Missing Data Mechanisms

Utrecht University Winter School: Missing Data in R

Kyle M. Lang

Department of Methodology & Statistics
Utrecht University

Outline

What are Missing Data?

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Not every empty cell is a missing datum.

- Quality-of-life ratings for dead patients in a mortality study
- Firm profitability after the company goes out of business
- Self-reported severity of menstrual cramping for men
- Empty blocks of data following "gateway" items

A Little Notation

$$
\begin{aligned}
Y &:= \text{An } N \times P \text{ Matrix of Arbitrary Data} \\
Y_{mis} &:= \text{The } \textit{missing} \text{ part of } Y \\
Y_{obs} &:= \text{The } \textit{observed} \text{ part of } Y \\
R &:= \text{An } N \times P \text{ response matrix} \\
M &:= \text{An } N \times P \text{ missingness matrix}
\end{aligned}
$$

The R and M matrices are complementary.

- $r_{np} = 1$ means y_{np} is observed; $m_{np} = 1$ means y_{np} is missing.
- $r_{np} = 0$ means y_{np} is missing; $m_{np} = 0$ means y_{np} is observed.
- M_p is the *missingness* of Y_p.

Missing Data Mechanisms

Missing Completely at Random (MCAR)

- $P(R|Y_{mis}, Y_{obs}) = P(R)$
- Missingness is unrelated to any study variables.

Missing at Random (MAR)

- $P(R|Y_{mis}, Y_{obs}) = P(R|Y_{obs})$
- Missingness is related to only the *observed* parts of study variables.

Missing not at Random (MNAR)

- $P(R|Y_{mis}, Y_{obs}) \neq P(R|Y_{obs})$
- Missingness is related to the *unobserved* parts of study variables.

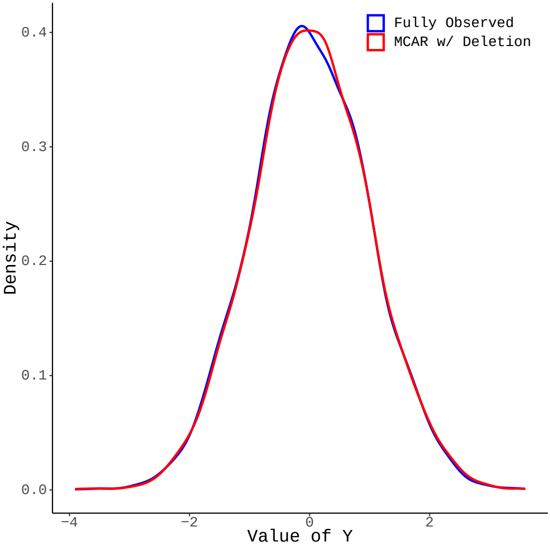
Simulate Some Toy Data

```
library(mvtnorm); library(dplyr); library(magrittr)

set.seed(235711)

nObs <- 5000 # Sample Size
pm   <- 0.3  # Proportion Missing

sigma <- matrix(c(1.0, 0.5, 0.3,
                  0.5, 1.0, 0.0,
                  0.3, 0.0, 1.0),
                ncol = 3)
dat0 <- rmvnorm(nObs, c(0, 0, 0), sigma) %>% data.frame()
colnames(dat0) <- c("x", "y", "z")

dat0 %$% cor(y, x)

[1] 0.4997145
```

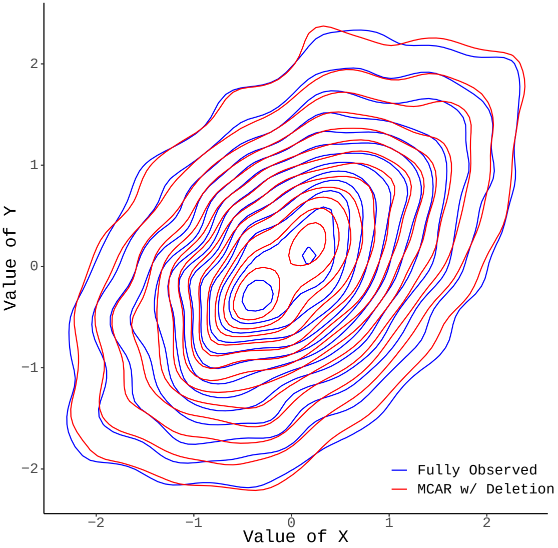
MCAR Example

```r
## Simulate MCAR Missingness:
m <- sample(1:nObs, size = pm * nObs)

## Impose MCAR missing on Y:
mcarData           <- dat0
mcarData[m, "y"] <- NA

## Check the correlation between X & Y:
mcarData %$% cor(y, x, use = "pairwise")

[1] 0.5195767
```

