A Very Brief Excursion into the Mathematics of National Security Decision Making

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Mathematical decision making—the use of mathematics to help make better decisions—experienced a golden age in the middle of the last century when national security challenges, industrial opportunities, and advances in computer technology accelerated innovation. During World War II statistical analyses improved bomber survivability and accurately predicted enemy tank strength.[5] At the same time a whole field of operations research was invented to solve wartime logistics challenges. Post-war research into game theory spurred a strategic shift toward nuclear deterrence. And as computers became more powerful and more widespread, more complex problems were able to be solved using tools like Monte Carlo campaign simulations and nonlinear optimization. With the current proliferation of machine learning, predictive modeling, and big data techniques, mathematical decision making is entering a second golden age. This article provides a brief introduction to mathematical decision making for the non-mathematician by examining two national security case studies: the U.S. Air Force's *Comprehensive Core Capability Risk Assessment Framework* (C3RAF)[1] and U.S. Agency for International Development and Dalberg Data Insight's *Outil de Surveillance Épidémiologique Guinéa* (OSEG).[7]

The U.S. Air Force must accomplish a complex set of tasks in order for it to achieve an objective. For example, to maintain a no-fly zone an F-15 fighter on patrol requires periodic air refueling, which must be organized by a command and control center, which in turn requires GPS provided by Air Force satellites and a functioning computer network. All of these systems are operated by personnel who the Air Force must recruit, educate, and train. A failure in any one of these operations can substantially degrade the effectiveness of the no-fly zone. Planners in the Pentagon's air staff have the challenge of informing trades across programs for a budget based on input from several major commands and identifying modes of potential failure across the Air Force. In a budget constrained environment, planners must compare thousands of choices, each one potentially quite different from every other one.

On the opposite side of the Atlantic, the Ministry of Health of Guinea faces a very different national security challenge: how to stop the spread of infectious diseases such as malaria and prevent a possible future Ebola outbreak. There is no question that combating these diseases is key to Guinea's development.[11] Malaria is the second leading cause of death in Guinea, accounting for fifteen percent of deaths. And during the 2014–2016 West African Ebola epidemic, Guinea alone had almost four thousand likely cases and 2500 deaths.[6] Daily migration of people who may carry a disease with them compounds the difficulty of disease surveillance. For example, a resident of Boffa carrying a virus or parasite may infect a resident of Kindia while both are staying in Conakry. It's a bit more complicated that this—malaria is carried from person to person by mosquitoes that feed at night and are more prevalent in some regions—but the principles are the same. When prioritizing interventions such as a quarantine or focused eradication campaign, decision makers in Guinea must be able to assess the the likelihood of transmission and prioritization of regions.

Mathematical decision making provides a common thread binding these two national security challenges together. In a complex system of systems it may be impossible to know the true impact of one broad sector on another, but there are several mathematical tools to help decision makers. These include linear systems, network theory, risk analysis, and centrality.

Linear Systems. In 1941 economist Wassily Leontief developed an early model of interdependency of goods traded between industries, work that would later win him a Nobel Prize.[19] Suppose we wish for an industry, say steel production, to meet consumer demand in goods or services. But other industries themselves also require that industry's output, in this case steel, as inputs. By accounting for each industry's input requirements of every other industry's outputs measured in units of production, we have a mathematical equation called the Leontief input-output model. Knowing all the demands, the model can be solved quite easily to determine the units of production for each industry.

We can use this same idea to model degradation and recovery of a city's or country's public works infrastructure by using units of operability instead of units of production. Suppose that all the sectors of a country—such as roads, water supply, electrical grids, and telecommunications—are running along quite nicely. But then that country experiences an incident—perhaps a natural disaster, terrorist attack, or epidemic—that degrade some or all sectors. An inoperability input–output equation models the reduction in functionality due to an outside impact.[13, 12] Failures in one infrastructure component can contribute to failure in another, which itself may lead to a series of cascading failures across all components. The 2008 financial crisis demonstrated that while individual components may have properly managed risk, each component may inherit risk from their dependence on others. When several high profile banks failed, their failures cascaded to other financial institutions that otherwise appeared to be at sufficiently low risk.[2]

Network Theory. In 1736 mathematician Leonard Euler proposed a puzzle asking whether it was possible to walk about the city of Königsberg crossing each of the seven bridges connecting the banks of the Pregel River and its two islands exactly once—no backtracking or double crossing. The solution was found by replacing the islands and mainland river banks with points called *nodes* and the bridges with line segments called *edges* that linked the nodes together. By counting the degree of each node—that is, the number of edges associated with each node—Euler was able confidently answer "no." This puzzle laid the foundation for the mathematical discipline called graph theory.

