Homework 5

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Problem 1.

1. Yes, this network is a polytree. Here is the definition of polytree, " A polytree is a directed acyclic graph whose underlying undirected graph is a tree". By definition, this network is a polytree.
2. To fully express the joint distribution,$2^5 - 1 = 31$ parameters are necessary. However, only $1 + 1 + 2 + 2 + 4 = 10$ parameters are necessary in the Bayesian network.
3. Yes. UFO landing on highway and Rain are conditionally independent, given not knowing the Traffic. However, in this case, both the Rain and Traffic are known. So the Rain and UFO are not independent, given the Traffic. Knowing that it's raining decreases the belief of UFO landing on the highway.
4. No. According to d-separation, Low-pressure system and UFO landing on highway are conditionally independent, given not knowing the intermediate nodes, Rain and Traffic. Here is the path of the nodes,
   UFO landing on highway $\rightarrow Traffic \leftarrow Rain \leftarrow$ **Low-pressure system**
   So knowing that there is a low-pressure system won't provide any information about the UFO landing on the highway.
5. Yes. Knowing that there is a low-pressure system increases the belief of Traffic by increasing the belief of the Rain. By d-separation, Low-pressure system would be conditionally independent with Traffic, given Rain. Here is the chain,
   Traffic $\leftarrow Rain \leftarrow$ **Lower-pressure system**
   However, in this case, Rain is not given, so low-pressure system and traffic are not conditionally independent.
6. a. $P(U) = \{P(U = 0) = \frac{0}{6} = 0, P(U = 1) = \frac{6}{6} = 1\}$
   b. After the Laplace smoothing with smoothing constant 1, $P(U) = \{P(U = 0) = \frac{0+1}{6+1+1} = \frac{1}{8}, P(U = 1) = \frac{6+1}{6+1+1} = \frac{7}{8}\}$
   c. The smoothed model would be a better model. Rare events like an UFO landing on the highway could be absent from the dataset. The smoothed one would be a better approximation to the real distribution, for it's not deeming the rare events impossible. Also as the number of samples increases, the deviation caused by smoothing will decrease.
7. a.

   $P(R = 1) = P(R = 1|L = 1)\cdot P(L = 1)+P(R = 1|L = 0)\cdot P(L = 0)$

   So $P(R = 1) = 0.4 \times 0.9 + (1 - 0.4) \times 0.1 = 0.42$

   $P(R = 0) = P(R = 0|L = 1)\cdot P(L = 1)+P(R = 0|L = 0)\cdot P(L = 0)$

   So $P(R = 1) = 0.4 \times (1 - 0.9) + (1 - 0.4) \times (1 - 0.1) = 0.58$

   $P(G = 1) = P(G = 1|R = 1)\cdot P(R = 1)+P(G = 1|R = 0)\cdot P(R = 0)$

So $P(G=1) = 0.9 \times 0.42 + 0.3 \times 0.58 = 0.552$

b.

$$P(G=1|L=0) = P(G=1|R=1)\cdot P(R=1|L=0) + P(G=1|R=0)\cdot P(R=0|L=0)$$

So $P(G=1|L=0) = 0.9 \times 0.1 + 0.3 \times (1-0.1) = 0.36$

c. Since T and G,L and R as well as L and G, are conditionally independent given R,

$$P(L=1,R=1,G=0,T=0,U=0) = P(L=1)\times P(R=1|L=1)$$
$$\times P(G=0|R=1) \times P(T=0|U=0,R=1) \times P(U=0)$$

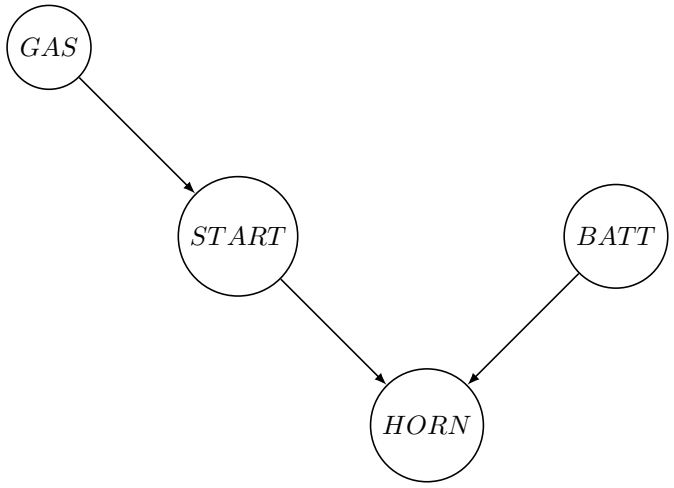
So,

$$P(L=1,R=1,G=0,T=0,U=0) = 0.4\times 0.9\times (1-0.9)\times (1-0.5)\times (1-0.01) = 0.01782$$

8. **Markov blanket for each random variable:**
U:{R,T}
L:{R}
R:{L,G,U,T}
T:{U,R}
G:{R}

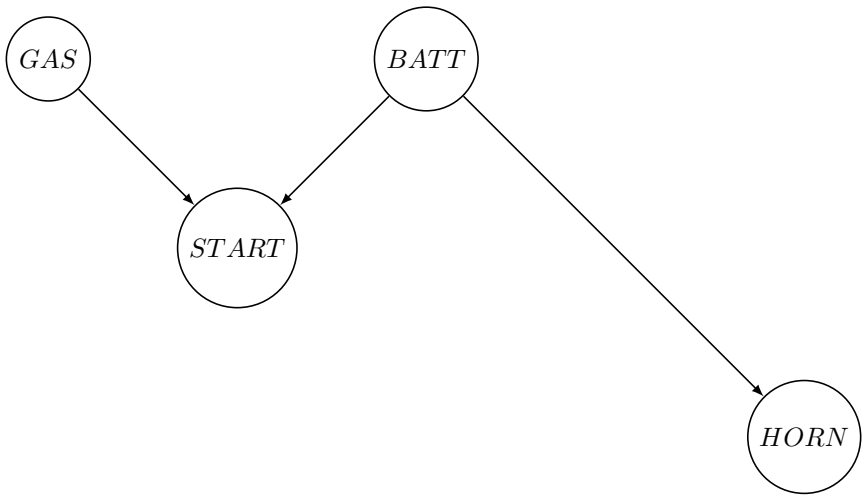
Problem 2.

1. $3 \cdot 2 \cdot 2 \cdot 3 - 1 = 35$ parameters are required to represent the full joint distribution.
2. Bayesian network generated with topological ordering: GAS,START,HORN, BATT



For certain type of car, the horn can only be blew after the car is started.

3. $(1 + 1 + 3 + 3 \times 2) = 11$ parameters are required to represent the distribution using the Bayesian network of part 2.
4. The best Bayesian network I could think of these random variables is given by topological order,GAS,START,BATT, HORN.
5. Bayesian network generated with topological ordering: GAS,START,BATT, HORN



Most of the cars nowadays need battery to start.

6. $(1 + 1 + 3 + 3 \times 3) = 14$ parameters are required to represent the distribution using the Bayesian network of part 5.