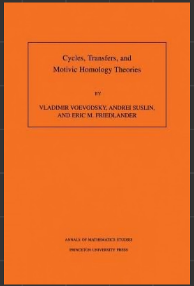
Hochschild Homology:

classical, topological, & motivic



USC Algebra Seminar, 15. IV. 22

Kyle Ormsby, Reed College

Outline

① Hochschild homology

② Topological & motivic variants

③ Computations over Spec $\mathbb{C}$

④ Two truths and a lie ?

Hochschild homology of mod-p motivic cohomology over algebraically closed fields

Bjørn Ian Dundas, Michael A. Hill, Kyle Ormsby, Paul Arne Østvær

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Abstract

We perform Hochschild homology calculations in the algebro-geometric setting of motives. The motivic Hochschild homology coefficient ring contains torsion classes which arise from the mod-p motivic Steenrod algebra and from generating functions on the natural numbers with finite non-empty support. Under the Betti realization, we recover Bökstedt's calculation of the topological Hochschild homology of finite prime fields.

1 Introduction

Let $\mathscr{R}$ be a motivic ring spectrum such as algebraic cobordism, homotopy algebraic K-theory, or motivic cohomology [31]. Working in the stable motivic homotopy category $\mathscr{SH}(F)$ of a field F, we define the motivic Hochschild homology $\mathbf{MHH}(\mathscr{R})$ of $\mathscr{R}$ as the derived tensor product

$$\mathscr{R} \wedge_{\mathscr{R}\wedge\mathscr{R}^{op}} \mathscr{R}. \tag{1}$$

The concepts of Hochschild homology for associative algebras and topological Hochschild homology for structured ring spectra inspire our constructions. In the event $\mathscr{R}$ is commutative one may equivalently to (1) form the tensor product in the category of commutative motivic ring spectra with the simplicial circle

$$S^1 \otimes \mathscr{R}. \tag{2}$$

The primary purpose of this paper is to calculate the homotopy groups of motivic Hochschild homology of $\mathbf{M}\mathbb{F}_p$ over algebraically closed fields — the Suslin-Voevodsky mod-p motivic cohomology ring spectrum for p any prime number. When the base field F admits an embedding into the complex numbers $\mathbb{C}$, the Betti realization functor allows us to compare our $\mathbf{MHH}$ calculations with Bökstedt's pioneering work on topological Hochschild homology of the corresponding topological Eilenberg-Mac Lane spectrum $\mathbf{H}\mathbb{F}_p$. In fact, our calculation for $\mathbf{MHH}$ specializes to the one for $\mathbf{THH}$ in [8]. Additively, $\mathbf{THH}(\mathbb{F}_p)$ splits as a restricted product of Eilenberg-MacLane spectra in the stable homotopy category. This is not the case, however, for $\mathbf{MHH}(\mathbb{F}_p)$, $\mathbf{M}\mathbb{F}_p$, and $\mathscr{SH}(F)$. The source of this extra layer of complexity is the abundance of τ-torsion elements in the coefficients. Here τ is a canonical class in the mod-p motivic cohomology of F, which maps to the unit element in singular cohomology under Betti realization.

We express the coefficient ring $\mathbf{MHH}_\star(\mathbb{F}_p)$ in terms of algebra generators τ, μ_i, $x_{S,f}$ arising from the mod-p motivic Steenrod algebra [17], [34], and generating endofunctions $f\colon \mathbb{N} \circlearrowleft$ with finite non-empty support containing some subset $S \subset \mathbb{N}$. The infinity of τ-torsion classes $x_{S,f}$ is not witnessed in $\mathbf{THH}_*(\mathbb{F}_p)$. For example, Kronecker delta functions give rise to such classes (in this case, S is either empty or a singleton set).

