

Modeling the Epidemic Spread of an H1N1 Influenza Outbreak in a Rural University Town With Vaccination

MATH 596 Final Project

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Background

- Project based on *Modelling the epidemic spread of an H1N1 influenza outbreak in a rural university town* by N.K. Vaidya et al.
- Data collected from Washington State University student health center.
- Original paper discussed these models without vaccination.
- We are interested in the relationship between introducing vaccination and the reproductive number.

Assumptions

Parameters used:

$$\gamma = \frac{1}{6} \quad \text{Recovery Rate}$$

$$\mu = 4.2 \times 10^{-3} \quad \text{Birth/Death Rate}$$

$$\beta = 5.45 \times 10^{-5} \quad \text{Disease Transmission Rate}$$

$$b = 0.15 \quad \text{Protection Rate}$$

$$c = 3.74 \times 10^{-6} \quad \text{Transmission Rate for Protected Population}$$

$$g = \frac{1}{6} \quad \text{Quarantine Exit Rate}$$

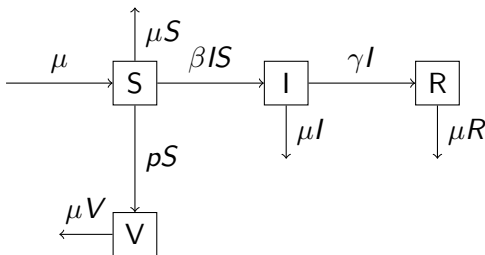
NOTE: μ is not part of original model. μ is based on an 8 month life expectancy. Our focus is on p .

INITIAL values:

$$S(0) = 18223, I(0) = 11, R(0) = 0, P(0) = 0, V(0) = 0, Q(0) = 0.$$

SIR Model with Vaccination Model

Let S be susceptible population, I be infected population, R be recovered population, and V be vaccinated population. μ is birth and death rate, p is vaccinated proportion, β is mass action, and γ is recovery rate.



SIR model with Vaccination Model

Let S be susceptible population, I be infected population, R be recovered population, and V be vaccinated population. μ is birth and death rate, p is vaccinated proportion, β is mass action, and γ is recovery rate.

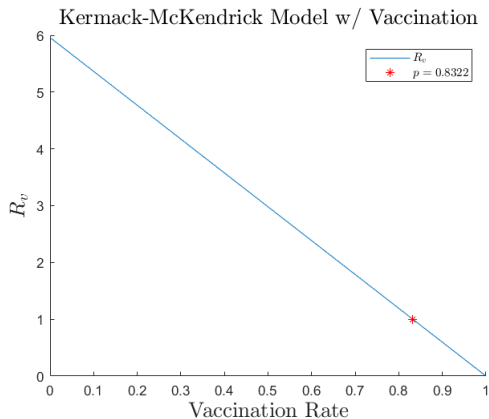
$$\frac{dS}{dt} = \mu - \mu S - pS - \beta IS$$

$$\frac{dI}{dt} = \beta IS - \mu I - \gamma I$$

$$\frac{dV}{dt} = pS - \mu V$$

$$\frac{dR}{dt} = \gamma I - \mu R$$

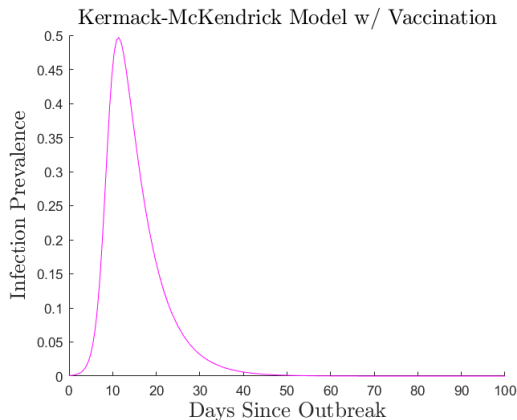
Reproduction Number (SIR Model)



$$R_0 = \frac{\beta(1-p)}{\gamma + \mu}$$

$\mu =$

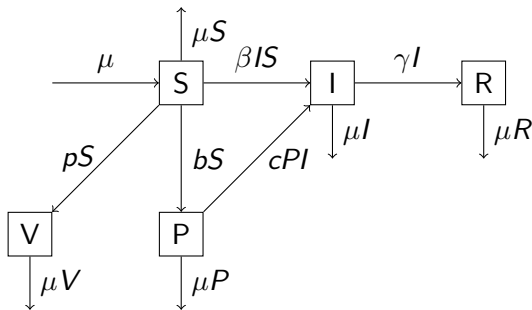
Data Fitting (SIR)



Protected Compartment with Vaccination Model

Let S be susceptible population, I be infected population, R be recovered population, V be vaccinated population, and P be protected compartment. μ is birth and death rate, p is vaccinated proportion, β is mass action from S to I , b is rate to switch from S to P , c is mass action between protected and infected, and γ is recovery rate.