MCAR Example



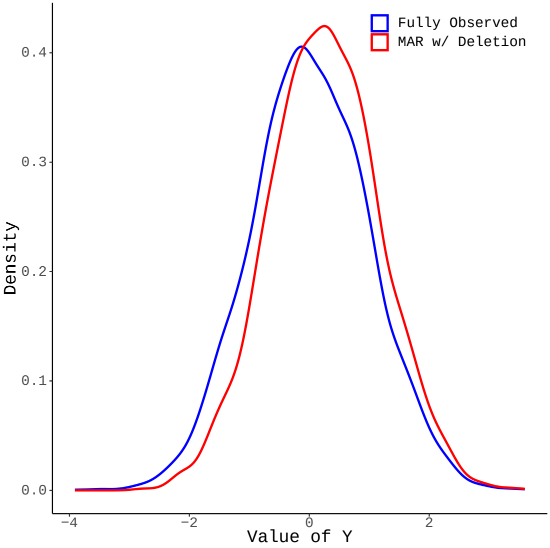
MCAR Example



MAR Example

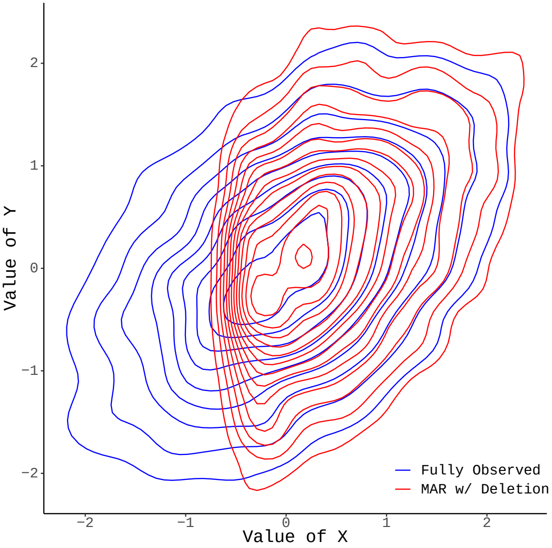
```r
## Simulate MAR Missingness:
m <- with(dat0, x < quantile(x, probs = pm))

## Impose MAR missing on Y:
marData            <- dat0
marData[m, "y"] <- NA

## Check the correlation between X & Y:
marData %$% cor(y, x, use = "pairwise")

[1] 0.3822143
```

MAR Example



Fully Observed
MAR w/ Deletion
0.4
0.3
0.2
0.1
0.0
Density
−4
−2
0
2
Value of Y

MAR Example

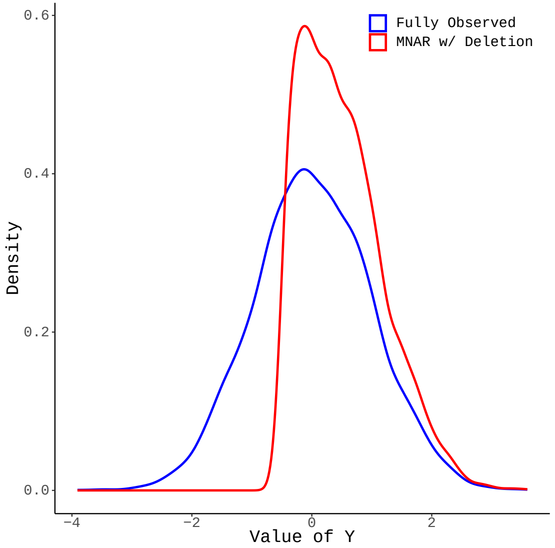


Value of Y
2
1
0
−1
−2
−2
−1
0
1
2
Value of X
Fully Observed
MAR w/ Deletion

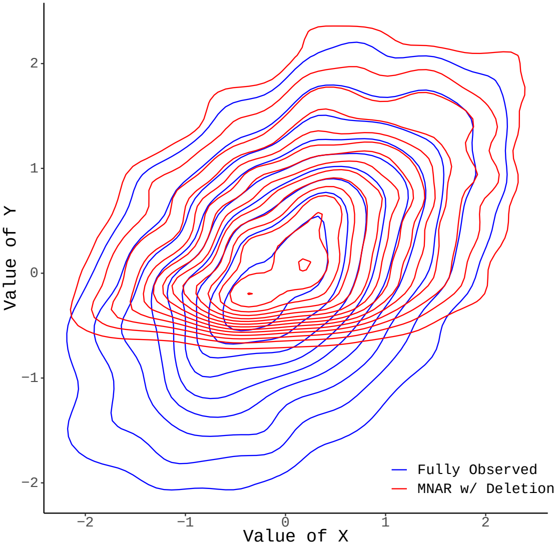
MNAR Example

```
## Simulate MNAR Missingness:
m <- with(dat0, y < quantile(y, probs = pm))

## Impose MNAR missing on Y:
mnarData            <- dat0
mnarData[m, "y"] <- NA

## Check the correlation between X & Y:
mnarData %$% cor(y, x, use = "pairwise")

[1] 0.3902962
```

MNAR Example



MNAR Example



Crucial Nuance

In our previous MAR example, ignoring the predictor of missingness actually produces *Indirect MNAR*.