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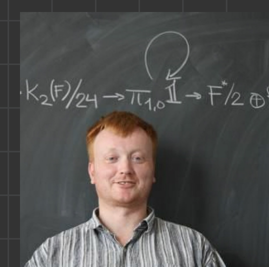
All work joint with Bjørn Dundas, Mike Hill, & Paul Arne Østvær



① Hochschild homology

Setup: k a field, A a commutative k-algebra

Hochschild complex ($\otimes = \otimes_k$):

$$(\star) \quad \cdots \longrightarrow A^{\otimes n+1} \longrightarrow A^{\otimes n} \longrightarrow \cdots \longrightarrow A^{\otimes 2} \longrightarrow A$$

$$a_0 \otimes a_1 \otimes \cdots \otimes a_n \longmapsto \sum_{i=0}^{n-1} (-1)^i a_0 \otimes \cdots \otimes a_{i-1} \otimes a_i a_{i+1} \otimes a_{i+2} \otimes \cdots \otimes a_n$$

$$+ (-1)^n a_n a_0 \otimes a_1 \otimes \cdots \otimes a_{n-1}.$$

E.g. $a \otimes b \otimes c \longmapsto ab \otimes c - a \otimes bc + ca \otimes b$.

Defn $\quad HH_n(A/k) = H_n(\star) = \mathrm{Tor}_n^{A \otimes A}(A, A)$

derived version of $A \otimes_{A \otimes A} A$

Hochschild-Kostant-Rosenburg Theorem

Suppose char $k = 0$ and A is smooth of finite type over k.

Then $\underbrace{HH_*(A/k)}\simeq \underbrace{\Omega^*_{A/k}}$.

de Rham complex: $\Omega_{A/k}$ corepresents k-linear derivations on A $\left(\mathrm{Hom}_k(\Omega_{A/k}, M) \cong \mathrm{Der}_k(A, M)\right)$, $\Omega^n_{A/k} = \Lambda^n_k \Omega_{A/k}$ with "exterior derivative" differential

Does $HH_*(A/k)$ carry a natural differential compatible with de Rham?

Yes, coming from an S^1-action.

Going in Circles

Recall $\underbrace{HH(A/k)}_{\text{Hochschild complex, previously } \circledast} = A \otimes^{\mathbb{L}}_{A\otimes A} A$. By Dold-Kan, this is the geometric realization of some simplicial commutative k-algebra.

Fact $HH(A/k) = \left| \text{pushout} \underbrace{\begin{pmatrix} A \amalg A \longrightarrow A \\ \downarrow \\ A \end{pmatrix}}_{\text{each } A \in [\Delta^{op}, CAlg_k] \text{ constant}} \right|$

$= |S^1 \otimes A|$, using ① $[\Delta^{op}, CAlg_k]$ is tensored over $[\Delta^{op}, Set]$,

② $S^1 = \text{pushout}\begin{pmatrix} S^0 \longrightarrow * \\ \downarrow \\ I \end{pmatrix}$.

Going in Circles (ct'd)

So $S^1 \otimes A$ carries an action of S^1 and pushed through Dold-Kan, we get an action of $C^*(S^1, k)$ on $HH(A/k)$

$$\Longrightarrow H_*(S^1; k) \cong k[\varepsilon]/(\varepsilon^2) \text{ acts on } HH_*(A/k).$$

$$|\varepsilon| = 1$$

The "Connes differential" is

$$\begin{aligned} HH_n(A/k) &\longrightarrow HH_{n+1}(A/k). \\ \eta &\longmapsto \varepsilon \cdot \eta \end{aligned}$$

Under the hypotheses of HKR, this corresponds to the de Rham differential.

② Topological Hochschild Homology

Instead of working over k (or $\mathbb{Z}$), let's work over $\mathbb{S}$.

sphere spectrum!

"Just" as one has

$$\begin{tikzcd} \mathbb{Z}\text{-mod} \arrow[r, bend left, "-\otimes_{\mathbb{Z}}\mathbb{R}"] & \mathbb{R}\text{-mod} \arrow[l, bend left, "\mathrm{res}"] \arrow[r, bend left, "-\otimes_{\mathbb{R}}\mathbb{C}"] & \mathbb{C}\text{-mod} \arrow[l, bend left, "\mathrm{res}"] \end{tikzcd}$$

we also have

$$\begin{tikzcd} \mathrm{Sp} \arrow[r, bend left, "-\otimes_{\mathbb{S}}\mathbb{Z} \,=\, -\wedge H\mathbb{Z}"] & D(\mathbb{Z}) \arrow[l, bend left, "H"] \end{tikzcd}.$$

derived category of Abelian groups

Sp ↳ symmetric monoidal ∞-category of spectra, $\otimes = \wedge$; constructed from pointed spaces by inverting suspension $\Sigma = S^1 \wedge -$.

