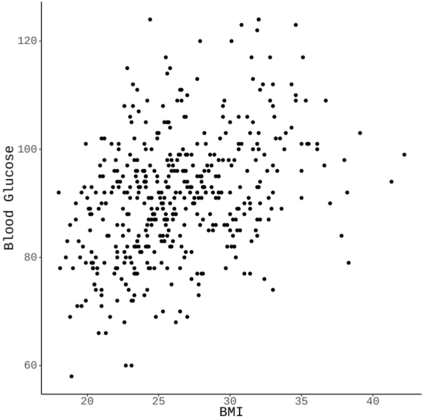
Pairwise Deletion (Available Case Analysis)

- Use only complete pairs of observations for analysis
  - Different samples sizes for different parameter estimates
  - Can cause computational issues

Bad Methods (These almost never work)

(Unconditional) Mean Substitution

- Replace Y_{mis} with $\bar{Y}_{obs}$
  - Negatively biases regression slopes and correlations
  - Attenuates measures of linear association



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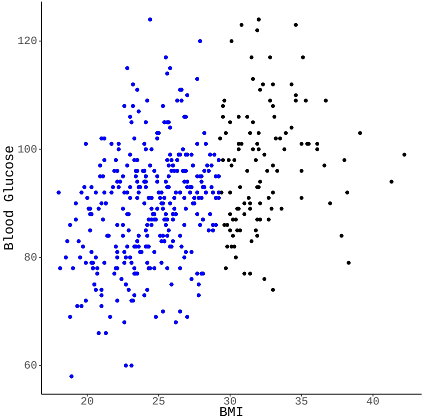

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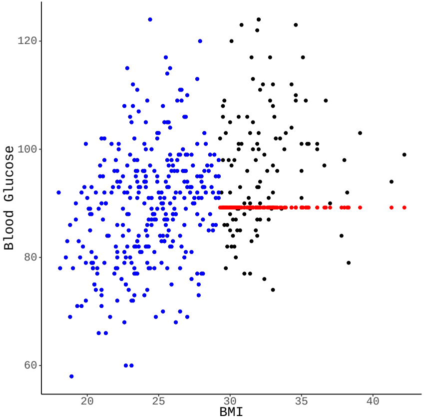

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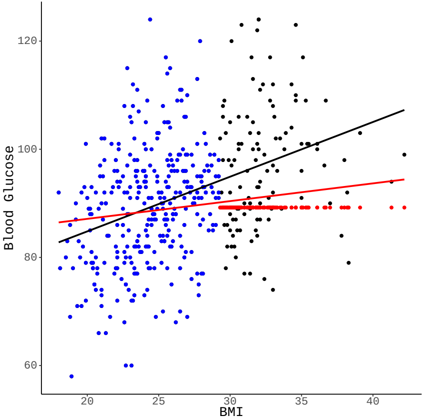
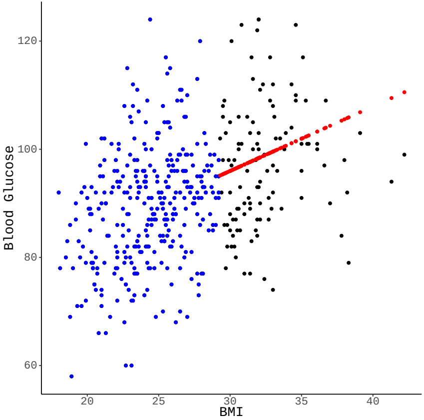


Bad Methods (These almost never work)

Deterministic Regression Imputation
(Conditional Mean Substitution)

- Replace Y_{mis} with $\widehat{Y}_{mis}$ from some regression equation
  - Positively biases regression slopes and correlations
  - Inflates measures of linear association



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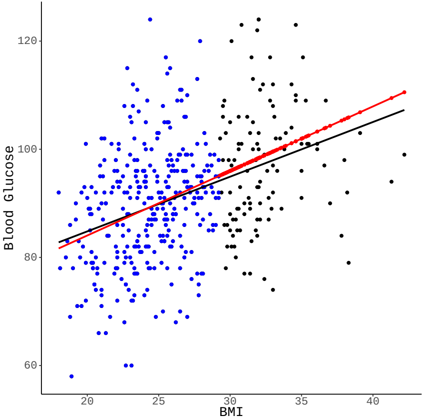

