

Introduction to Structural Equation Modeling using lavaan

Intro and regression

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Outline of this lecture

SEM

SAPI

Lavaan commands

Regression

Path model

Steps

Technical output

Steps Ctd.

Model fit

The end

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Observed variable

Construct = latent (unmeasured) variable / factor

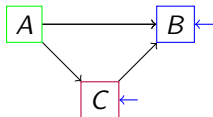
Direct relationship

Covariance

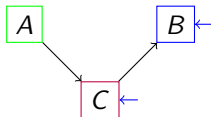
Variance

Residual variance / measurement error

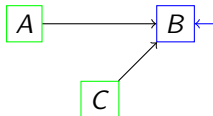
Path models



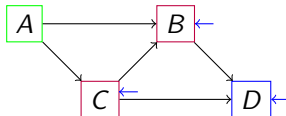
Partial mediation



Full mediation

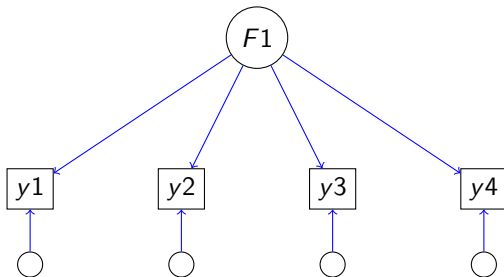


Multiple regression



More complex path models

Measurement models



SEM

- From theory to model (and path diagram)
- Compare

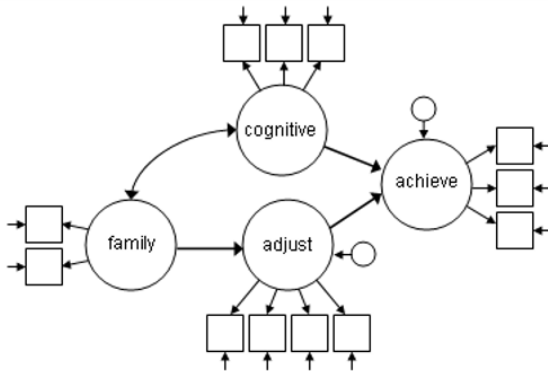


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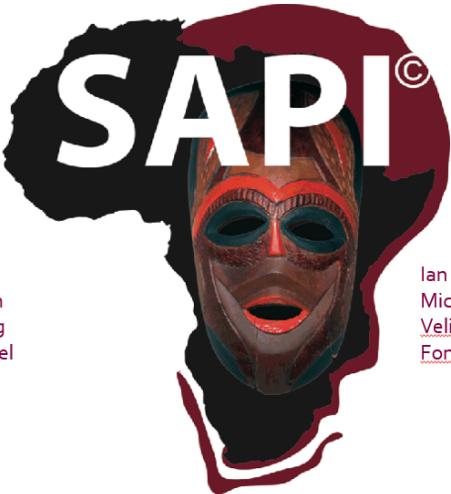
Technical output

Steps Ctd.

Model fit

The end

Example: South African Personality Inventory Project (SAPI)



Carin Hill
 Leon Jackson
 Deon Meiring
 J. Aleweyn Nel

Ian Rothmann
 Michael Temane
 Velichko H. Valchev
 Fons J. R. van de Vijver

SAPI details for this lecture

Our data:

- Selection of 1000 participants.
- Selection of 7 out of 262 personality items:
 - Regression:
 - Outcome: Q77, I enjoy telling funny stories
 - Predictor: Age
 - Path model:
 - Outcome: Q196, I make others laugh
 - Mediator: Q77, I enjoy telling funny stories
 - Predictor: Age
 - Later, we use: Age, ReadAb, Q44, Q63, and Q76.

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1. Loading data into R

For example, in case of a .txt file:

```
data_sapi <- read.table("Sapi.txt", header = T)

data_sapi[sapply(data_sapi,
  function(x) as.character(x) %in% c("-999") )] <- NA
```

3. Draw your model



4. Specify your model



- Translate \rightarrow into \sim .
- Translate 'residual variance notation' into $\sim\sim$.
Note = residual variance = a variance = special case covariance.
- Translate \leftrightarrow (= covariance) into $\sim\sim$.
- Do not forget about intercepts: Translate into \sim .
- Translate 'circled arrow' (= variance) into $\sim\sim$.
Note = variance = special case covariance.
- Do not forget about means: Translate into \sim .

