

Homework Set 1: ASTRO 530: Stellar Atmospheres*

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1. COMPUTING AND PLOTTING: PLANCK FUNCTION

For this problem, we write our own Planck function and plot it. The Planck function we are writing receives an input wavenumber (μm^{-1}), which can be a vector, and temperature (K), and it outputs spectral radiance B_ν . For part a, we are meant to write our own function inside of a callable module and compare it to the Astropy library's function, `BlackBody`. For part b, we plot the spectral radiance numerous ways and compare outputs of different temperatures.

1.1. Coding the Planck Function

We write our own Planck function and compare it to the `astropy.modeling.physical_models.BlackBody` (hereafter `BlackBody`) function. When writing our own function, we want it to fit the Planck function given wavenumber and temperature. We start with the Planck function

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{\frac{h\nu}{ek_BT} - 1} \quad (1)$$

which compares frequency and temperature. Using the equation for wavenumber,

$$\tilde{\nu} = 1/\lambda = \nu/c, \quad (2)$$

we can change Eq. 1 into

$$B_\nu(T) = 2hc\tilde{\nu}^3 \frac{1}{(e^{\frac{hc\tilde{\nu}}{kT}} - 1)} \quad (3)$$

which uses wavenumber and temperature. This can be transcribed into python which takes in a single value or vector of values for wavenumber in units of per microns (μm^{-1}) and a single temperature value in units of kelvin (K).

Looking at the `BlackBody` function, it initially takes temperature in any units to form a class, then it needs wavelengths to output values of spectral radiance. Because we want to input wavenumber instead of wavelength, we use the `".to(u.cm, equivalence = 'spectral()")` command to change our wavenumbers into wavelength for the function to work.

To compare our Planck function to `BlackBody`, we plot both in Fig. 1 and in Table 1. This plot overlaps our Planck function over `BlackBody` function along wavenumber at a temperature of 7000 K. As seen in the plot, our function continually overlaps at all wavenumbers perfectly. The residuals in Fig. 1 and in Table 1 show a difference on the order of $1\text{e-}20$ which is almost nothing and can be seen as machine precision. Using NumPy, the machine precision of float variables is $2\text{e-}16$ which is greater than that residual.

1.2. Plotting the Planck Function

We want to make three plots that plot the spectral radiance (B_ν) against wavenumber ($\tilde{\nu}$). Each plot shows the the spectral radiance for temperatures of 10000 K, 7000 K, and 3000K. Fig. 2 is in linear space where $\tilde{\nu}$ is in the range of 0 to $12 \mu\text{m}^{-1}$. Fig. 3 is in log-linear space where $\tilde{\nu}$ is in same range as linear space and the spectral radiance is in log space alone. Fig. 4 is in loglog space where $\log\tilde{\nu}$ is in the range of -1.0 to 1.2.

