

PROBLEM 1. You have nine math books. Five of them are yellow Springer-Verlag texts and four are gray Cambridge University Press texts.

- (a) How many ways are there to arrange the books, left to right, along a shelf?
- (b) What if the yellow books need to stay together (but their ordering is still important)?
- (c) What if, in addition, the gray books need to stay together (and ordering within each color group is important)?



PROBLEM 2. A domino is a list of two, not necessarily distinct, numbers  $a, b$  where each of  $a$  and  $b$  are between 0 and 6, inclusive. We consider the pairs  $a, b$  and  $b, a$  to be the same.

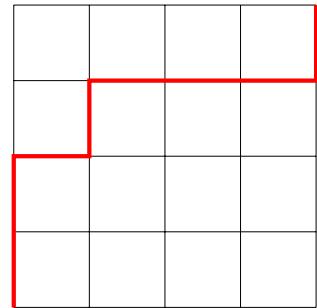
- (a) How many dominoes are there?
- (b) Say two dominoes *match* if they share at least one number. Thus, a matching pair will have the form

$$[a|b] [b|c]$$

where  $a, b, c$  are numbers between 0 and 6, inclusive. How many pairs of matching dominoes are there (where the order of the pair of dominoes does not count)? [Hints: A *double* is a domino with a repeated number, e.g., [4|4]. Why can't a matching pair consist of two doubles? Break the problem into two cases depending on whether a double occurs.]



PROBLEM 3. A path on a square grid is called monotonic if it proceeds only by single steps right or up. On a  $4 \times 4$  (or  $n \times k$ ) grid, how many distinct monotonic paths go from the bottom left corner to the top right corner?



*Challenge*

- (a) In Problem 1, what if the only restriction is that the colors appear in a symmetrical pattern about the central book? [Hint: Let  $g$  stand for gray and  $y$  for yellow. Suppose the first four books have the color pattern  $ggyy$ . What is the rest of the pattern? How many arrangements have this color pattern? How many possible color patterns are there for the first four books?]

Challenge problems are optional and should only be attempted after completing the previous problems.