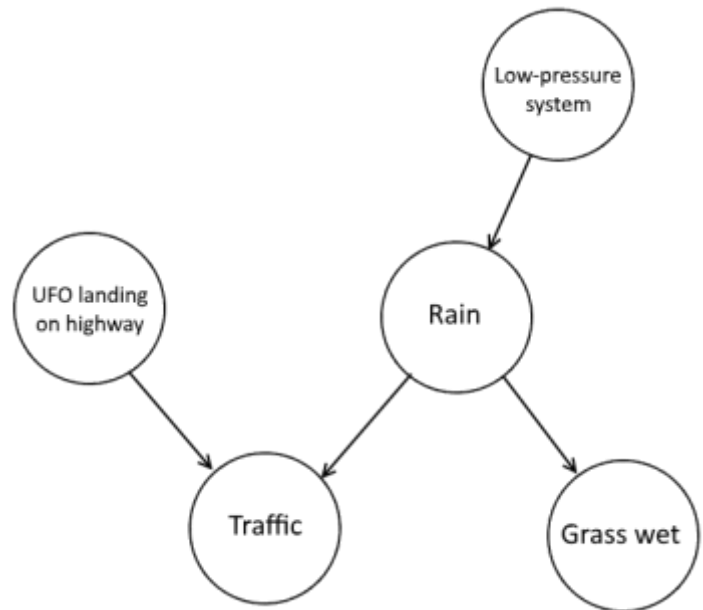


Problem 1:

Use the Bayesian network to the right. Let (L)=Low-pressure system, (R)=Rain, (U)=UFO landing on highway, (T)=Traffic, and (G)=Grass wet. Assume all random variables are binary.



1. Is this network a polytree? If so, how do you know? If not, why not?
2. How many parameters are necessary to fully express the joint distribution?
How many parameters are necessary in the Bayesian network?
3. You checked Google Maps and there is traffic. Would finding out if it is also raining affect your belief about whether a UFO has landed on the highway? Justify your answer using conditional independence rules.
4. You check your barometer and you see that there is low pressure outside. Does this give you any information about whether a UFO landed on the highway? Justify your answer using conditional independence rules.
5. Does knowing there is low pressure give you any information about whether there is traffic? Justify your answer using conditional independence rules.
6. Assume that we observe the data on the right, and we want to estimate the parameters of this network:
 - a. Compute the most likely distribution $P(U)$ with no smoothing.
 - b. Compute $P(U)$ with Laplace smoothing with smoothing constant 1.
 - c. Which of these would you expect to be a better model of the real world? (Use your imagination – remember that we have modeled the UFO landing as a possible event in our problem.) Explain.

L	R	G	U	T
1	1	1	0	1
0	1	1	0	0
0	0	1	0	0
1	0	0	0	0
1	0	0	0	1
0	1	0	0	1

7. For the next parts, use the following as the parameters of the network:

$$P(L = 1) = 0.4$$

$$P(R = 1 \mid L = 1) = 0.9$$

$$P(R = 1 \mid L = 0) = 0.1$$

$$P(G = 1 \mid R = 1) = 0.9$$

$$P(G = 1 \mid R = 0) = 0.3$$

$$P(U = 1) = 0.01$$

$$P(T = 1 \mid U = 1, R = 1) = 1$$

$$P(T = 1 \mid U = 1, R = 0) = 0.9$$

$$P(T = 1 \mid U = 0, R = 1) = 0.5$$

$$P(T = 1 \mid U = 0, R = 0) = 0.1$$

- a. Compute $P(G = 1)$.
- b. Compute $P(G = 1 \mid L = 0)$.
- c. Compute the joint probability $P(L = 1, R = 1, G = 0, T = 0, U = 0)$.

8. Specify the Markov blanket for each random variable in this Bayesian network.

Problem 2:

A car is modeled as four discrete random variables: GAS (Full, Medium, Empty); START (Yes, No); HORN (Working, Not working); BATT (Charged, Partial, Dead). They are random variables but they interact as one would expect: the horn tends to work when there is battery charge and the car is more likely to start with gas and battery.

1. How many parameters are required to represent the full joint distribution?
2. Given the ordering: GAS, START, HORN, BATT, draw the Bayesian Network. You may optionally explain any non-intuitive decisions.
3. How many parameters are required to represent the distribution using your Bayesian network of part 2?
4. What is the best Bayesian network ordering you can think of for these random variables?
5. Draw another Bayesian network using the ordering you came up with in part 4. You may optionally explain any non-intuitive decisions.
6. How many parameters are required to represent the distribution using your network of part 5?