Topological Hochschild Homology (ct'd)

Every spectrum X has Abelian homotopy groups $\pi_n X$, $n \in \mathbb{Z}$.

For $C \in D(\mathbb{Z})$, $\pi_n HC = H_n(C)$.

Given a commutative ring object $A \in D(\mathbb{Z})$ (usual ring, dga, ...)

Bökstedt defined

$$\boxed{THH(A) := HA \wedge_{HA \wedge HA} HA = S^1 \otimes HA.}$$

(In fact, may form $THH(E_\infty\text{-ring})$ in this fashion.)

Then $THH_n(A) := \pi_n THH(A)$.

By Serre finiteness, if $A \supseteq \mathbb{Q}$, then $THH_*(A) \cong HH_*(A/\mathbb{Z})$.

Applications

① Improved de Rham theory in characteristic p.

Thm (Hesselholt) k a perfect field of char $p>0$, R a smooth k-algebra, then $THH_n(R) \cong \bigoplus_{i\geq 0} \Omega^{n-2i}_{R/k}$ as R-modules.

Work of Bhatt-Morrow-Scholze et al in p-adic Hodge theory.

② Trace methods: $\exists$ Dennis/Bökstedt trace

$$\underbrace{K(R)}_{\text{algebraic K-theory spectrum of } R} \longrightarrow THH(R)$$

which factors through $TC(R)$, the topological cyclic homology of R. Dundas-McCarthy: cyclotomic trace is an "infinitessimal p-adic equivalence".

Computations

Since $THH(A) = HA \underset{HA\wedge HA}{\wedge} HA$ and $\pi_* HA = A$ (in deg 0),

we have a Tor-spectral sequence

$$E^2_{h,t} = \mathrm{Tor}_{h,t}^{\pi_* HA\wedge HA}(A,A) \Longrightarrow THH_{h+t}(A)$$

and $d^r : E^r_{h,t} \longrightarrow E^r_{h-r,\, t+r-1}$.

For $A = \mathbb{F}_2$, $\pi_* H\mathbb{F}_2 \wedge H\mathbb{F}_2 = \mathcal{A}_* = \mathbb{F}_2[\xi_1, \xi_2, \xi_3, \dots]$, $|\xi_i| = 2^i - 1$

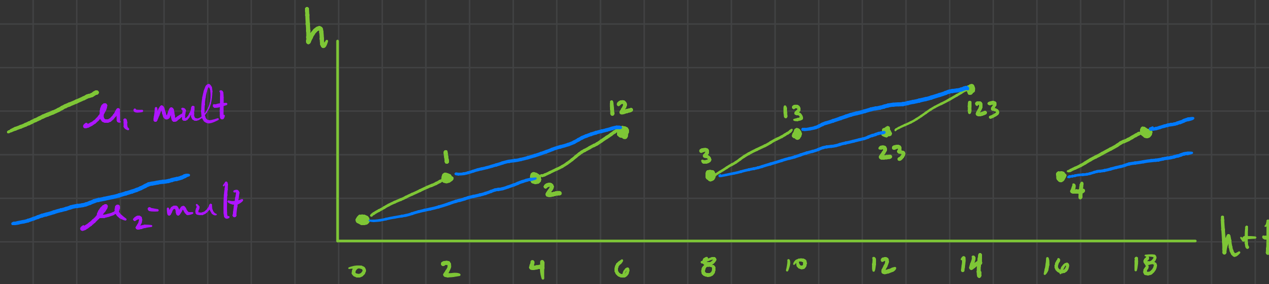
is the mod-2 dual Steenrod algebra.

Thus $E^2 \cong \Lambda_{\mathbb{F}_2}(\mu_1, \mu_2, \mu_3, \dots)$ with $|\mu_i| = (1, 2^i - 1)$.