4. Specify your model: regression SAPI



- Translate \rightarrow into \sim .
- Translate 'residual variance notation' into $\sim\sim$.
Note = residual variance = a variance = special case covariance.
- Translate \leftrightarrow (= covariance) into $\sim\sim$.
- Do not forget about intercepts: Translate into \sim .

```
mod.regr <- '
  Q77 ~ Age # regression
  Q77 ~~ Q77 # residual variance
  Q77 ~ 1   # intercept
'
```


4. Specify your model: regression general

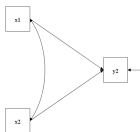
A regression model is specified as follows:
dependent \sim predictor1 + predictor2 + etc.

Additionally, one needs to specify that the error (of the dependent variable) has a variance:
 $\text{dependent} \sim \text{dependent}$

If needed, one can specify to include an intercept:
dependent ~ 1

Note that one include intercept directly:
dependent ~ 1 + predictor1 + predictor2 + etc.

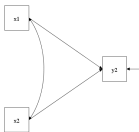
4. Specify your model: regression answer



```
mod.regr_DIY <- '
  y2 ~ 1 + x1 + x2  # intercept + regression
  y2 ~~ y2          # residual variance

  # Make it a habit to also specify:
  x1 ~ 1            # mean x1
  x2 ~ 1            # mean x2
  x1 ~~ x1          # variance x1
  x1 ~~ x2          # covariance x1 and x2
  x2 ~~ x2          # variance x2
'
```

4. Specify your model: regression answer 2



```
mod.regr_DIY <- '
  y2 ~ 1 + x1 + x2 # intercept + regression
  y2 ~~ y2         # residual variance

# Make it a habit to also specify:
x1 + x2 ~ 1        # means predictors - DO specify this!
x1 + x2 ~~ x1      # (co)variance predictors
x2 ~~ x2           # variance x2
'
```

Note: As opposed to what I say in the video, do specify the means for the predictors explicitly; otherwise they will be set to 0.

4. Specify your model: regression answer general

```
y ~ 1 + x_1 + x_2 + ... + x_k # intercept + regression
y ~~ y                        # residual variance
```

```
# (co)variances of exogenous variables:
```

```
x_1 + x_2 + ... + x_k ~~ x_1
      x_2 + ... + x_k ~~ x_2
                        ...
                        x_k ~~ x_k
```


5. Fit the regression model: All code

```
# Data
data_sapi <- read.table("Sapi.txt", header = T)
data_sapi[apply(data_sapi,
  function(x) as.character(x) %in% c("-999"))] <- NA

# Model
mod.regr <- '
  Q77 ~ Age # regression
  Q77 ~~ Q77 # residual variance
  Q77 ~ 1 # intercept
'

# Fit model
fit.regr <- lavaan(model = mod.regr, data = data_sapi)
```



```
lavaanPlot(model = fit.regr,
  graph_options = list(overlap = "true",
    fontsize = "20"),
  node_options = list(shape = "box",
    fontname = "Helvetica"),
  edge_options = list(color = "grey"),
  coefs = T, stand = T, covs = T,
)
```

Plot with tidySEM

```
library(tidySEM)
if (!require("tidySEM")) install.packages("tidySEM")
```

```
graph_sem(fit.regr)
```

Plot with semPlot

```
if (!require("semPlot")) install.packages("semPlot")
library(semPlot)
```

```
semPaths(fit.regr, "par", weighted = FALSE, nCharNodes = 7,
        shapeMan = "rectangle", sizeMan = 8, sizeMan2 = 5)
```

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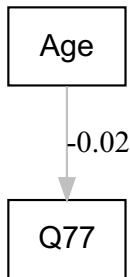
Model fit

The end


```
fit.regr <- lavaan(model = mod.regr, data = data_sapi)
```

Plot with lavaanPlot

```
lavaanPlot(model = fit.regr,
  graph_options = list(overlap = "true",
    fontsize = "20"),
  node_options = list(shape = "box",
    fontname = "Helvetica"),
  edge_options = list(color = "grey"),
  coefs = T, stand = F, covs = T,
)
```



