

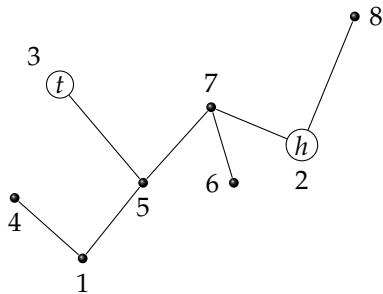
PROBLEM 1. Recall that a forest is an acyclic graph, *i.e.*, a graph which contains no cycle as a subgraph. Suppose  $G$  is a forest with 50 vertices and 44 edges. How many connected components does the  $G$  have?

PROBLEM 2.

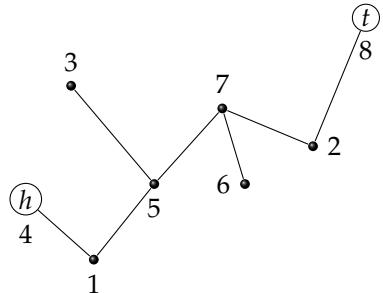
- (a) Find the three unlabeled trees with five vertices.
- (b) Use these unlabeled trees to count the number of (labeled) trees with five vertices.

PROBLEM 3. Determine the functions  $[8] \rightarrow [8]$  associated with the following vertebrates:

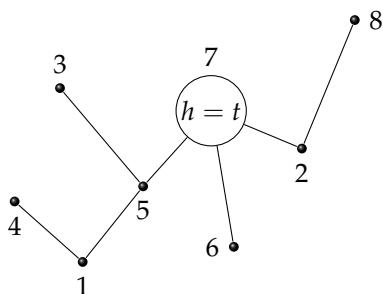
(a)



(b)



(c)



PROBLEM 4. Find the vertebrates associated with the following functions

(a)

$i$	1	2	3	4	5	6	7	8	9
$f(i)$	4	6	5	2	9	1	7	4	3

(b)

$i$	1	2	3	4	5	6	7	8	9
$f(i)$	2	3	1	5	6	1	8	8	8

PROBLEM 5. Characterize the vertebrates associated with functions  $[n] \rightarrow [n]$  which are permutations (i.e., bijective).