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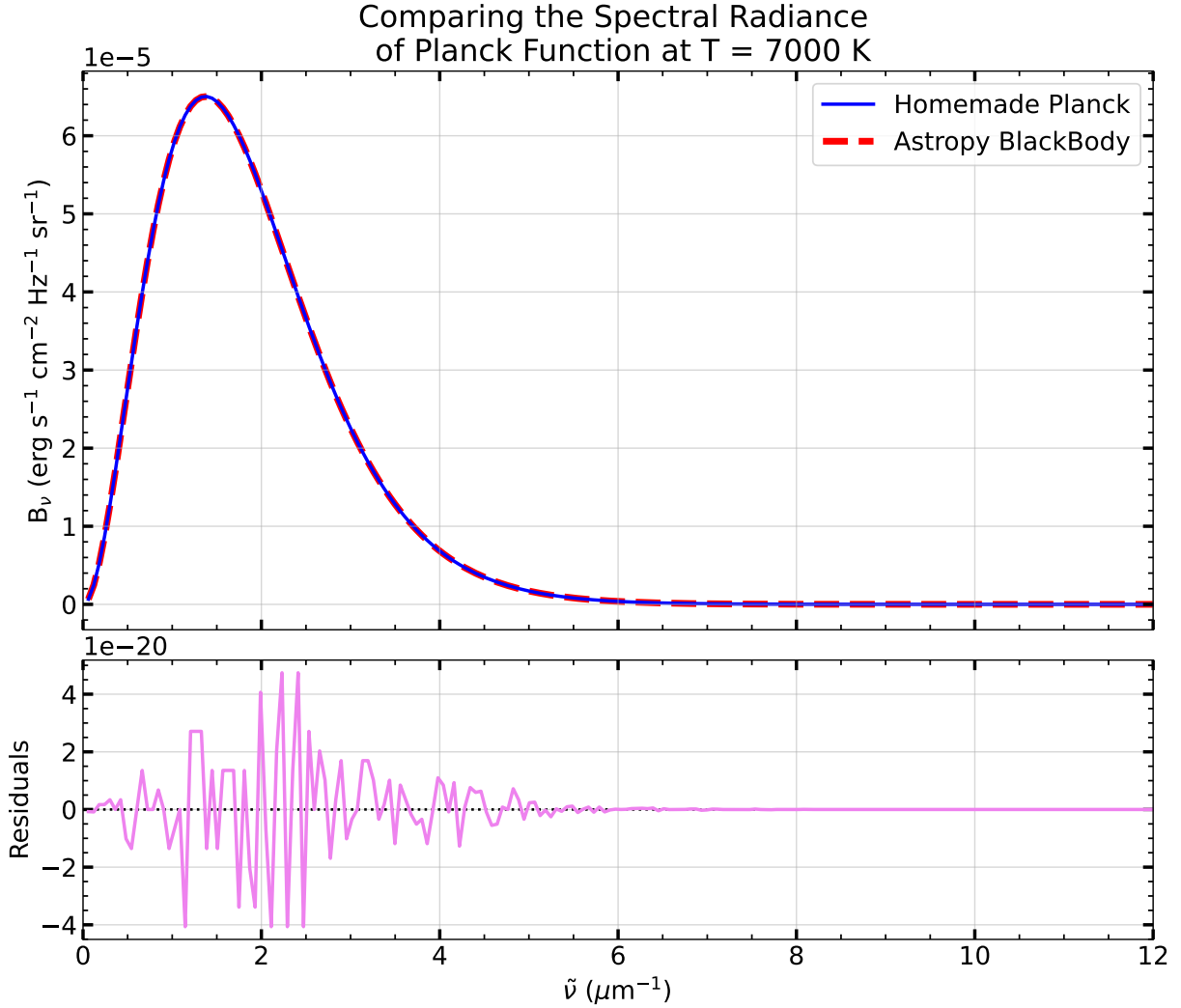


Figure 1. Plot comparing our Planck function to BlackBody function at a temperature of 7000 K. *Upper Panel* is the overlapping spectral radiances where wavenumber is plotted on the x-axis and spectral radiance is plotted on the y-axis. *Lower Panel* is the residual plot of our spectral radiance minus the BlackBody spectral radiance. Our Planck function is a solid blue line, BlackBody function is a dashed red line, and residuals are violet lines with black circles.

We thank everyone in the current first year cohort for their words of insight while navigating the perils of Astropy and Python. their feedback was invaluable in solving several problems.

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Software: Astropy (Astropy Collaboration et al. 2013, 2018, 2022), NUMPY (Harris et al. 2020), Matplotlib (Hunter 2007).

This work made use of Astropy:¹ a community-developed core Python package and an ecosystem of tools and resources for astronomy (Astropy Collaboration et al. 2013, 2018, 2022).

