

# Moderation

Utrecht University Winter School: Regression in R



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# Outline

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## Moderation

Testing Moderation

## Probing Interactions

## Categorical Moderators



# Moderation

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So far we've been discussing *additive models*.

- Additive models allow us to examine the partial effects of several predictors on some outcome.
  - The effect of one predictor does not change based on the values of other predictors.

Now, we'll discuss *moderation*.

- Moderation allows us to ask *when* one variable,  $X$ , affects another variable,  $Y$ .
  - We're considering the conditional effects of  $X$  on  $Y$  given certain levels of a third variable  $Z$ .



# Equations

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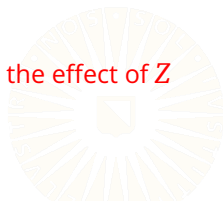
In additive MLR, we might have the following equation:

$$Y = \beta_0 + \beta_1 X + \beta_2 Z + \varepsilon$$

This additive equation assumes that  $X$  and  $Z$  are independent predictors of  $Y$ .

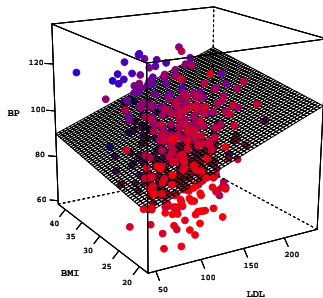
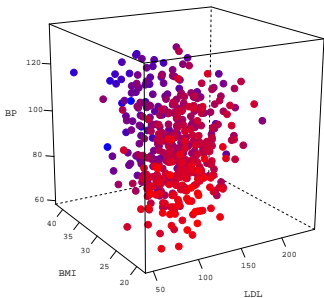
When  $X$  and  $Z$  are independent predictors, the following are true:

- $X$  and  $Z$  *can* be correlated.
- $\beta_1$  and  $\beta_2$  are *partial* regression coefficients.
- The effect of  $X$  on  $Y$  is the same at **all levels** of  $Z$ , and the effect of  $Z$  on  $Y$  is the same at **all levels** of  $X$ .



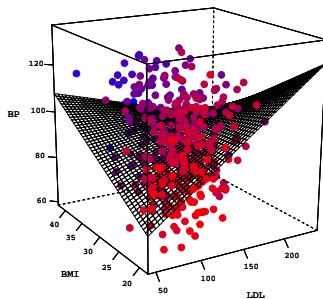
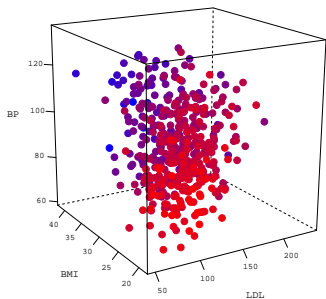
# Additive Regression

The effect of  $X$  on  $Y$  is the same at **all levels** of  $Z$ .



# Moderated Regression

The effect of  $X$  on  $Y$  varies **as a function** of  $Z$ .



# Equations

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The following derivation is adapted from Hayes (2018).

- When testing moderation, we hypothesize that the effect of  $X$  on  $Y$  varies as a function of  $Z$ .
- We can represent this concept with the following equation:

$$Y = \beta_0 + f(Z)X + \beta_2Z + \varepsilon \quad (1)$$



# Equations

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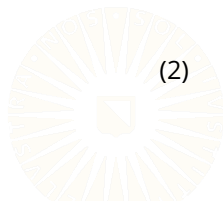
The following derivation is adapted from Hayes (2018).

- When testing moderation, we hypothesize that the effect of  $X$  on  $Y$  varies as a function of  $Z$ .
- We can represent this concept with the following equation:

$$Y = \beta_0 + f(Z)X + \beta_2Z + \varepsilon \quad (1)$$

- If we assume that  $Z$  linearly (and deterministically) affects the relationship between  $X$  and  $Y$ , then we can take:

$$f(Z) = \beta_1 + \beta_3Z \quad (2)$$





# Equations

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- Substituting Equation 2 into Equation 1 leads to:

$$Y = \beta_0 + (\beta_1 + \beta_3 Z)X + \beta_2 Z + \varepsilon$$



# Equations

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- Substituting Equation 2 into Equation 1 leads to:

$$Y = \beta_0 + (\beta_1 + \beta_3 Z)X + \beta_2 Z + \varepsilon$$

- Which, after distributing  $X$  and reordering terms, becomes:

$$Y = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 XZ + \varepsilon$$



# Testing Moderation

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Now, we have an estimable regression model that quantifies the linear moderation we hypothesized.

$$Y = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 XZ + \varepsilon$$

- To test for significant moderation, we simply need to test the significance of the interaction term,  $XZ$ .
  - Check if  $\hat{\beta}_3$  is significantly different from zero.



