Calculating grades of multiple choice exams

October 13, 2020

This document describes how grades of multiple choice (MC) exams are calculated at Tilburg University, as of October 13, 2020.

The knowledge score

After the examiner has specified the number of items on the MC exam, *L,* and the number of answering categories for each item, *a*, the *guessing score, g,* (i.e., the expected number of correct answers on the MC exam when randomly guessing all answers) is defined as $g = L/a$. Using a student's number of correct answers on the test, *X*, the guessing score, and the number of items on the test, the student's *knowledge score, k,* is calculated as

$$k = max\left(\frac{X - L/a}{L - L/a}, 0\right).$$

In words, the student's knowledge score, *k,* indicates how much (s)he scores above chance level as a proportion between 0 (if *X* is equal to the guessing score, *g*) and 1 (if *X* is equal to the maximum score, *L*). The "*max*" in the formula indicates that if the student answered less items correctly than can be expected by chance (i.e., if $X < g$), then *k* is set to 0.

Calculating the grade

The values of two parameters are needed to transform the knowledge score into an unrounded grade; the minimum grade, *min*, and the knowledge score required to pass the exam, here called *pass*. The minimum grade is set to 0 for all TiSEM exams and 1 for the exams of all other TiU faculties. The default and recommended value of *pass* is 0.55, which means that a student's knowledge score should be at least 0.55 to pass the exam. For example, if a test has 40 items with 4 categories, then $pass = k = 0.55$ corresponds to $X = 26.5$. The examiner can choose another value for *pass* than 0.55, although it is not recommended to lower the value of *pass*.

The formula for calculating the unrounded grade is a piecewise linear equation:

If $k < pass$	then	$grade = min + k \times (5.5 - min)/pass$
If $k \geq pass$	then	$grade = 5.5 + \frac{k - pass}{1 - pass} \times 4.5$

Figure 1 on the next page illustrates graphically how the unrounded grade is calculated for TiSEM and other TiU faculties using this formula, assuming $pass = 0.55$. If the examiner chooses another value of *pass*, then point *Q* in the graph will shift along the horizontal line at $grade = 5.5$. More specifically, the vertical bar at $k = 0.55$ shifts to that value of *pass* while the lower line segment (for $k < pass$) will end at the new position of *Q*, and the upper line segment ($k \geq pass$) will originate at *Q*. See below Figure 1 for an extended example with two values of *pass* (0.55 and 0.7) for an MC exam with 40 items and 4 categories.

Finally, the unrounded grade is rounded to the "nearest half" excluding 5.5 (…, 4, 4.5, 5, 6, 6.5, 7, …). Unrounded grades in the interval [4.75, 5.499] are rounded to 5, whereas unrounded grades in the interval [5.5, 6.249] are rounded to 6.