Table 1. Spectral Radiance Comparison

$\tilde{\nu}^a$	PF ^{b,c}	BB ^{c,d}	Residuals ^d
0.0	nan	nan	nan
0.5	2.767e-05	2.767e-05	3.388e-21
1.0	5.834e-05	5.834e-05	-6.776e-21
1.5	6.438e-05	6.438e-05	-2.711e-20
2.0	5.298e-05	5.298e-05	6.776e-21
2.5	3.663e-05	3.663e-05	2.711e-20
3.0	2.257e-05	2.257e-05	-2.033e-20
3.5	1.280e-05	1.280e-05	-5.082e-21
4.0	6.836e-06	6.836e-06	0.000e+00
4.5	3.482e-06	3.482e-06	-1.016e-20
5.0	1.709e-06	1.709e-06	2.753e-21
5.5	8.140e-07	8.140e-07	-1.059e-22
6.0	3.782e-07	3.782e-07	-6.353e-22
6.5	1.720e-07	1.720e-07	-2.382e-22
7.0	7.689e-08	7.689e-08	-3.970e-23
7.5	3.384e-08	3.384e-08	6.617e-24
8.0	1.470e-08	1.470e-08	3.309e-24
8.5	6.308e-09	6.308e-09	-2.151e-23
9.0	2.679e-09	2.679e-09	-1.696e-23
9.5	1.128e-09	1.128e-09	0.000e+00
10.0	4.706e-10	4.706e-10	1.551e-24
10.5	1.949e-10	1.949e-10	-6.721e-25
11.0	8.020e-11	8.020e-11	1.292e-26
11.5	3.279e-11	3.279e-11	0.000e+00
12.0	1.333e-11	1.333e-11	-4.847e-26

This table is available in its entirety in a machine-readable form in the online journal. A portion is shown here for guidance regarding its form and content.

^a Wavenumber in units of μm^{-1} .

^b Our Planck Function in units of spectral radiance B_ν ($\text{erg s}^{-1} \text{ cm}^{-2} \text{ Hz}^{-1} \text{ sr}^{-1}$).

^c Temperature is set to 7000 K.

^d BlackBody function results in units of spectral radiance B_ν ($\text{erg s}^{-1} \text{ cm}^{-2} \text{ Hz}^{-1} \text{ sr}^{-1}$).

^e Residuals between PF and BB.

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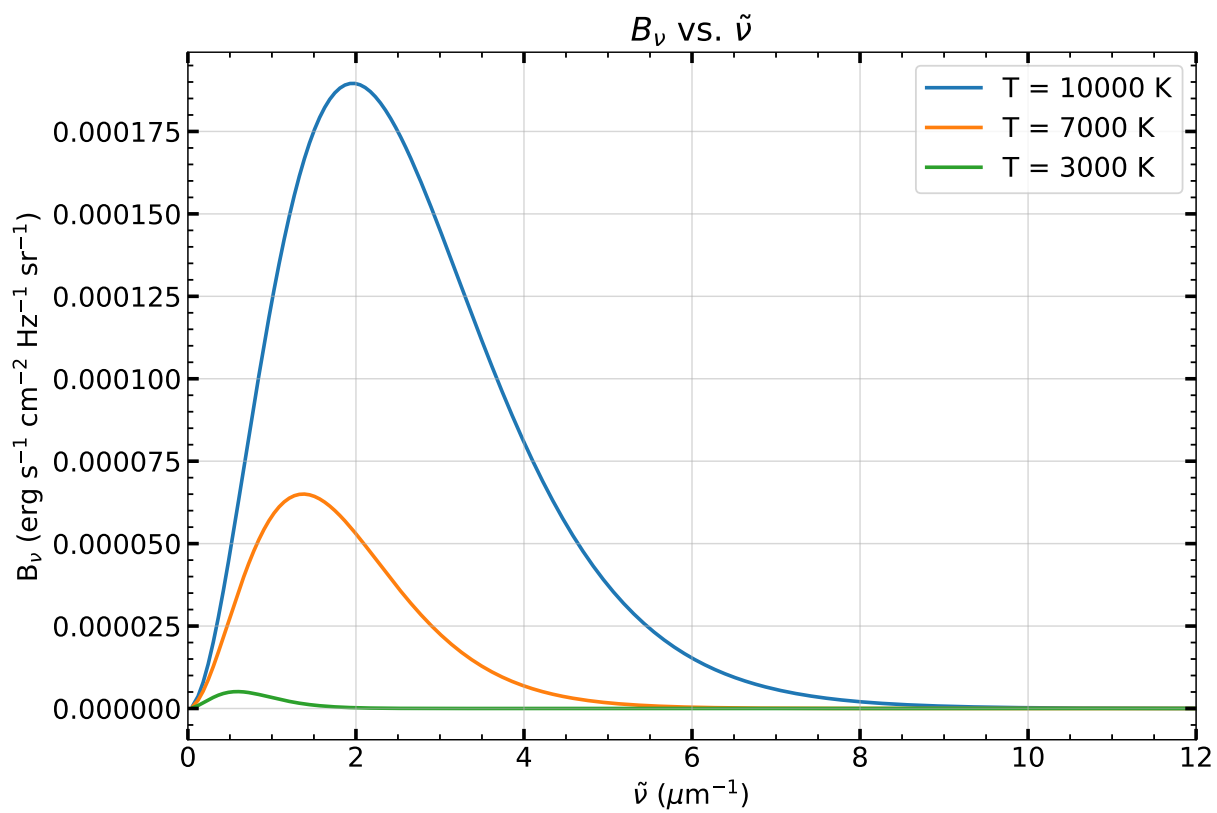


