1.1. Measured Astrometric Values

There are multiple ways to visually represent an astrometric system, and thus the measured values must be clearly understood to ensure they are correct before using them as inputs to the SMODT orbital model and plotting routines.

The orientation and definition of x and y depends on the choice of view of the binary system. The two most common of these are the telescope and the naked eye view, seen in Figure 1 compared to a standard Cartesian coordinate system. As direct observations of binary systems result in images matching the orientation of the telescope's view, and that in many different situations angles are commonly measured from the positive "x" axis, the combination of these two led to the standard conventional coordinate system given by a) in Figure 2; thus, E=y and N=x. This convention can also be seen used in Figure 3, showing the geometric meaning of the Thiele-Innes elements along with the apparent and true orbital ellipses of an example binary system. While that is the conventional coordinate system for measuring astrometric data, when plotting the data along with the predicted orbit more recently people follow that of b) in Figure 2. Therefore, as long as the proper coordinate systems are being used the appropriate x and y values can be calculated with the equations (33a & 33b).

To help avoid the possible confusion due to the different definitions of x and y, the more easily understood ϕ and ρ, with associated errors, are used as the input data coordinates. The models and post-processing plotting routines will then appropriately handle the required conversions.

In the case where the data is measured as the difference in Right Ascension (α) and Declination (δ) of the companion from the primary star, in units of [$''$], these match the E=y and N=x respectively. This allows for direct comparison of the data to the values calculated in equations (31a) and (31b). If these units need to be converted to ϕ and ρ, then the following conversions can be used.

The respective x and y values from the measured position angle (ϕ) and separation angle (ρ) are given by:

$$x_{TH-I} = \rho\cos(\phi) = y_{plot} = North = \delta \tag{1a}$$

$$y_{TH-I} = \rho\sin(\phi) = x_{plot} = East = \alpha \tag{1b}$$

with ϕ in units of [radians] and ρ in [$''$].

The error in the data values in the new coordinate system would then be:

$$\sigma_x = \Delta x_{TH-I} = x_{TH-I}\sqrt{\left(\frac{\Delta\rho}{\rho}\right)^2 + \left(\frac{\cos(\phi+\Delta\phi)-\cos(\phi)}{\cos(\phi)}\right)^2} \tag{2a}$$

$$\sigma_y = \Delta y_{TH-I} = y_{TH-I}\sqrt{\left(\frac{\Delta\rho}{\rho}\right)^2 + \left(\frac{\sin(\phi+\Delta\phi)-\sin(\phi)}{\sin(\phi)}\right)^2} \tag{2b}$$

If the astrometric values measured are as East and North, both in units of[″], the following equations can be used to convert them to those of ϕ and ρ:

$$\rho = \sqrt{(East)^2 + (North)^2} \tag{3a}$$

$$\phi = arctan\left(\frac{East}{North}\right) \tag{3b}$$

with matching errors:

$$\sigma_\rho = \Delta\rho = \rho\left(\frac{|\Delta North \times North| + |\Delta East \times East|}{North^2 + East^2}\right) \tag{4a}$$

$$\sigma_\phi = \Delta\phi = \left|\frac{East}{North}\right| \frac{\sqrt{\left(\frac{\Delta East}{East}\right)^2 + \left(\frac{\Delta North}{North}\right)^2}}{1+\left(\frac{East}{North}\right)^2} \tag{4b}$$