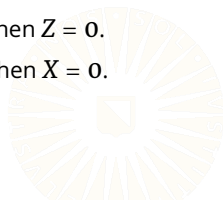
# Interpretation

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Given the following equation:

$$Y = \hat{\beta}_0 + \hat{\beta}_1 X + \hat{\beta}_2 Z + \hat{\beta}_3 XZ + \hat{\varepsilon}$$

- $\hat{\beta}_3$  quantifies the effect of  $Z$  on the focal effect (the  $X \rightarrow Y$  effect).
  - For a unit change in  $Z$ ,  $\hat{\beta}_3$  is the expected change in the effect of  $X$  on  $Y$ .
- $\hat{\beta}_1$  and  $\hat{\beta}_2$  are *conditional effects*.
  - Interpreted where the other predictor is zero.
  - For a unit change in  $X$ ,  $\hat{\beta}_1$  is the expected change in  $Y$ , when  $Z = 0$ .
  - For a unit change in  $Z$ ,  $\hat{\beta}_2$  is the expected change in  $Y$ , when  $X = 0$ .



# Example

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Still looking at the *diabetes* dataset.

- We suspect that patients' BMIs are predictive of their average blood pressure.
- We further suspect that this effect may be differentially expressed depending on the patients' LDL levels.



# Example

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```
## Focal Effect:
```

```
out0 <- lm(bp ~ bmi, data = dDat)
```

```
partSummary(out0, -c(1, 2))
```

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	61.9973	3.6659	16.91	<2e-16
bmi	1.2379	0.1371	9.03	<2e-16

Residual standard error: 12.72 on 440 degrees of freedom

Multiple R-squared: 0.1563, Adjusted R-squared: 0.1544

F-statistic: 81.54 on 1 and 440 DF, p-value: < 2.2e-16

# Example

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```
## Additive Model:
```

```
out1 <- lm(bp ~ bmi + ldl, data = dDat)  
partSummary(out1, -c(1, 2))
```

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	59.26577	3.91281	15.147	< 2e-16
bmi	1.16567	0.14156	8.235	2.08e-15
ldl	0.04016	0.02056	1.953	0.0515