Figure 2. Plot of spectral radiance (B_ν) vs. wavenumber ($\tilde{\nu}$), in linear space for three temperatures. 10000 K is a solid blue line, 7000 K is a solid orange line, and 3000 K is a solid green line.

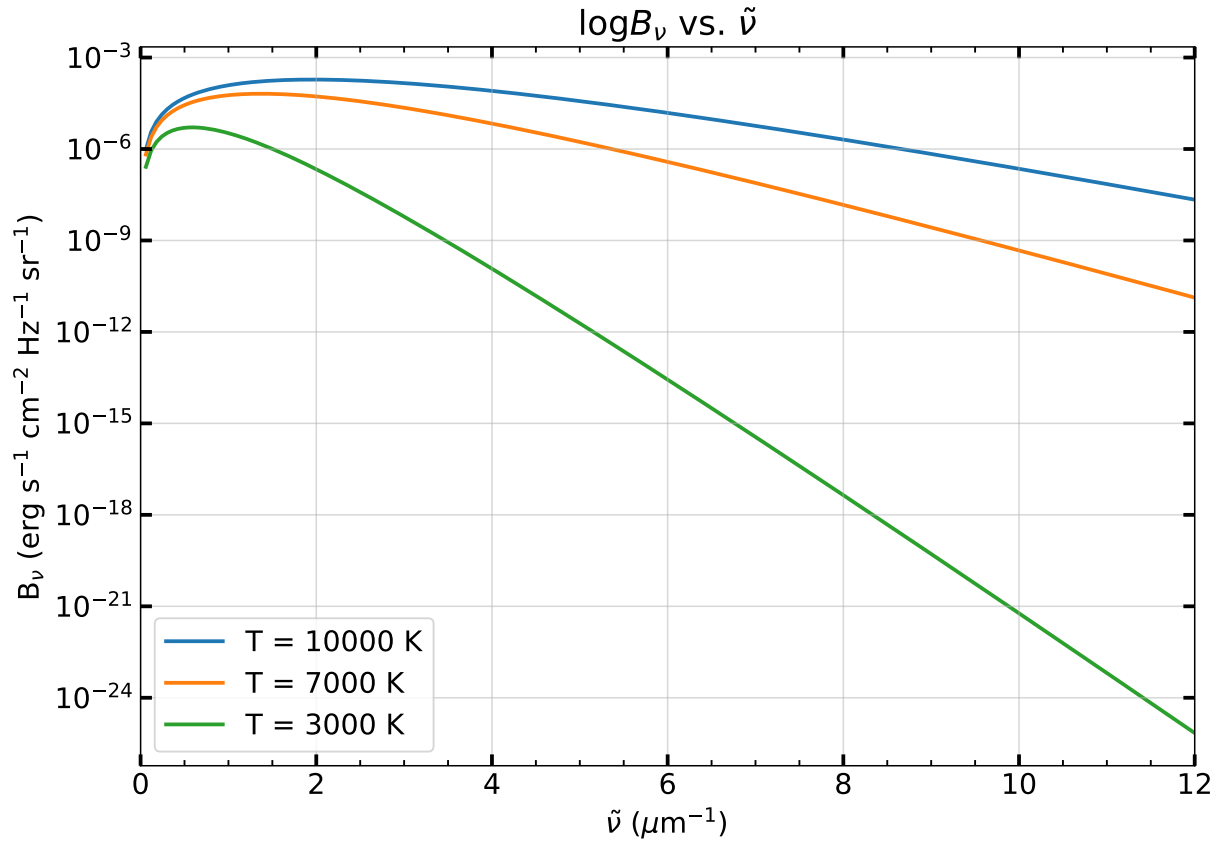


Figure 3. Plot of spectral radiance (B_ν) vs. wavenumber ($\tilde{\nu}$). in log-linear space for three temperatures. 10000 K is a solid blue line, 7000 K is a solid orange line, and 3000 K is a solid green line.