Regression equation

```
parameterEstimates(fit.regr)[c(1,2,3),] # or ...[1:3,]
```

##	lhs	op	rhs	est	se	z	pvalue	ci.lower	ci.upper
## 1	Q77	~	Age	-0.022	0.004	-5.275	0	-0.030	-0.014
## 2	Q77	~~	Q77	1.184	0.068	17.536	0	1.052	1.317
## 3	Q77	~1		4.275	0.135	31.657	0	4.011	4.540

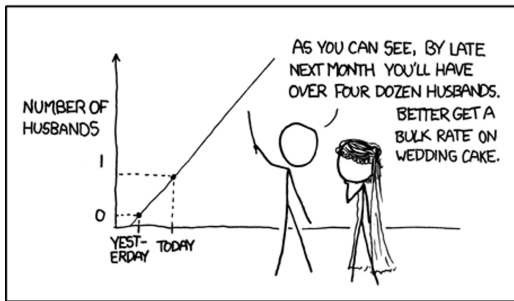
General regression equation (for person i):

$$Q77_i = B_0 + B_1 * Age_i + e_i, \text{ with } e_i \sim N(0, \sigma^2).$$
$$\text{Or: } \hat{Q}_{77j} = B_0 + B_1 * Age_j.$$

Regression equation (for person i):

$$Q77_i = 4.275 - 0.022 * Age_i + e_i, \text{ with } e \sim N(0, 1.184).$$

MY HOBBY: EXTRAPOLATING



REGRESSION

Summary regression model in lavaan Ctd.

R^2 = R-squared = proportion of shared/explained variance:

```
inspect(fit.regr, 'r2')
```

Q77

```
## 0.043
```

Intermezzo: Correlation Age and Q77 and R^2 in R

```
corr <- cor(data_sapi[, 2], data_sapi[, 9],
            use = "complete.obs")

round(corr, 3)    # = correlation between Age and Q77

## [1] -0.208
```

```
round(corr^2, 3) # =  $R^2$ 

## [1] 0.043

#  $R^2$  = proportion of shared/explained variance.
# This is the  $R^2$  in a regression with one predictor.
```

In a regression with one outcome and one predictor,
the R^2 is the square of their correlation.

Intermezzo: Correlation Age and Q77 and R^2 in R Ctd.

Remark:

I would not use lavaan for correlations, but you can:

```
mod.corr <- '
  Q77 ~~ Q77
  Age ~~ Age
  Q77 ~~ Age # covariance
  # If standardized, then correlation
'

fit.corr <- lavaan(model = mod.corr, data = data_sapi)
parameterEstimates(fit.corr, stand=T)[3,'std.all']

## [1] -0.2080665
```

Message intermezzo:

In a regression with one outcome and one predictor, the R^2 is the square of their correlation.

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Analyzing the SAPI data: Path model (Mediation)

Variables:

- Outcome: Q196, I make others laugh
- Mediator: Q77, I enjoy telling funny stories
- Predictor: Age



Specification path model in lavaan

```

model.path <- '
  # regressions
  Q77 ~ Age
  Q196 ~ Q77

  # residual variances
  Q77 ~~ Q77
  Q196 ~~ Q196

  # variance of predictor
  Age ~~ Age
  # mean of predictor
  Age ~ 1

  ## intercepts
  Q77 ~ 1
  Q196 ~ 1

```

Plot with lavaanPlot

```
lavaanPlot(model = fit_path,  
  graph_options = list(overlap = "true",  
                        fontsize = "20"),  
  node_options = list(shape = "box",  
                       fontname = "Helvetica"),  
  edge_options = list(color = "grey"),  
  coefs = T, stand = F, covs = T,  
  ) # Note: stand = F
```



Summary path model in lavaan

```
summary(fit_path, fit.measures = TRUE,  
        ci = TRUE, rsquare = TRUE)
```

Estimates

```
parameterEstimates(fit_path)[,-7] # i.e., without p-value
```

##	lhs	op	rhs	est	se	z	ci.lower	ci.upper
## 1	Q77	~	Age	-0.021	0.004	-4.971	-0.029	-0.013
## 2	Q196	~	Q77	0.451	0.027	16.724	0.398	0.504
## 3	Q77	~~	Q77	1.179	0.067	17.479	1.047	1.312
## 4	Q196	~~	Q196	0.545	0.031	17.479	0.484	0.606
## 5	Age	~~	Age	108.692	6.219	17.479	96.504	120.881
## 6	Age	~1		30.612	0.422	72.580	29.785	31.439
## 7	Q77	~1		4.248	0.136	31.175	3.981	4.516
## 8	Q196	~1		2.153	0.102	21.167	1.954	2.352

Summary path model in lavaan Ctd.