Residual standard error: 12.68 on 439 degrees of freedom

Multiple R-squared: 0.1636, Adjusted R-squared: 0.1598

F-statistic: 42.94 on 2 and 439 DF, p-value: < 2.2e-16

# Example

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```
## Moderated Model:
```

```
out2 <- lm(bp ~ bmi * ldl, data = dDat)
partSummary(out2, -c(1, 2))
```

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	14.480616	14.291677	1.013	0.311514
bmi	2.867825	0.541312	5.298	1.86e-07
ldl	0.448771	0.127160	3.529	0.000461
bmi:ldl	-0.015352	0.004716	-3.255	0.001221

Residual standard error: 12.54 on 438 degrees of freedom

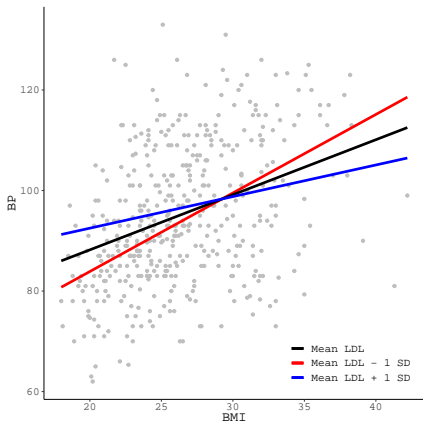
Multiple R-squared: 0.1834, Adjusted R-squared: 0.1778

F-statistic: 32.78 on 3 and 438 DF, p-value: < 2.2e-16



# Visualizing the Interaction

We can get a better idea of the patterns of moderation by plotting the focal effect at conditional values of the moderator.

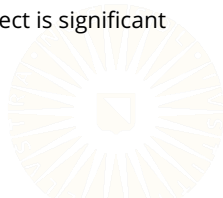


# Probing the Interaction

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A significant estimate of  $\beta_3$  tells us that the effect of  $X$  on  $Y$  depends on the level of  $Z$ , but nothing more.

- The plot on the previous slide gives a descriptive illustration of the pattern, but does not support statistical inference.
  - The three conditional effects we plotted look different, but we cannot say much about how they differ with only the plot and  $\hat{\beta}_3$ .
- This is the purpose of *probing* the interaction.
  - Try to isolate areas of  $Z$ 's distribution in which  $X \rightarrow Y$  effect is significant and areas where it is not.



# Probing the Interaction

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The most popular method of probing interactions is to do a so-called *simple slopes* analysis.

- Pick-a-point approach
- Spotlight analysis

In simple slopes analysis, we test if the slopes of the conditional effects plotted above are significantly different from zero.

- To do so, we test the significance of *simple slopes*.



# Simple Slopes

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Recall the derivation of our moderated equation:

$$Y = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 XZ + \varepsilon$$

We can reverse the process by factoring out  $X$  and reordering terms:

$$Y = \beta_0 + (\beta_1 + \beta_3 Z)X + \beta_2 Z + \varepsilon$$

Where  $f(Z) = \beta_1 + \beta_3 Z$  is the linear function that shows how the relationship between  $X$  and  $Y$  changes as a function of  $Z$ .

$f(Z)$  is the *simple slope*.

- By plugging different values of  $Z$  into  $f(Z)$ , we get the value of the conditional effect of  $X$  on  $Y$  at the chosen level of  $Z$ .

# Significance Testing of Simple Slopes

The values of  $Z$  used to define the simple slopes are arbitrary.

- The most common choice is:  $\{(\bar{Z} - SD_Z), \bar{Z}, (\bar{Z} + SD_Z)\}$
- You could also use interesting percentiles of  $Z$ 's distribution.

The standard error of a simple slope is given by:

$$SE_{f(Z)} = \sqrt{SE_{\beta_1}^2 + 2Z \cdot \text{COV}(\beta_1, \beta_3) + Z^2 SE_{\beta_3}^2}$$

So, you can test the significance of a simple slope by constructing a Wald statistic or confidence interval using  $\hat{f}(Z)$  and  $SE_{f(Z)}$ :

$$t = \frac{\hat{f}(Z)}{SE_{f(Z)}}, \quad CI = \hat{f}(Z) \pm t_{crit} \times SE_{f(Z)}$$



# Interaction Probing

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When probed the interaction with simple slopes analysis:

1. Choose interesting values of the moderator,  $Z$ .
2. Check the significance of the focal effect,  $X \rightarrow Y$ , at the  $Z$  values chosen in Step 1.
3. Use the results from Step 2 to get an idea of where in  $Z$ 's distribution the focal effect is or is not significant.



# Interaction Probing

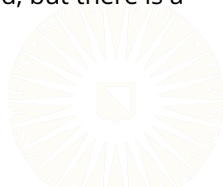
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When probed the interaction with simple slopes analysis:

1. Choose interesting values of the moderator,  $Z$ .
2. Check the significance of the focal effect,  $X \rightarrow Y$ , at the  $Z$  values chosen in Step 1.
3. Use the results from Step 2 to get an idea of where in  $Z$ 's distribution the focal effect is or is not significant.

We saw manual calculations for the the quantities needed, but there is a simpler way:

- CENTERING



# Centering

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Centering shifts the scale of a variable up or down by subtracting a constant (e.g., the variable's mean) from each of its observations.

- The most familiar form of center is *mean centering*.
- We can center on any value.
  - When probing interactions, we can center  $Z$  on the interesting values we choose to define the simple slopes.
  - Due to the interpretation of conditional effects, running the model with  $Z$  centered on a specific value automatically provides a test of the simple slope for that value of  $Z$ .





# Probing via Centering

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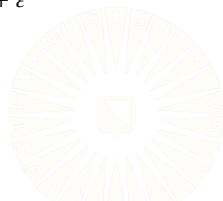
Say we want to do a simple slopes analysis to test the conditional effect of  $X$  on  $Y$  at three levels of  $Z = \{Z_1, Z_2, Z_3\}$ .

- All we need to do is fit the following three models:

$$Y = \beta_0 + \beta_1 X + \beta_2 (Z - Z_1) + \beta_3 X (Z - Z_1) + \varepsilon$$

$$Y = \beta_0 + \beta_1 X + \beta_2 (Z - Z_2) + \beta_3 X (Z - Z_2) + \varepsilon$$

$$Y = \beta_0 + \beta_1 X + \beta_2 (Z - Z_3) + \beta_3 X (Z - Z_3) + \varepsilon$$



# Probing via Centering

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Say we want to do a simple slopes analysis to test the conditional effect of  $X$  on  $Y$  at three levels of  $Z = \{Z_1, Z_2, Z_3\}$ .

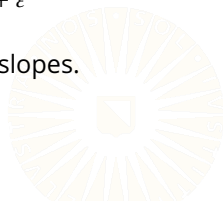
- All we need to do is fit the following three models:

$$Y = \beta_0 + \beta_1 X + \beta_2(Z - Z_1) + \beta_3 X(Z - Z_1) + \varepsilon$$

$$Y = \beta_0 + \beta_1 X + \beta_2(Z - Z_2) + \beta_3 X(Z - Z_2) + \varepsilon$$

$$Y = \beta_0 + \beta_1 X + \beta_2(Z - Z_3) + \beta_3 X(Z - Z_3) + \varepsilon$$

- The default output for  $\hat{\beta}_1$  provides tests of the simple slopes.



# Example

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Create transformed predictors by centering on critical values of the moderator,  $Z_{LDL}$ .