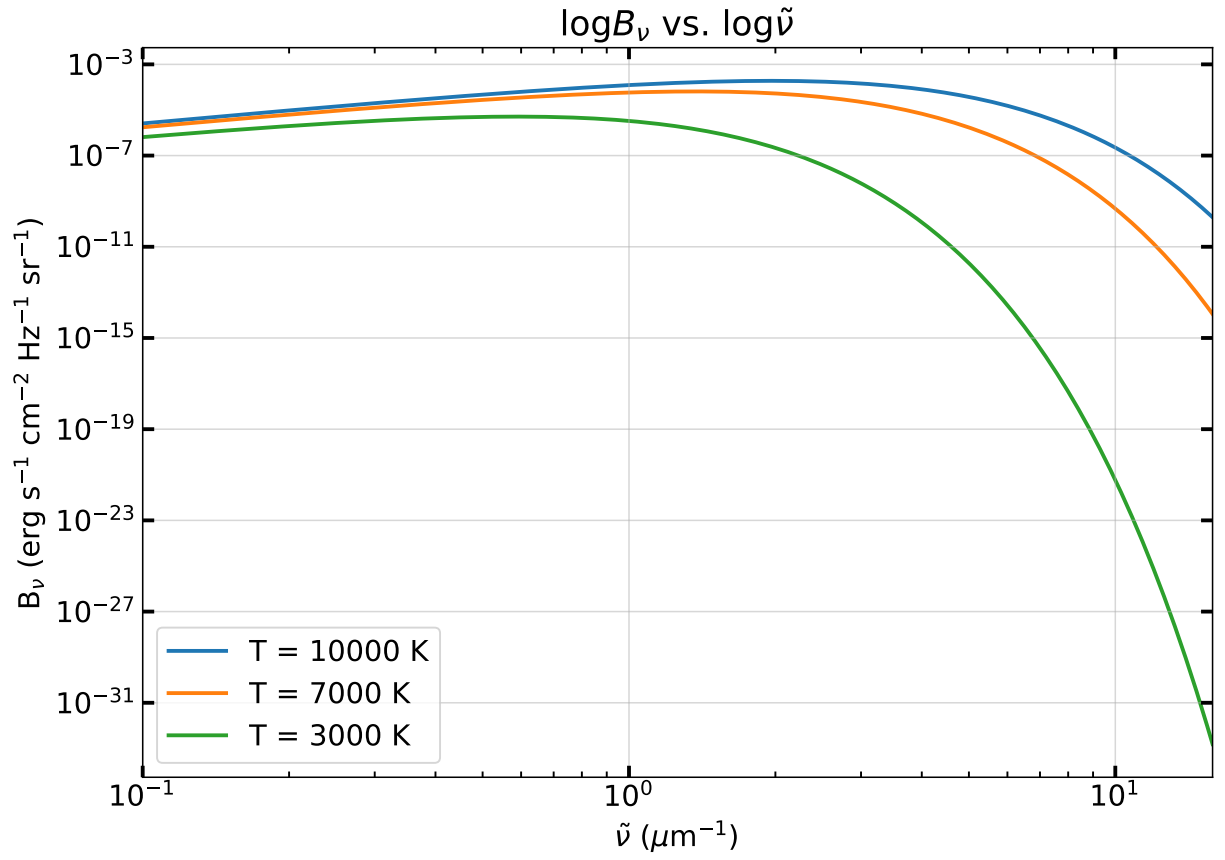


Figure 4. Plot of spectral radiance (B_ν) vs. wavenumber ($\tilde{\nu}$), in loglog space for three temperatures. 10000 K is a solid blue line, 7000 K is a solid orange line, and 3000 K is a solid green line.