Pictorially ...

Computations (ct'd)

$E^2 \cong \Lambda_{\mathbb{F}_2}(\mu_1, \mu_2, \mu_3, \ldots)$ with $|\mu_i| = (1, 2^i - 1)$



Note ① One class in each even degree.

② Differentials go 1 left & down, so $E^2 = E^\infty$.

Fact By a power operations argument, $\mu_i^2 = \mu_{i+1} \in THH_*(\mathbb{F}_2)$.

Thm (Bökstedt) $THH_*(\mathbb{F}_p) = \mathbb{F}_p[\mu]$, $|\mu| = 2$ for all primes p.

⟨new⟩ # Motivic Hochschild Homology

Now move from spectra to motivic spectra and replace HA with MA, the motivic Eilenberg-Mac Lane spectrum.

Morel-Voevodsky:

$$[Sm/k, [\Delta^{op}, Set]] + \text{Nisnevich \& } \mathbb{A}^1\text{-localize} = \text{motivic spaces } Spc_k$$

$$\left(Sm/k \xrightarrow{y} \cdot \xleftarrow{const} [\Delta^{op}, Set] \right)$$

$$Spc_k \rightsquigarrow_{\text{invert } \mathbb{P}^1 \wedge -} \text{motivic spectra } Sp_k$$

Motivic Hochschild Homology (ct'd)

$$\begin{tikzcd} \mathbb{A}^1 \setminus 0 \arrow[r] \arrow[d] \arrow[dr, phantom, "\ulcorner", very near end] & \mathbb{A}^1 \simeq * \arrow[d] \\ * \simeq \mathbb{A}^1 \arrow[r] & \mathbb{P}^1 \end{tikzcd} \implies \mathbb{P}^1 \simeq \underbrace{S^1}_{\text{simplicial circle}} \wedge \underbrace{(\mathbb{A}^1 \setminus 0)}_{\text{geometric circle}} .$$

Upshot Bigraded spheres $S^{m,n} := (S^1)^{\wedge m-n} \wedge (\mathbb{A}^1 \setminus 0)^{\wedge n}$

(in $S^{m,n}$: m = total degree, n = weight)

Thus we have bigraded homotopy groups

$$\pi_{m,n} X := [S^{m,n}, X]$$

for $X \in Sp_k$.

Note Need homotopy sheaves to detect weak equivs.

Motivic Hochschild Homology (ct'd)

Important (co)homology theories are representable in Sp_k:

- MA = motivic cohomology with coefficients in A
- KGL = (homotopy) algebraic K-theory
- KQ = Hermitian K-theory
- MGL = algebraic cobordism

We define the motivic Hochschild homology of a commutative ring A to be the motivic spectrum

$$\boxed{MHH(A) := MA \underset{MA \wedge MA}{\wedge} MA = S^1 \otimes MA}$$

with coefficients $MHH_{*,*}(A) = \bigoplus_{m,n \in \mathbb{Z}} \pi_{m,n} MHH(A)$.

③ Computations over $\mathbb{C}$

Fix k algebraically closed, $A = \mathbb{F}_p$, $p \neq \mathrm{char}(k)$.

Theorem 1.1. *Over an algebraically closed field of exponential characteristic $\mathsf{e}(F) \neq p$, there is an algebra isomorphism*

$$\mathbf{MHH}_\star(\mathbb{F}_p) \cong \mathbb{F}_p[\tau, \mu_i, \mathsf{x}_{S,f}]_{i\in\mathbb{N},\,(S\subset \operatorname{supp} f,\, f\colon \mathbb{N}\circlearrowleft)}/\mathcal{I} \tag{3}$$

with the ideal of relations

$$\mathcal{I} = \left(\begin{array}{c} \mu_i^p - \tau^{p-1}\mu_{i+1}, \\ \tau^{p-1}\mathsf{x}_{S,f}, \\ \mathsf{x}_{S,f}\cdot \mathsf{x}_{T,g} - \displaystyle\sum_{u\in \operatorname{supp}(f+g) - S\cup T} \epsilon_u \cdot \mathsf{x}_{S\cup T\cup\{u\}, f+g} \end{array} \right).$$