R^2 = R-squared = proportion of shared/explained variance:

```
inspect(fit_path, 'r2')
```

```
##      Q77   Q196
```

```
## 0.039 0.314
```

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Step by step (repetition): Step 1

Load data into R

Step 2

Data screening

e.g., check measurement levels, missing data (notation), and correlations.

Not part of lavaan,
plenty of information on the internet.

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Step 4

Specify your model into lavaan syntax
i.e., Translate your drawing into lavaan syntax

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Number of model parameters - SAPI regression

Number of model parameters (unknowns):

```
fitMeasures(fit.regr, "npar")  
## npar  
##      3
```

Which ones can be inspected in the technical output:

```
lavInspect(fit.regr)
```

More explanation later on.

For now: 3 model parameters estimated.

Number of unique elements

Note: Raw data can be summarized by

- a variance matrix (S)
- a mean vector (m)

Number of unique elements (knowns):

- Unique elements in variance matrix = $p(p + 1)/2$
- Possibly: Unique elements in mean = p

with p the number of variables in the model.

In SAPI regression ($p = 2$):

- Unique elements S : $p(p + 1)/2 = 2 * 3/2 = 3$
- Unique elements m : $p = 2$

Number of model parameters = $3 + 2 = 5$.

Degrees of freedom - SAPI regression

Degrees of freedom (df) =

Number of unique elements - Number of model parameters =

$$5 - 3 = 2.$$

BUT:

```
fitMeasures(fit.regr, "df") #  $df = 0$ 
```

```
## df
```

0

SAPI regression: Fully specified

```
mod.regr_full <- '
  Q77 ~ Age # regression
  Q77 ~~ Q77 # residual variance
  Q77 ~ 1    # intercept

  # (co)variances and means predictor(s)
  Age ~~ Age # variance predictor
  Age ~ 1    # mean predictor
  #
  # Estimated by lavaan by default,
  # but now also part of lavInspect()

'

fit.regr_full <- lavaan(model = mod.regr_full,
                        data = data_sapi)
```

SAPI regression: Same estimates of course

```
parameterEstimates(fit.regr)[,1:7]
```

##	lhs	op	rhs	est	se	z	pvalue
## 1	Q77	~	Age	-0.022	0.004	-5.275	0
## 2	Q77	~~	Q77	1.184	0.068	17.536	0
## 3	Q77	~1		4.275	0.135	31.657	0
## 4	Age	~~	Age	111.304	0.000	NA	NA
## 5	Age	~1		30.706	0.000	NA	NA

```
parameterEstimates(fit.regr_full)[,1:7]
```

##	lhs	op	rhs	est	se	z	pvalue
## 1	Q77	~	Age	-0.022	0.004	-5.275	0
## 2	Q77	~~	Q77	1.184	0.068	17.536	0
## 3	Q77	~1		4.275	0.135	31.657	0
## 4	Age	~~	Age	111.304	6.347	17.536	0
## 5	Age	~1		30.706	0.425	72.177	0

Number of model parameters - SAPI regression

Now, number of model parameters: 5, see

```
fitMeasures(fit.regr_full, "npar")  
  
## npar  
##      5
```

Then, indeed, $df = 5 - 5 = 0$.