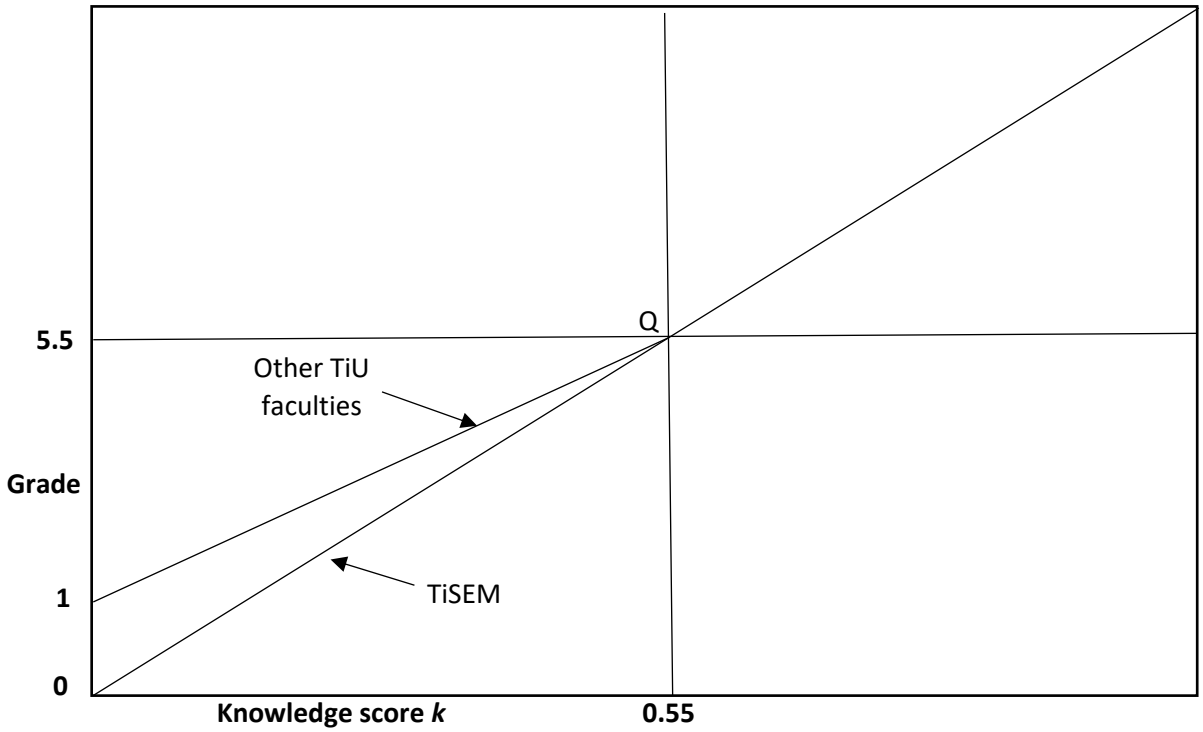

Figure 1: Grade, before rounding, as a function of knowledge score, *k,* for TiSEM and other TiU faculties, assuming that a pass corresponds to the TiU default knowledge score of 0.55.

Extended example

The table below provides the rounded grade as a function of the students' numbers of correct answers (2nd column) for a MC exam with 40 questions, 4 categories, minimum grade 1, and required passing scores equal to 0.55 or 0.7. The rounded grade corresponding to *pass* = 0.55 can be found in column 7, whereas the rounded grade corresponding to *pass* = 0.7 can be found in the last column.

			pass=0.55		pass=0.7	
	X	knowledge k	unrounded grade	grade	unrounded grade	grade
[1,]	5	0.00000000	1.000000	1.0	1.000000	1.0
[2,]	6	0.00000000	1.000000	1.0	1.000000	1.0
[3,]	7	0.00000000	1.000000	1.0	1.000000	1.0
[4,]	8	0.00000000	1.000000	1.0	1.000000	1.0
[5,]	9	0.00000000	1.000000	1.0	1.000000	1.0
[6,]	10	0.00000000	1.000000	1.0	1.000000	1.0
[7,]	11	0.03333333	1.272727	1.5	1.214286	1.0
[8,]	12	0.06666667	1.545455	1.5	1.428571	1.5
[9,]	13	0.10000000	1.818182	2.0	1.642857	1.5
[10,]	14	0.13333333	2.090909	2.0	1.857143	2.0
[11,]	15	0.16666667	2.363636	2.5	2.071429	2.0
[12,]	16	0.20000000	2.636364	2.5	2.285714	2.5
[13,]	17	0.23333333	2.909091	3.0	2.500000	2.5
[14,]	18	0.26666667	3.181818	3.0	2.714286	2.5
[15,]	19	0.30000000	3.454545	3.5	2.928571	3.0
[16,]	20	0.33333333	3.727273	3.5	3.142857	3.0
[17,]	21	0.36666667	4.000000	4.0	3.357143	3.5
[18,]	22	0.40000000	4.272727	4.5	3.571429	3.5
[19,]	23	0.43333333	4.545455	4.5	3.785714	4.0
[20,]	24	0.46666667	4.818182	5.0	4.000000	4.0

```
[21,] 25 0.50000000  5.090909   5.0  4.214286    4.0
[22,] 26 0.53333333  5.363636   5.0  4.428571    4.5
[23,] 27 0.56666667  5.666667   6.0  4.642857    4.5
[24,] 28 0.60000000  6.000000   6.0  4.857143    5.0
[25,] 29 0.63333333  6.333333   6.5  5.071429    5.0
[26,] 30 0.66666667  6.666667   6.5  5.285714    5.0
[27,] 31 0.70000000  7.000000   7.0  5.500000    6.0
[28,] 32 0.73333333  7.333333   7.5  6.000000    6.0
[29,] 33 0.76666667  7.666667   7.5  6.500000    6.5
[30,] 34 0.80000000  8.000000   8.0  7.000000    7.0
[31,] 35 0.83333333  8.333333   8.5  7.500000    7.5
[32,] 36 0.86666667  8.666667   8.5  8.000000    8.0
[33,] 37 0.90000000  9.000000   9.0  8.500000    8.5
[34,] 38 0.93333333  9.333333   9.5  9.000000    9.0
[35,] 39 0.96666667  9.666667   9.5  9.500000    9.5
[36,] 40 1.00000000 10.000000  10.0 10.000000   10.0
```