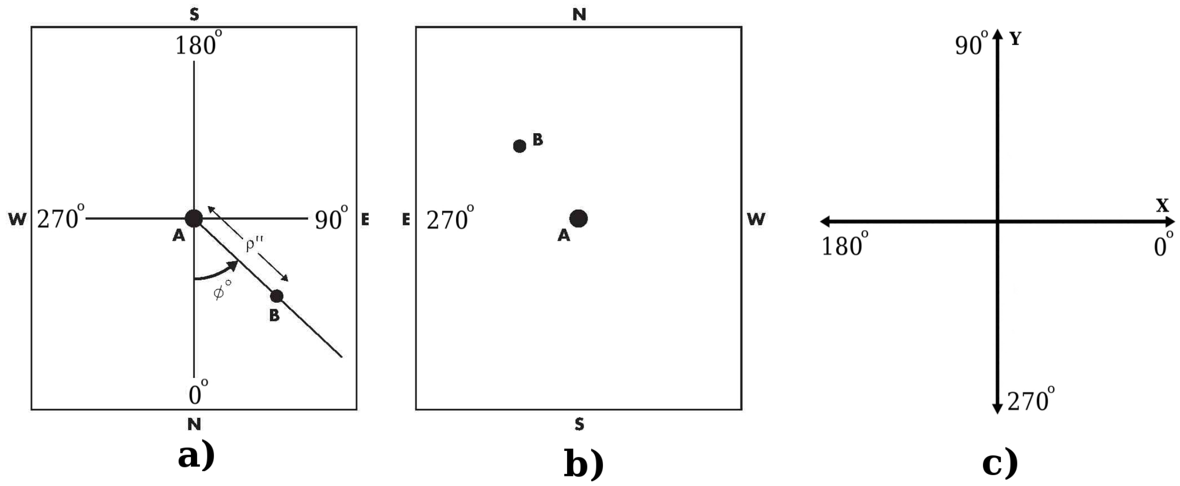

Fig. 1.— View of a binary system through a) A telescope, b) The naked eye, compared to c) The Cartesian coordinate system. Argyle (2004).

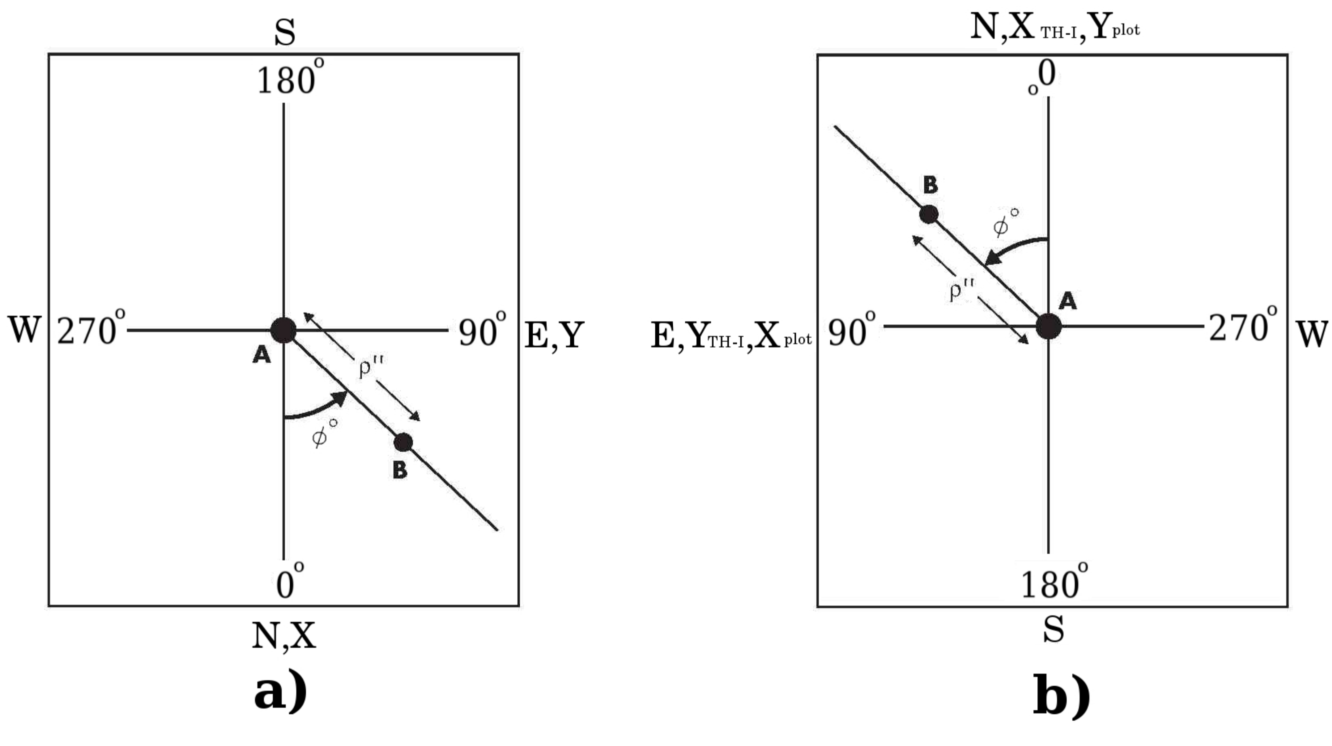

Fig. 2.— Conventional orientations and coordinates for a) measuring and b) plotting the astrometric values of a binary system.