Figure: Protected Compartment with Vaccination Model



Protected Compartment with Vaccination Model

Let S be susceptible population, I be infected population, R be recovered population, V be vaccinated population, and P be protected compartment. μ is birth and death rate, p is vaccinated proportion, β is mass action from S to I , b is rate to switch from S to P , c is mass action between protected and infected, and γ is recovery rate.

$$\frac{dS}{dt} = \mu - pS - bS - \beta IS - \mu S$$

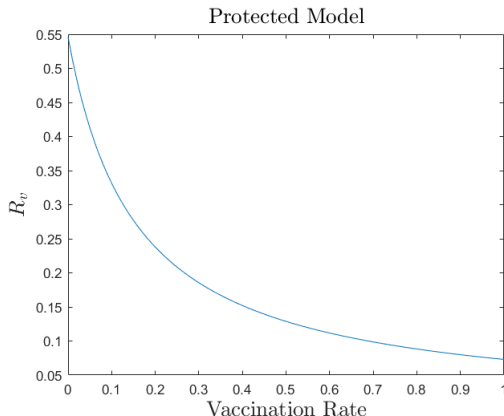
$$\frac{dI}{dt} = \beta IS + cPI - \gamma I - \mu I$$

$$\frac{dP}{dt} = bS - cPI - \mu P$$

$$\frac{dV}{dt} = pS - \mu V$$

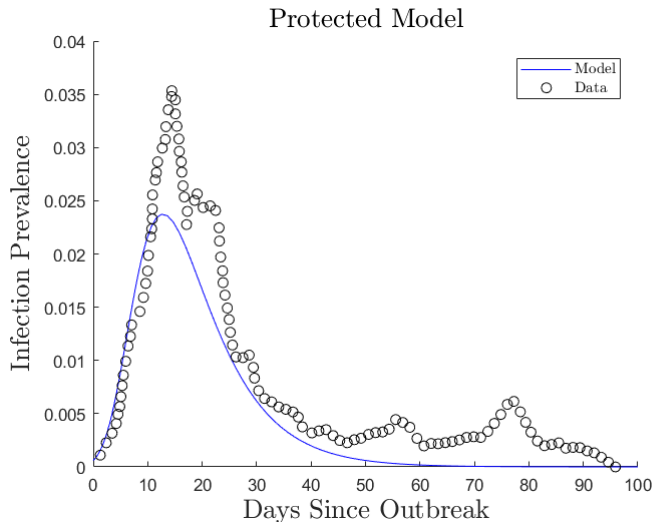
$$\frac{dR}{dt} = \gamma I - \mu R$$

Reproduction Number (Protected)



$$R_0 = \left(\frac{1}{\gamma + \mu} \right) \left(\frac{B\mu + bc}{\mu + b + p} \right)$$

Data Fitting (Protected)



Remarks about Protected Model

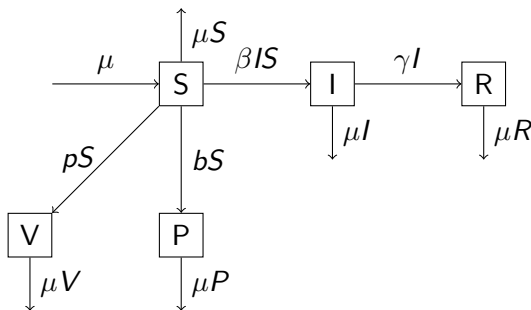
Over the course of 1 year,

- if $p = 0$ vaccination, there are 1913 new infections.
- if $p = .25$ vaccination, there are 637 new infections.
- if $p = .50$ vaccination, there are 553 new infections.

Alternate Protected with Vaccination Model

Let S be susceptible population, I be infected population, R be recovered population, V be vaccinated population, and P be protected compartment. μ is birth and death rate, p is vaccinated proportion, β is rate of transmission from S to I , b is rate to switch from S to P , and γ is recovery rate.

Figure: Protected Compartment with Vaccination Model



Alternate Protected with Vaccination Model

Let S be susceptible population, I be infected population, R be recovered population, V be vaccinated population, and P be protected compartment. μ is birth and death rate, p is vaccinated proportion, β is rate of transmission from S to I , b is rate to switch from S to P , and γ is recovery rate.