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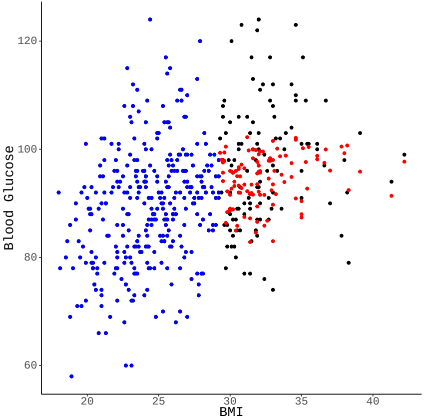

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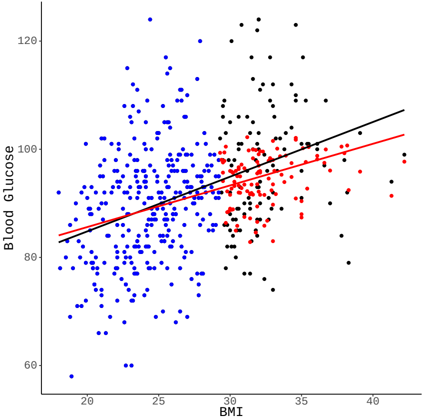


Bad Methods (These almost never work)

General Issues with Deletion-Based Methods

- Biased parameter estimates unless data are MCAR
- Generalizability issues

General Issues with Simple Single Imputation Methods

- Biased parameter estimates even when data are MCAR
- Attenuates variability in any treated variables

Bad Methods (These almost never work)

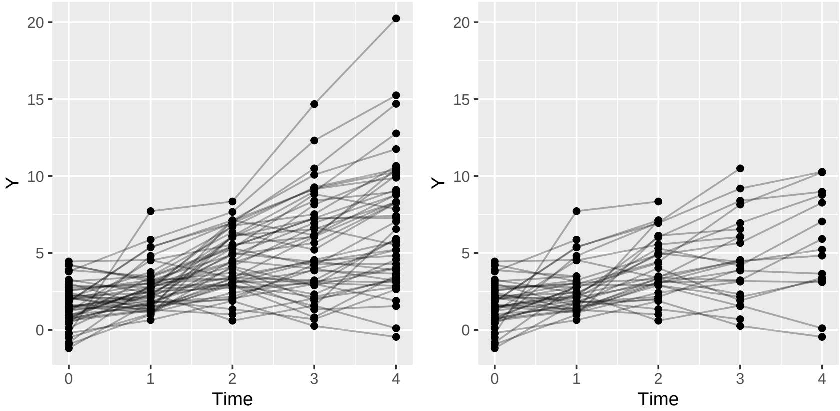
Averaging Available Items (Person-Mean Imputation)

- Compute aggregate scores using only available values
  - Missing data must be MCAR
  - Each item must contributes equally to the aggregate score

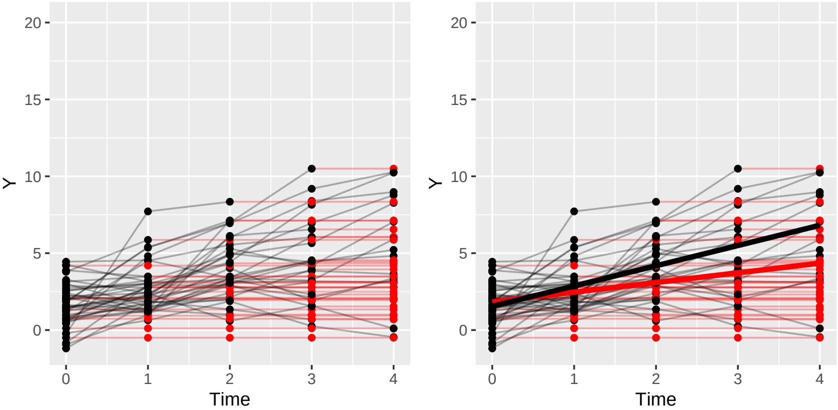
Last Observation Carried Forward (LOCF)

- Replace post-dropout values with the most recent observed value
  - Assume that dropouts would maintain their last known values
  - Attenuates estimates of growth/development

LOCF



LOCF



OK Methods (These work in some situations)

```
Error in `[.data.frame`(dat2, ,
c("bmi", "glu")):  undefined
columns selected
Error in complete(miceS, 1):
object 'miceS' not found
Error in datS[!mVec, ] <- NA:
object 'datS' not found
Error in fortify(data):  object
'datS' not found
Error:  geom_point requires the
following missing aesthetics:  x
and y
```