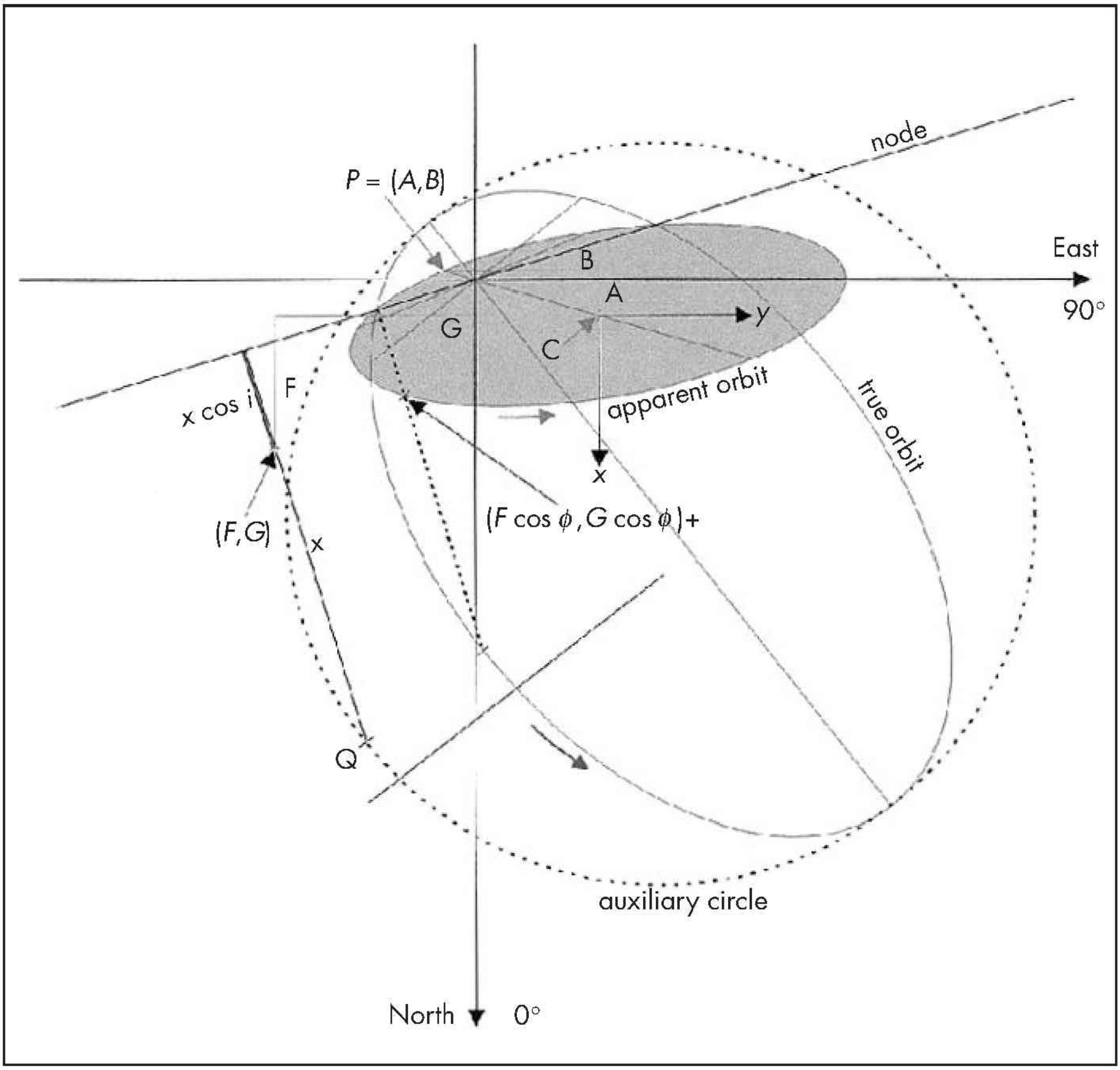

Fig. 3.— Plot of apparent and true orbital ellipses, and the Thiele-Innes elements, Argyle (2004).