```
zMean    <- mean(dDat$lcl)
zSD       <- sd(dDat$lcl)
dDat$zCen <- dDat$lcl - zMean
dDat$zHi  <- dDat$lcl - (zMean + zSD)
dDat$zLo  <- dDat$lcl - (zMean - zSD)
```



# Example

Test the simple slope of  $X_{BMI} \rightarrow Y_{BP}$  at 1 SD below the mean of  $Z_{LDL}$ .

```
out2.1 <- lm(bp ~ bmi*zLo, data = dDat)
partSummary(out2.1, -c(1, 2))
```

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	52.637886	4.764883	11.047	< 2e-16
bmi	1.562525	0.185667	8.416	5.59e-16
zLo	0.448771	0.127160	3.529	0.000461
bmi:zLo	-0.015352	0.004716	-3.255	0.001221

Residual standard error: 12.54 on 438 degrees of freedom

Multiple R-squared: 0.1834, Adjusted R-squared: 0.1778

F-statistic: 32.78 on 3 and 438 DF, p-value: < 2.2e-16

The estimated slope for  $bmi$ ,  $\hat{\beta}_1 = 1.563$ , is the simple slope.

# Example

Test the simple slope of  $X_{BMI} \rightarrow Y_{BP}$  at the mean of  $Z_{LDL}$ .

```
out2.2 <- lm(bp ~ bmi*zCen, data = dDat)
partSummary(out2.2, -c(1, 2))
```

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	66.286409	3.812480	17.387	< 2e-16
bmi	1.095631	0.141679	7.733	7.27e-14
zCen	0.448771	0.127160	3.529	0.000461
bmi:zCen	-0.015352	0.004716	-3.255	0.001221

Residual standard error: 12.54 on 438 degrees of freedom

Multiple R-squared: 0.1834, Adjusted R-squared: 0.1778

F-statistic: 32.78 on 3 and 438 DF, p-value: < 2.2e-16

The estimated slope for  $bmi$ ,  $\hat{\beta}_1 = 1.096$ , is the simple slope.

# Example

Test the simple slope of  $X_{BMI} \rightarrow Y_{BP}$  at 1 SD above the mean of  $Z_{LDL}$ .

```
out2.3 <- lm(bp ~ bmi*zHi, data = dDat)
partSummary(out2.3, -c(1, 2))
```

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	79.934933	6.023132	13.271	< 2e-16
bmi	0.628736	0.216369	2.906	0.003848
zHi	0.448771	0.127160	3.529	0.000461
bmi:zHi	-0.015352	0.004716	-3.255	0.001221

Residual standard error: 12.54 on 438 degrees of freedom

Multiple R-squared: 0.1834, Adjusted R-squared: 0.1778

F-statistic: 32.78 on 3 and 438 DF, p-value: < 2.2e-16

The estimated slope for  $bmi$ ,  $\hat{\beta}_1 = 0.629$ , is the simple slope.

# Compare Approaches

The manual and the centering approaches give identical answers, barring rounding errors:

	Z Low	Z Center	Z High
Manual	1.562525	1.095631	0.628736
Centering	1.562525	1.095631	0.628736

## Simple Slopes

	Z Low	Z Center	Z High
Manual	0.185667	0.141679	0.216369
Centering	0.185667	0.141679	0.216369

## Standard Errors

# Categorical Moderators

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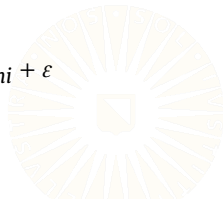
Categorical moderators encode *group-specific* effects.

- E.g., if we include *sex* as a moderator, we are modeling separate focal effects for males and females.

Given a set of codes representing our moderator, we specify the interactions as before:

$$Y_{total} = \beta_0 + \beta_1 X_{inten} + \beta_2 Z_{male} + \beta_3 X_{inten} Z_{male} + \varepsilon$$

$$Y_{total} = \beta_0 + \beta_1 X_{inten} + \beta_2 Z_{lo} + \beta_3 Z_{mid} + \beta_4 Z_{hi} \\ + \beta_5 X_{inten} Z_{lo} + \beta_6 X_{inten} Z_{mid} + \beta_7 X_{inten} Z_{hi} + \varepsilon$$





# Example

---