Here the support of f is a finite non-empty subset of the natural numbers and $S \subset \operatorname{supp} f \subset \mathbb{N}$ does not contain the minimal element of $\operatorname{supp} f$. The coefficient $\epsilon_u \in \mathbb{F}_p$ is given explicitly in Definition 2.12. The algebra generators have bidegrees given by $|\tau| = (0, -1)$, $|\mu_i| = (2p^i, p^i - 1)$, and

$$|\mathsf{x}_{S,f}| = (|S| + 1)(-1, p-1) + p \sum_{j\in \operatorname{supp} f} f(j)(2p^j, p^j - 1).$$

③ Computations over $\mathbb{C}$ (ct'd)

Fix k algebraically closed, $A = \mathbb{F}_p$, $p \neq \mathrm{char}(k)$.

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$$|\mathsf{x}_{S,f}| = (|S|+1)(-1, p-1) + p \sum_{j \in \mathrm{supp} f} f(j)(2p^j, p^j - 1).$$

Definition 2.12. For functions $f, g\colon \mathbb{N} \circlearrowleft$ with finite support and non-empty finite sets $S, T \subseteq \mathbb{N}$ define $K_{S,T,f,g} \in \mathbb{F}_p$ by

$$K_{S,T,f,g} = \left(\prod_{s \in S} \binom{fs - 1 + gs}{fs - 1}\right)\left(\prod_{t \in T} \binom{ft + gt - 1}{ft}\right)\left(\prod_{c \notin S \cup T} \binom{fc + gc}{fc}\right)$$

if $(S, f), (T, g) \in J$ and $S \cap T = \emptyset$, and set $K_{S,T,f,g} = 0$ otherwise. Moreover, we define

$$\epsilon_{u,S,T,f,g} = K_{S \cup \{u\}, T \cup \{t_{f+g}\}, f+g} + K_{S \cup \{t_{f+g}\}, T \cup \{u\}, f+g}.$$

Yikes!

Goals ① Shape of the computation when $p = 2$.

② Consequences in motivic homotopy.

Computation Strategy

Input $\pi_{**} M\mathbb{F}_2 = \mathbb{F}_2[\tau]$, $|\tau| = (0,-1)$

$$\mathcal{A}_{**} = \pi_{**} M\mathbb{F}_2 \wedge M\mathbb{F}_2 \cong \mathbb{F}_2[\tau, \xi_1, \xi_2, \dots, \tau_0, \tau_1, \dots] / (\tau_i^2 - \tau\xi_{i+1} \mid i \geq 0)$$

$$|\xi_i| = (2^{i+1}-2, 2^i - 1), \quad |\tau_i| = (2^{i+1}-1, 2^i-1)$$

Step 1 Calculate étale motivic Hochschild homology

$$MHH_{**}(\mathbb{F}_2)[\tau^{-1}] \cong \mathbb{F}_2[\tau^{\pm 1}, \mu_0, \mu_1, \dots] / (\mu_i^2 - \tau\mu_{i+1} \mid i \geq 0)$$

$$\cong THH_*(\mathbb{F}_2)[\tau^{\pm 1}].$$

Step 2 Calculate mod-τ MHH

$$MHH_*(\mathbb{F}_2)/\tau \cong \Gamma_{\mathbb{F}_2}(\bar{\mu}_0, \bar{\mu}_1, \dots) \otimes \Lambda_{\mathbb{F}_2}(\bar{\lambda}_1, \bar{\lambda}_2, \dots).$$

divided powers algebra

Computation Strategy

Step 3 τ-torsion in $MHH_{**}(\mathbb{F}_2)$ injects into $MHH_{**}(\mathbb{F}_2)/\tau$ with image that of the τ-Bockstein.

Step 4 Give a presentation of τ-torsion in $MHH_{**}(\mathbb{F}_2)$ in terms of generators $x_{S,f}$ where $f: \mathbb{N} \longrightarrow \mathbb{N}$ has finite support and $S \subseteq \operatorname{supp} f$.