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QUESTION: What happens if we ignore the predictor of missingness, but that predictor is independent of our study variables?

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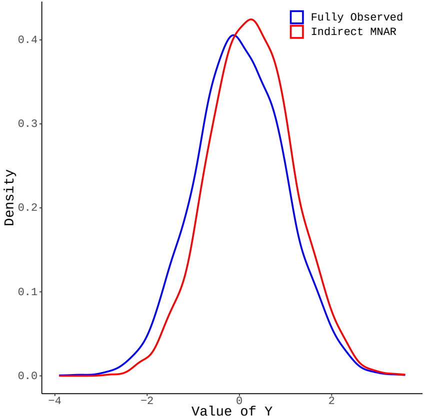
QUESTION: What happens if we ignore the predictor of missingness, but that predictor is independent of our study variables?

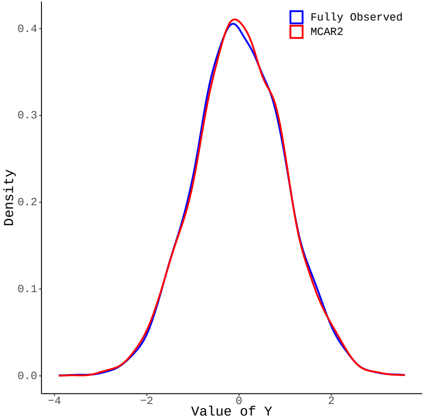
```
m <- with(dat0, z < quantile(z, probs = pm))

mcarData2           <- dat0
mcarData2[m, "y"] <- NA

mcarData2 %$% cor(y, x, use = "pairwise")

[1] 0.5118075
```

ANSWER: We get back to MCAR :)

Crucial Nuance

The missing data mechanisms are not simply characteristics of an incomplete dataset; we also need to account for the analysis.





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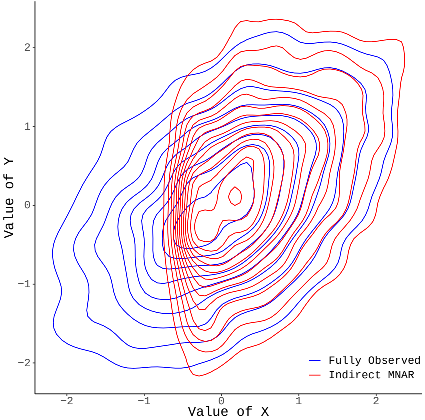

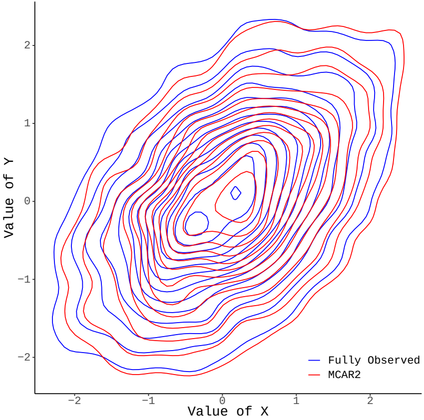

Testing the Missing Data Mechanism

We cannot fully test the MAR or MNAR assumptions.

- To do so would require knowing the values of the missing data.
- We can find observed predictors of missingness.
  - Use classification algorithms to predict missingness from Y_{obs}.
  - We can never know that we have discovered all MAR predictors.
- In practice, MAR and MNAR live on the ends of a continuum.
  - Our missing data problem exists at some unknown point along this continuum.
  - We can do a lot to nudge our problem towards the MAR side.

Testing the Missing Data Mechanism

We can (partially) test the MCAR assumption.

- With MCAR, the missing data and the observed data should have the same distribution.
- We can test for MCAR by testing the distributions of *auxiliary variables*, $\mathbf{Z}$.
    - Use a t-test to compare the subset of Z_p that corresponds to Y_{mis} to the subset corresponding to Y_{obs}.
    - The Little (1988) MCAR test is a multivariate version of this.

These procedures actually test if the data are *observed* completely at random.

References

Little, R. J. A. (1988). A test of missing completely at random for multivariate data with missing values. *Journal of the American Statistical Association*, *83*(404), 1198–1202. doi: 10.1080/01621459.1988.10478722