Nowadays, graph theory is used to describe any number of things. When describing the internet, nodes are websites and edges are hyperlinks connecting those websites together. For counterinsurgency, nodes are individuals to be influenced or eliminated, and edges are the relationships between those individuals (belonging to the same clan, being teacher and student, or even simply sharing a tattoo).[8] For a threat network, nodes can be any "tangible elements. . . that can be targeted for action," like financiers, smuggling routes, and terrorist training camps. Edges are the "behavioral, physical, or functional relationship" between the nodes, used to help identify centers of gravity.[15] Network theory provides an additional layer of utility on top of graph theory by assigning values to the nodes and edges. While graph theory lets us describe the nature of a system, network theory lets us calculate with it.

The Air Force is partitioned into forty-two *core capabilities*, each one with potentially billions of dollars in annual budget that influence each other and the national military objectives. Some core capabilities such as strategic attack or nuclear strike work to directly achieve Air Force mission objectives. These mission capabilities are supported by other enabling capabilities, such as space operations and air refueling. Underpinning the success of all these capabilities are foundational capabilities such as education and research and development. The nodes for the Air Force network are these forty-two core capabilities, and their values are the likelihood of that core capability failing to accomplish its mission outcomes. The edges are the roughly eighteen hundred dependencies of one core capability on another, and their values are the impacts of not meeting those outcomes.

In 1927 mathematicians W. O. Kermack and A. G. McKendrick formulated a simple model for disease transmission, now known as the SIR model, by placing individuals of a population into three buckets—susceptible individuals, infectious individuals, and recovered individuals–and tracking the change of individuals from one bucket to another over time.[18] With a slight modification, the Kermack–McKendrick theory can be combined with network theory to model the spread of infectious disease across multiple geographic regions. For the Guinea Ministry of Health trying to stop the spread of disease the network nodes are Guinea's thirty-four prefectures, and the edges are interprefecture travel. The node values are the incidence rate of infection in the prefecture. The edge values are likelihood that a resident from one prefecture is currently in another prefecture.

Risk Analysis. Stated simply, risk is the expected outcome of uncertain events. Military risk is typically assessed in terms of the probability of failing to meet mission objectives and the consequences of failing to do so.[4] Epidemiological risk can be the infection rate for some susceptible population. Mathematically, risk is precisely the expected value of loss, and in context of network theory, it is the sum of the node values times their associated edge values.

To calculate the node values the Air Force Risk Assessment Framework approximates the demand distribution required for that capability in a specific scenario. The demand distribution may include many random variables such as variability in maintenance, fleet availability, crew rest, weather conditions, adversary response, rules of engagement, and so forth. Mathematically (by the central limit theorem), the combination of many random variables can be well approximated by the bell-shaped, normal distribution. Even if specifications of a capability are well known, knowing the probability of failure is highly uncertain, because data is simply unavailable due to the infrequency of armed conflicts. Because of this, the demand distribution must be carefully determined by subject matter experts. This task can be especially difficult for operational experts unfamiliar with probability distributions. Instead, many subject matter experts find it more intuitive to instead define "goalposts" of the demand, the upper and lower bounds of what they perceive as plausible.[9] A capability at or above the "success" goalpost is virtually guaranteed to succeed; one that is at the "failure" goalpost is all but guaranteed to fail. Mathematically, this approximation of the normal distribution by a uniform distribution introduces at minimum an eight percent error and eliminates black swan scenarios.

Air Force expertise resides largely within the context of a single core capability. Experts may overestimate the impact of their core capability to others while underestimating the magnitude of support it requires—a common cognitive bias called the availability heuristic. Other times, experts may overestimate or underestimate across the board—another cognitive bias called anchoring. These biases can be countered using cumulative voting. Cumulative voting, sometimes known as dot voting or fractional voting, gives each voter a number of points, coins, or sticky dots. A voter could give all, some, or none of their points to any one candidate but must spend all of them across all candidates. Each core capability expert is given one hundred points to allocate to the other capabilities proportional to the level of their support on that expert's core capability. Likewise, each core capability expert is given another one hundred points to allocate to the other capabilities proportional to the level of the expert's core capability's support to them. Because cumulative voting method only provides the expert's relative assessment, it requires an additional mathematical technique known as maximum-likelihood estimation, commonly used in machine learning. Solving such a constrained nonlinear problem is itself not trivial.

Calculating the epidemiological risk in Guinea uses a very different approach to computing edge values—a big data approach. The edge values of the epidemiological risk network—the flow of people from region to region–are the expected amount of time residents from a prefecture spends in any prefecture, including their own. To measure this value, the OSEG project gathered mobile phone location data of users throughout Guinea. Data from the local telecom included the number of mobile phone users traveling in which regions and for how long. The data were aggregated to protect privacy, preserve security, and respect data sovereignty. There are still critical data questions. For example, are people with mobile phones more or less likely to travel than those without? Are those who are infectious more or less likely to travel?