Step 5 Combine étale and τ-torsion computations via a pullback square.

Step 1 **Étale MHH**

$$\pi_{**} M\mathbb{F}_2 \wedge M\mathbb{F}_2[\tau^{-1}] \cong \mathbb{F}_2[\tau^{\pm 1}, \xi_1, \xi_2, \dots, \tau_0, \tau_1, \dots] / (\tau_i^2 - \tau\xi_{i+1} \mid i \geq 0)$$

$$\cong \mathbb{F}_2[\tau^{\pm 1}, \tau_0, \tau_1, \dots], \quad |\tau_i| = (2^{i+1}-1, 2^i - 1)$$

so the Tor-spectral sequence takes the form

$$E^2 = \mathrm{Tor}_{*,*,*}^{\mathbb{F}_2[\tau^{\pm 1}, \tau_0, \tau_1, \dots]}\left(\mathbb{F}_2[\tau^{\pm 1}], \mathbb{F}_2[\tau^{\pm 1}]\right)$$

$$\cong \Lambda_{\mathbb{F}_2}(\mu_0, \mu_1, \dots)[\tau^{\pm 1}] \Longrightarrow \mathrm{MHH}_{**}(\mathbb{F}_2)[\tau^{\pm 1}]$$

with $|\mu_i| = (1, 2^{i+1}-1, 2^i - 1)$.

- Degree considerations $\Rightarrow E^2 = E^\infty$.
- Power operations $\Rightarrow \mu_i^2 = \tau\mu_{i+1}$.

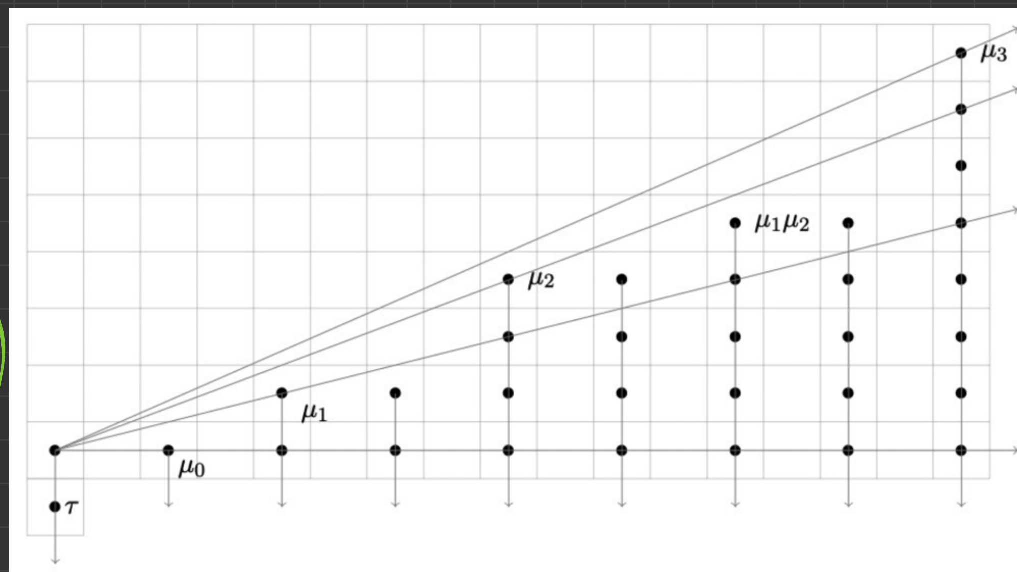
Step 1 **Étale MHH** (ct'd)