This implies that we have a saturated model:

Number of unique elements = Number of model parameters.
 $S_{yy}, S_{yx}, S_{xx}, m_y, m_x$ vs $\alpha, \beta, \sigma_\epsilon^2, S_{xx}, m_y$

Technical output

Which model parameters are estimated can be inspected in the technical output:

```
lavInspect(fit.regr_full)
```

Technical output - SAPI regression (Part 1)

```
lavInspect(fit.regr_full)[1:3]
```

```
## $lambda
##      Q77 Age
## Q77    0   0
## Age    0   0
##
## $theta
##      Q77 Age
## Q77    0
## Age    0   0
##
## $psi
##      Q77 Age
## Q77    2
## Age    0   4
```

Technical output - SAPI regression (Part 2)

```
lavInspect(fit.regr_full)[4:6]
```

```
## $beta
```

Q77 Age

```
## Q77      0      1
```

```
## Age      0      0
```

##

\$nu

```
##      intrcp
```

Q77 0

```
## Age      0
```

##

```
## $alpha
```

```
##      intrcp
```

Q77 3

```
## Age      5
```

Technical output - Model parameters

- λ
- θ
- ψ
- β
- ν
- α

Note: Enumeration/Indication of parameters, not parameter values!

Technical output: General overview

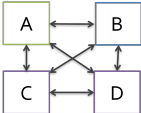
Measurement equation: $y_i = \nu + \Lambda\eta_i + \epsilon_i, \quad \epsilon_i \sim N(\mathbf{0}, \Theta)$

Structural equation: $\eta_i = \alpha + B\eta_i + \zeta_i, \quad \zeta_i \sim N(\mathbf{0}, \Psi)$

Modeled covariance matrix: $\Sigma = \Theta + \Lambda\Psi\Lambda^T$

- ν (**nu**), vector with means/intercepts of observed variables.
- Λ (**Lambda**), matrix with factor loadings relating observed variables to latent variables.
- Θ (**Theta**), variance-covariance matrix of residuals of observed variables (ϵ_i).
- α (**alpha**), vector with intercepts/means of latent variables.
- B (**Beta**), matrix with structural parameters (β 's).
- Ψ (**Psi**), variance-covariance matrix of residuals of latent variables (ζ_i): variances of exogenous variables and residual variances of endogenous variables.

Saturated model ($p = 4$) - with means

	Data				Model	
	A	B	C	D		
S =	A	σ^2_A				
	B	σ_{BA}	σ^2_B			
	C	σ_{CA}	σ_{BC}	σ^2_C		
	D	σ_{DA}	σ_{DB}	σ_{DC}	σ^2_D	

Number of model parameters:

- cross relations / covariances: 6
- variances: 4
- means: 4

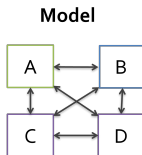
$$(6+4)+4 = 14$$

Saturated model ($p = 4$) - with means

Data

	A	B	C	D
A	σ^2_A			
B	σ_{BA}	σ^2_B		
C	σ_{CA}	σ_{BC}	σ^2_C	
D	σ_{DA}	σ_{DB}	σ_{DC}	σ^2_D

S =



Total unique elements:

$$10 + 4 = 14$$

Number of model parameters:

$$(6 + 4) + 4 = 14$$

Hence, $df = 14 - 14 = 0$.

```
# Variances & Covariances for residuals of observed variables
lavInspect(fit_sat)$theta
```

```
##      Age ReadAb  Q44  Q63
## Age      1
## ReadAb   5      2
## Q44      6      8      3
## Q63      7      9     10      4
```

```
# Means / Intercepts of observed variables
lavInspect(fit_sat)$nu
```

```
##          intrcp
## Age          11
## ReadAb       12
## Q44          13
## Q63          14
```

Example saturated model ($p = 4$) Ctd.

```
fitMeasures(fit_sat,
            c("npars", "logl", "unrestricted.logl",
              "chisq", "pvalue",
              "cfi", "tli", "rmsea"))

##           npars           logl  unrestricted.logl
##      14.000      -4498.184      -4498.184
##           pvalue           cfi           tli
##           NA           1.000           1.000
```

Note: At the end, more details about model fit.

Baseline model ($p = 4$)

- Variables: A, B, C, and D
 - How many unique elements?
 - $4(4+1)/2 + 4 = 14$
- Baseline model = Unrelated variables
 - How many model parameters?
 - 4 variances and 4 means = 8 parameters
- How many df?
 - $df = 14 - 8 = 6$

A

B

C

D

Example baseline model ($p = 4$) Ctd.

```
fitMeasures(fit_base4,
            c("npars", "logl", "unrestricted.logl",
              "chisq", "pvalue",
              "cfi", "tli", "rmsea"))

##           npars           logl  unrestricted.logl
##           8.000          -4530.256          -4498.184
##           pvalue           cfi           tli
##           0.000           0.000           0.000
```

No deviation from the baseline model,
because we estimated it: CFI = 0 and TLI = 0.