```
## Load data:
socSup <- readRDS(paste0(dataDir, "social_support.rds"))

## Focal effect:
out3 <- lm(bdi ~ tanSat, data = socSup)
partSummary(out3, -c(1, 2))
```

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	24.4089	5.3502	4.562	1.54e-05
tanSat	-0.8100	0.3124	-2.593	0.0111

Residual standard error: 9.278 on 93 degrees of freedom

Multiple R-squared: 0.06742, Adjusted R-squared: 0.05739

F-statistic: 6.723 on 1 and 93 DF, p-value: 0.01105

# Example

---

```
## Estimate the interaction:
```

```
out4 <- lm(bdi ~ tanSat * sex, data = socSup)
partSummary(out4, -c(1, 2))
```

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	20.8478	6.2114	3.356	0.00115
tanSat	-0.5772	0.3614	-1.597	0.11372
sexmale	14.3667	12.2054	1.177	0.24223
tanSat:sexmale	-0.9482	0.7177	-1.321	0.18978

Residual standard error: 9.267 on 91 degrees of freedom

Multiple R-squared: 0.08955, Adjusted R-squared: 0.05954

F-statistic: 2.984 on 3 and 91 DF, p-value: 0.03537

# Example

---

On the last slide, the estimated slope for `tanSat`,  $\hat{\beta}_1 = -0.577$ , is the simple slope for females.

- To estimate the simple slope for males, we simply change the reference group of the `sex` factor and re-estimate the model.

```
## Test the 'male' simple slope by changing reference group:
```

```
socSup$sex2 <- relevel(socSup$sex, ref = "male")
```

```
## Re-estimate the interaction:
```

```
out5 <- lm(bdi ~ tanSat * sex2, data = socSup)
```

# Example

```
partSummary(out5, -c(1, 2))
```

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	35.2146	10.5067	3.352	0.00117
tanSat	-1.5254	0.6201	-2.460	0.01579
sex2female	-14.3667	12.2054	-1.177	0.24223
tanSat:sex2female	0.9482	0.7177	1.321	0.18978

Residual standard error: 9.267 on 91 degrees of freedom

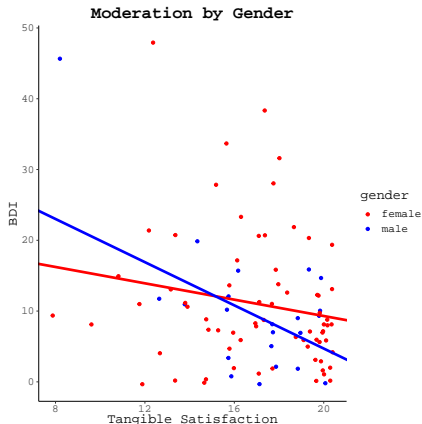
Multiple R-squared: 0.08955, Adjusted R-squared: 0.05954

F-statistic: 2.984 on 3 and 91 DF, p-value: 0.03537

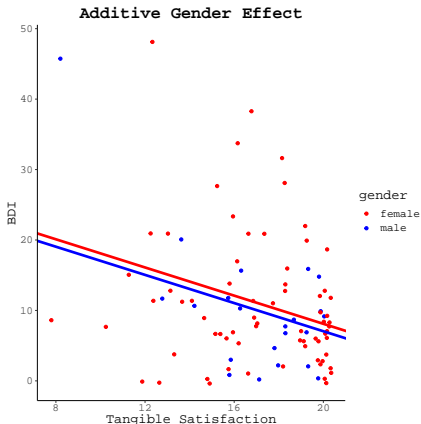
The estimated slope for tanSat,  $\hat{\beta}_1 = -1.525$ , is now the simple slope for males.

# Visualizing Categorical Moderation

$$\hat{Y}_{BDI} = 20.85 - 0.58X_{tsat} + 14.37Z_{male} - 0.95X_{tsat}Z_{male}$$



$$\hat{Y}_{BDI} = 28.10 - 1.00X_{tsat} - 1.05Z_{male}$$



# References

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Hayes, A. F. (2018). *Introduction to mediation, moderation, and conditional process analysis: A regression-based approach* (2nd ed.). New York: Guilford Press.