Upshot $MHH_{**}(\mathbb{F}_2)[\tau^{-1}] \cong \mathbb{F}_2[\tau^{\pm 1}, \mu_0, \mu_1, \dots] / (\mu_i^2 - \tau\mu_{i+1} \mid i \geq 0)$

$$\cong \mathbb{F}_2[\tau^{\pm 1}, \mu_0]$$

$\mathrm{im}(MHH_{**}(\mathbb{F}_2)$

$\to (\quad '' \quad)[\tau^{-1}])$



Step 2 **Mod-τ MHH**

$\mathcal{A}_{**}/\tau \cong \mathbb{F}_2[\xi_1, \xi_2, \ldots] \otimes \Lambda_{\mathbb{F}_2}(\tau_0, \tau_1, \ldots)$ so the Tor-spectral sequence takes the form

$$E^2 = \mathrm{Tor}^{\mathcal{A}_{**}/\tau}_{*,*,*}(\mathbb{F}_2, \mathbb{F}_2)$$

divided powers algebra

$$\cong \Lambda_{\mathbb{F}_2}(\bar{\lambda}_1, \bar{\lambda}_2, \ldots) \otimes \Gamma_{\mathbb{F}_2}(\bar{\mu}_0, \bar{\mu}_1, \ldots) \Longrightarrow \mathrm{MHH}_{**}(\mathbb{F}_2)/\tau.$$

Advanced degree yoga $\Longrightarrow$ $E^2 = E^\infty$ with no hidden ext'ns.

Upshot

$$\mathrm{MHH}_{**}(\mathbb{F}_2)/\tau \cong \Lambda_{\mathbb{F}_2}(\bar{\lambda}_1, \bar{\lambda}_2, \ldots) \otimes \Gamma_{\mathbb{F}_2}(\bar{\mu}_0, \bar{\mu}_1, \ldots).$$

(See arXiv: 2204.0041 for Steps 3-5.)

④ Consequences

Since $THH(\mathbb{F}_p)$ is an $H\mathbb{F}_p$-module, we get the splitting

$$THH(\mathbb{F}_p) \simeq \bigvee_{i\geq 0} \Sigma^{2i} H\mathbb{F}_p .$$

The τ-torsion in $MHH_{**}(\mathbb{F}_p)$ implies that this fails wildly in $Sp_{\mathbb{C}}$:

$MHH(\mathbb{F}_p)$ is not a free $M\mathbb{F}_p$-module.

In Sp, the following are true:

1. $H\mathbb{F}_2$ is a Thom spectrum of an E_2-map with target $\Omega^2 S^3$
2. $THH(\mathrm{Thom}_1) = \mathrm{Thom}_2 \underset{\eta_{top}=0}{=} \mathrm{Thom}_1 \wedge B(\mathrm{base}_1)_+$.
3. $B\Omega^2 S^3 \simeq \bigvee(\text{spheres})$ stably.

Consequences (ct'd)

Potential motivic analogues:

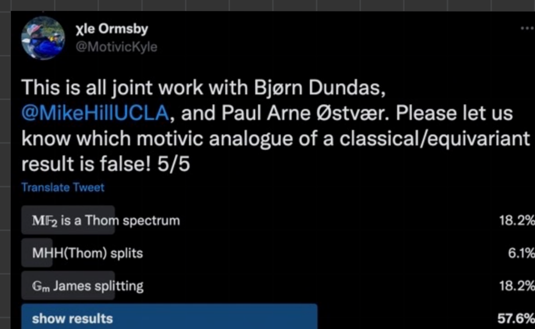
① $M\mathbb{F}_2$ is a Thom spectrum over $\Omega^{2,1}S^{3,1}$.

② $MHH(\mathrm{Thom}_1) \simeq \mathrm{Thom}_1 \wedge B(\mathrm{base}_1)_+$.

③ $B\Omega^{2,1}S^{3,1}$ is a wedge of spheres stably,

i.e. $\Sigma^\infty \Omega^{1,1}\Sigma^{1,1}S^2$ satisfies "$\mathbb{G}_m$-James splitting".

Thm (Dundas, Hill, O, Østvær) At most two of these are true!


Kyle Ormsby
@MotivicKyle

This is all joint work with Bjørn Dundas, @MikeHillUCLA, and Paul Arne Østvær. Please let us know which motivic analogue of a classical/equivariant result is false! 5/5

Translate Tweet

$M\mathbb{F}_2$ is a Thom spectrum	18.2%
MHH(Thom) splits	6.1%
G_m James splitting	18.2%
show results	57.6%

Thank You!

Questions welcome.