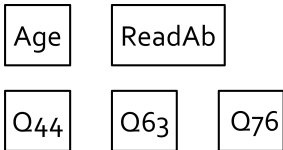
Note: At the end, more details about model fit.

Saturated model ($p = 5$): DIY

- Variables: A, B, C, D, and E
 - How many unique elements?
 -
 -
 -
- Saturated model = All variables related
 - How many model parameters?
 -
 -
 -
 -
- How many df?
 -

Baseline model ($p = 5$): DIY

- Variables: A, B, C, D, and E
 - How many unique elements?
 -
- Baseline model = Unrelated variables
 - How many model parameters?
 -
- How many df?
 -



Baseline model ($p = 5$): Answer

- Variables: A, B, C, D, and E
 - How many unique elements?
 - $5(5+1)/2 + 5 = 20$
- Baseline model = Unrelated variables
 - How many model parameters?
 - 5 variances and 5 means = 10 parameters
- How many df?
 - $df = 20 - 10 = 10$

Age

ReadAb

Q44

Q63

Q76

```
lavInspect(fit_base5)$theta
```

```
lavInspect(fit_base5)$nu
```

##	intrcp
## Age	6
## ReadAb	7
## Q44	8
## Q63	9
## Q76	10

Example baseline model ($p = 5$) Ctd.

```
fitMeasures(fit_base5,
  c("npars", "logl", "unrestricted.logl",
    "chisq", "pvalue",
    "cfi", "tli", "rmsea"))
```

	npars	logl	unrestricted.logl
##	10.000	-5440.660	-5360.694
	pvalue	cfi	tli
##	0.000	0.000	0.000

Note: At the end, more details about model fit.

From model to technical output



```
lavInspect(fit_example)$theta
```

```
##      Age ReadAb Q44
## Age      3
## ReadAb    1      4
## Q44      2      0      5
```

```
lavInspect(fit_example)$nu
```

```
##          intrcp
## Age          6
## ReadAb       7
## Q44          8
```

No relation between A and C. Hence, zero in Theta matrix.

Step by step - there are more: Step 10

Understand all errors/warnings

Note: Not all warning messages have to be fixed, but know what the problem is.

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11. Handling missing data: FIML

Note: Will be discussed in detail in Missing Data lecture.

Model needs to be fully specified (elsewise error):

```
mod.regr_full <- '
  Q77 ~ Age # regression
  Q77 ~~ Q77 # residual variance
  Q77 ~ 1    # intercept

  # (co)variances and means predictor(s)
  Age ~~ Age # variance predictor
  Age ~ 1    # mean predictor
'

fit.regr_fiml <- lavaan(model = mod.regr_full, data = data_sapi,
                        missing='fiml', fixed.x=FALSE)

## Warning: lavaan->lav_data_full():
##      some cases are empty and will be ignored:  453 535.
```

11. Handling missing data: FIML Ctd.

Previous estimates

```
parameterEstimates(fit.regr)[c(1,2,3),]
```

##	lhs	op	rhs	est	se	z	pvalue	ci.lower	ci.upper
## 1	Q77	~	Age	-0.022	0.004	-5.275	0	-0.030	-0.014
## 2	Q77	~~	Q77	1.184	0.068	17.536	0	1.052	1.317
## 3	Q77	~1		4.275	0.135	31.657	0	4.011	4.540

FIML estimates

```
parameterEstimates(fit.regr_fiml)[c(1,2,3),]
```

##	lhs	op	rhs	est	se	z	pvalue	ci.lower	ci.upper
## 1	Q77	~	Age	-0.021	0.004	-5.341	0	-0.029	-0.013
## 2	Q77	~~	Q77	1.133	0.052	21.829	0	1.031	1.234
## 3	Q77	~1		4.219	0.126	33.524	0	3.972	4.465

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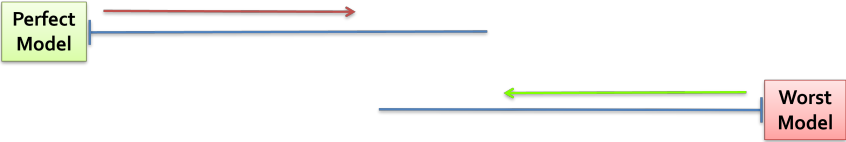
The end

Model Test and Fit Indices

- Log-likelihood ratio (Chi-Square) test
- RMSEA
- CFI/TLI
- Chi-Square Test of Model Fit for the Baseline Model
- SRMR
- Information Criteria: AIC, BIC

A good fit index...