$$\frac{dS}{dt} = \mu - pS - bS - \beta IS - \mu S$$

$$\frac{dI}{dt} = \beta IS - \gamma I - \mu I$$

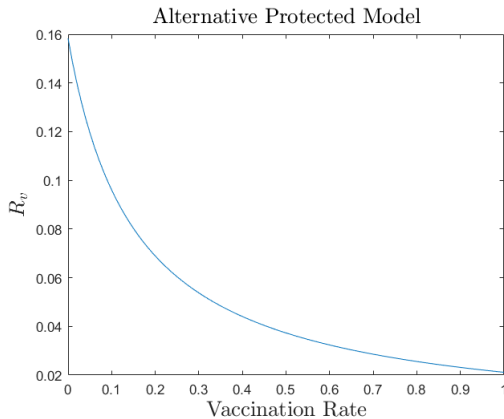
$$\frac{dP}{dt} = bS - \mu P$$

$$\frac{dV}{dt} = pS - \mu V$$

$$\frac{dR}{dt} = \gamma I - \mu R$$

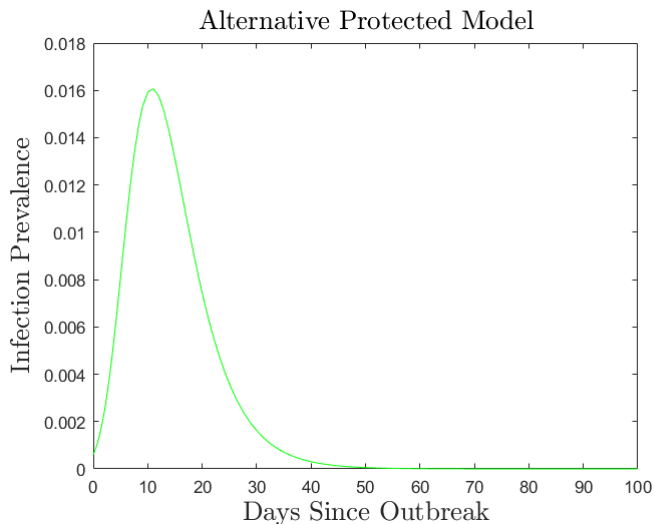
NOTE: No cPI in Alternate Protected model.

Reproduction Number (Alternate)



$$R_0 = \left(\frac{1}{\gamma + \mu} \right) \left(\frac{B\mu}{\mu + b + p} \right)$$

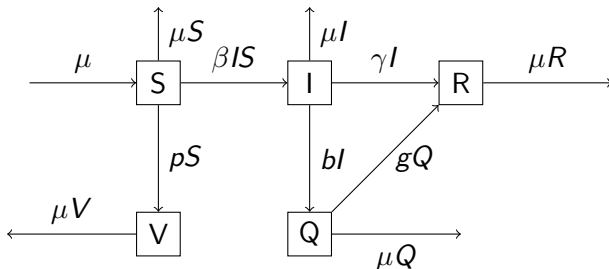
Data Fitting (Alternate)



Quarantine with Vaccination Model

Let S be susceptible population, I be infected population, R be recovered population, V be vaccinated population, and Q be quarantined compartment. μ is birth and death rate, p is vaccinated proportion, β is rate of transmission from S to I , b is rate to switch from S to Q , g is isolated recovery rate, and γ is recovery rate.

Figure: Quarantine Model with Vaccination



Quarantine with Vaccination Model

$$\frac{dS}{dt} = \mu - pS - \mu S - \beta IS$$

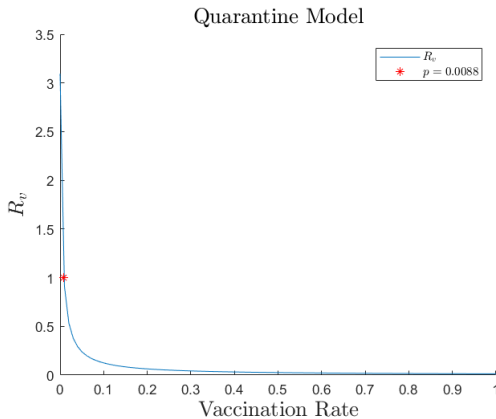
$$\frac{dI}{dt} = \beta IS - \beta I - \gamma I - \mu I$$

$$\frac{dQ}{dt} = bI - \mu Q - gQ$$

$$\frac{dR}{dt} = \gamma I - \mu R$$

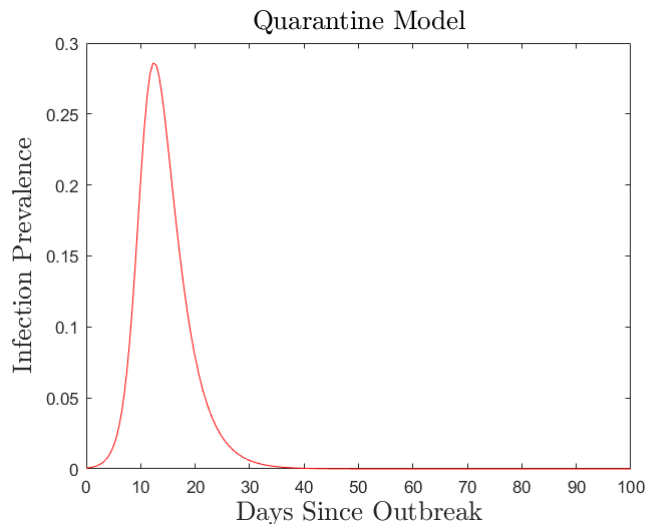
$$\frac{dV}{dt} = pS - \mu V$$

Reproduction Number (Quarantine)



$$R_0 = \left(\frac{\beta}{b + \gamma + \mu} \right) \left(\frac{\mu}{\mu + p} \right)$$

Data Fitting (Quarantine)



- For every model, when p increases, the reproduction number R_0 decreases.
- For the Alternate and Quarantine Models, the fit is off because the conditions were very different.
- We used different values for b and c which may be the reason for $R_0 < 1$.

Summary