Node values are infection rates as reported by the Ministry of Health in each prefecture. While there is no current Ebola outbreak, notional node values can be used to simulate the effective risk during a future possible outbreak and conduct contingency planning.

Centrality. Who is the best person in your LinkedIn network to have coffee with when you're trying to get a job? What is the center of gravity in a threat network? Which individual ought to be eliminated from a terror cell? Working in resource constrained environments requires that decision makers prioritize options. In networks that prioritization can be made using centrality. *Centrality* is a measure of influence of nodes within a network—status in a social network, superspreaders of a disease, and top websites in an internet search. There are several types of centrality. One of the simplest forms, called degree centrality, counts the number of incident edges on a node, the direct connections. Degree centrality is a good indicator of how active an individual is in a network, and for a social network, degree centrality is the number of friends a person has. Betweenness, another centrality measure, is the number of connections that flow through a node in a network. An individual such as a liaison, with high betweenness centrality, greatly influences what information flows through the network. Katz centrality [17] extends degree centrality to include not only nearest neighbors but also higher-order nodes—"I know a guy who knows a guy."

As higher-order nodes become more and more important, we arrive at something called eigenvector centrality. Eigenvectors are solutions to what mathematicians aptly call an eigenvector problem: finding the decay, growth, or steady-state behaviors of feedback loops. A feedback loop is any system where the output is fed back as its input and can be classified as negative, positive, or neutral. In negative feedback loops, perturbations eventually die off. These are what keep a bicycle from falling over. In positive feedback loops, they grow exponentially. Positive feedback loops are what occasionally give a microphone that annoying high-pitched squeal and contribute to the climatologists' nightmare scenario. They also model the virtuous investment circle that explains Moore's law of transistor doubling. And in neutral feedback loops perturbations seek a steady-state solution, never quite dying off and never quite growing. To explain neutral feedback, take the following thought experiment. Sit a million monkeys in front of a million computers, each computer set to a random webpage. If the monkeys randomly click from hyperlink to hyperlink to hyperlink, on which websites will they likely find themselves? That probability distribution is the eigenvector centrality for the internet—something that forms the basis for Google's patented PageRank algorithm.[21]

Centrality answers a relevant question about the network. The most relevant question for Air Force planners might be "what core capabilities impact a strategic objective the most?" In this case, centrality measures the influence, even indirect, of a core capability on achieving military objectives. It tells us how changing the inherent risk of an activity changes the effective risk in objectives. And for the Guinea Ministry of Health, the most relevant question might be "what regions are the most likely the sources of infection?" So, a good centrality measure is one that captures the flow of people through the prefectures. In essence it tells us how much a change in the local infection rate affects infection rates in other regions.

A Second Golden Age. If innovation golden ages are born out of new challenges and new opportunities, then mathematical decision making is entering a second golden age. This is an era of unprecedented accelerating change—global warming threatens food and health security, automation threatens the workforce, disinformation campaigns threaten democracy, and cyber terrorism threatens economic security. The impacts of emerging and disruptive technologies—like blockchain, machine learning, cheap sensors, and mobile crowd sourcing—on national security are not well known. And the effects of nascent technologies such as quantum computing, artificial general intelligence, and deep reinforcement learning are largely speculative.

Admiral William A. Owens, former Vice Chairman of the Joint Chief of Staff, lamented in 1996 that the military is "more adept at seeing the individual trees than that vast forest of military capability (the system-of-systems) which the individual systems are building for our fighting forces." [20] Twenty-two years later, the US military continues to focus at the tree level. Decisions are becoming "so complex and changing so rapidly that it is almost impossible for decision makers to gather and understand the information required to make and implement coherent policy. At the same time, consequences of incoherent policies are so serious."[10] In this information chaos, leaders and policy makers often mistakenly base their decisions on heuristics and cognitive biases rather than evidence.[16] Indeed, the US Navy is spending millions of dollars to study "spidey sense."[3] Instead research must focus on novel methodologies of incorporating objective evidence into decision making, especially in changing and uncertain environments.

Wicked problems are problems that are so complex and so interconnected that their solutions become ambiguous and even contradictory.[22, 14] Poverty and global climate change are good examples. Because the problem is interconnected, there are multiple value conflicts, and fixing one part of the system may break another part. Moreover, any attempt at a solution to wicked problems may be costly, affording little opportunity for adaptive learning. Furthermore, data is often incomplete. Every solution to a wicked problem is unique by nature. These solutions will require new insights and innovative methodologies for gaining these insights such as artificial intelligence, machine learning, and crowd sourcing. Machine learning methods can combine geospatial and social media data in novel ways or use real-time data from mobile phones or inexpensive sensors to shorten the decision-making feedback loop. These new techniques may lead to collective intelligence systems and complex anticipatory decision tools. Therein lies the need for new mathematics and an understanding of its application.

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