- Not sensitive to sample size
- Sensitive to discrepancies between modeled data and observed data



- Reflects the degree of parsimony of the model

Log-likelihood ratio test

- $-2\log\frac{L_0}{L_a} = -2\log L_0 + 2\log L_a$
- Statistic is Chi-square distributed
- Model test: unrestricted model is model that led to sample variance-covariance matrix
- $H_0 : \chi^2 = 0$ (perfect fit)



- + actual test for fit
- sensitive to sample size
- inflated with non-normal data

SRMR (Standardized Root Mean Square Residual)

$$SRMR = \sqrt{\frac{\sum_{i=1}^p \sum_{j=1}^p (\frac{s_{ij} - \hat{\sigma}_{ij}}{s_{ii}s_{jj}})}{p(p+1)/2}} = \sqrt{\frac{\text{sum of squared std residuals}}{\text{size var-covar matrix}}}$$

Perfect Model

Absolute index:

- Index of discrepancy between model and sample statistics
 - - sensitive to sample size
 - - insensitive to model complexity
- Rule of thumb: max .08

Relative indices: AIC/BIC

- Information theoretic:
Combine goodness of fit χ^2 and parsimony (nr. of parameters k).



- AIC (Akaike Information Criterion)
 - $AIC = \chi^2 + 2k$
- BIC (Bayesian Information Criterion)
 - $BIC = \chi^2 + k * \ln(n)$
- No rule of thumb, values depend on actual data set.
- Choose the model with the **lowest** AIC and/or BIC.

Model Comparison

- Nested models (i.e., one model is a special case of the other):
Chi-square difference test / AIC / BIC
- Non-nested models:
AIC / BIC

In lavaan: use `anova(fit.model1, fit.model2)`.

If and only if the covariance matrix is equal over models

same participants, same variables!

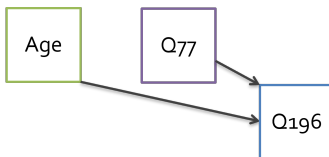
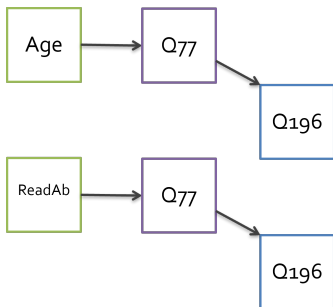
Model Comparisons: Nested models

Nested models: one model is a special case of the other.



Model Comparisons: Nested models?

- Nested?
- Chi-square test vs AIC/BIC?



Model Comparisons: AIC / BIC

Suitable for comparing

- both nested and non-nested models.
- more than two models.

Choose the model with the **lowest** AIC and/or BIC.

To be discussed in the Model Selection lecture

- Quantify relative support via IC weights.

```
library(devtools) # Make sure you have Rtools
install_github("rebeccakuiper/ICweights")
library(ICweights) # ?IC.weights
# Or:
library(restriktor) # ?calc_ICweights
```

- Extension AIC for order-restricted hypotheses (GORIC and GORICA).

```
library(restriktor) # ?goric
```


Summary

- Short SEM overview
- Simple analyses in lavaan
- Step-by-step checklist
- Model fit

Take home message

- Know what's happening:
 - Check whether unique elements - estimated = df
 - Get to know lavaan defaults
 - Read warnings (!)
- Make use of FIML (if possible)

Thanks & How to proceed

Thanks for listening!

Are there any questions?

- Ask fellow participant on course platform.
- Ask teacher during Q&A (or via course platform).
- See if making the lab exercises help.
- Check the lavaan tutorial: e.g.,
<https://lavaan.ugent.be/tutorial/inspect.html>.
- Do not forget that Google is your best friend :-).

You can start working on the lab exercises.