Stochastic Regression Imputation

- Fill Y_{mis} with $\widehat{Y}_{mis}$ plus some random noise.
  - Produces unbiased parameter estimates and predictions
  - Computationally efficient
  - Attenuates standard errors
  - Makes CIs and prediction

OK Methods (These work in some situations)

Stochastic Regression Imputation

- Fill Y_{mis} with $\widehat{Y}_{mis}$ plus some random noise.
  - Produces unbiased parameter estimates and predictions
  - Computationally efficient
  - Attenuates standard errors
  - Makes CIs and prediction intervals too narrow

```
Error in FUN(X[[i]], ...):
object 'bmi' not found
```

OK Methods (These work in some situations)

Nonresponse Weighting

- Weight the observed cases to correct for nonresponse bias
  - Popular in survey research and official statistics
  - Only worth considering with *Unit Nonresponse*
  - Doesn't make any sense with *Item Nonresponse*

Expectation Maximization

Good Methods (These almost always work)

Multiple Imputation (MI)

- Replace the missing values with M plausible estimates
  - Essentially, a repeated application of stochastic regression imputation (with a particular type of regression model)
  - Produces unbiased parameter estimates and predictions
  - Produces “correct” standard errors, CIs, and prediction intervals
  - Very, very flexible
  - Computationally expensive

Good Methods (These almost always work)

What happens when we apply MI to our previous MAR example?

```
## Estimate imputation model:
miceOut1 <- mice(data      = data.frame(y3, x),
                 m         = 100,
                 maxit     = 1,
                 method    = c("norm", ""),
                 printFlag = FALSE)

Error in data.frame(y3, x):  object 'y3' not found

## Replace missing values with imputations:
impList1 <- list()
for(m in 1 : miceOut1$m)
    impList1[[m]] <- complete(miceOut1, m)

Error in eval(expr, envir, enclos):  object 'miceOut1' not found
```

Good Methods (These almost always work)

```
## Estimate M correlations:
corList <-lapply(impList1,
                 FUN = function(impDat)
                     cor(impDat$x, impDat$y3)
                 )

## Pool estimates:
mean(unlist(corList))

[1] NA
```

The MI-based parameter estimate looks good.

- MI produces unbiased estimates of the parameter when data are MAR.

Good Methods (These almost always work)

```
Error in density.default(y):  'x' contains missing values
Error in impList1[[m]]:  subscript out of bounds
Error in plot.window(...):  need finite 'xlim' values
```

Good Methods (These almost always work)

Error in im

Good Methods (These almost always work)

What about applying MI to our MNAR example?

```
## Estimate imputation model:
miceOut2 <- mice(data      = data.frame(y4, x),
                 m         = 100,
                 maxit     = 1,
                 method    = c("norm", ""),
                 printFlag = FALSE)

Error in data.frame(y4, x):  object ’y4’ not found

## Replace missing values with imputations:
impList2 <- list()
for(m in 1 : miceOut2$m)
    impList2[[m]] <- complete(miceOut2, m)

Error in eval(expr, envir, enclos):  object ’miceOut2’ not found
```

Good Methods (These *almost* always work)

```
## Estimate M correlations:
corList2 <-lapply(impList2,
                  FUN = function(impDat)
                      cor(impDat$x, impDat$y4)
                  )

## Pool estimates:
mean(unlist(corList2))

[1] NA
```

The MI-based parameter estimate is still biased.

- MI cannot correct bias in parameter estimates when data are MNAR.

Good Methods (These *almost* always work)

```
Error in density(y4, na.rm = TRUE): object 'y4' not found
Error in impList2[[m]]:  subscript out of bounds
Error in plot.window(...):  need finite 'xlim' values
```

Good Methods (These *almost* always work)

Error in im

Good Methods (These almost always work)

Bayesian Modeling

- Treat missing values as just another parameter to be estimated
  - Models can be directly estimated in the presence of missing data
    - Essentially, runs MI behind-the-scenes during model estimation
  - The predictors of nonresponse must be included in the model, somehow
  - Computationally expensive

Good Methods (These almost always work)

Full Information Maximum Likelihood (FIML)

- Adjust the objective function to only consider the observed parts of the data
  - Models are directly estimated in the presence of missing data
  - The predictors of nonresponse must be included in the model, somehow
  - Unless you write your own optimization program, FIML is only available for certain types of models
  - In linear regression models, FIML cannot treat missing data on predictors (if the predictors are taken as fixed)

References

Little, R. J. A. (1988). Missing-data adjustments in large surveys. *Journal of Business & Economic Statistics*, *6*(3), 287–296. doi: 10.1080/07350015.1988.10509663