

**MATH 111: CALCULUS**  
**HOMEWORK DUE FRIDAY WEEK 11**

Make sure to review the homework instructions in the syllabus before writing your solutions. In particular, show your work and write in complete sentences (but also aim for concise explanations).

*Problem 1.* Suppose  $f: [a, b] \rightarrow \mathbb{R}$  is an integrable function. Let  $L_n$  and  $R_n$  denote the left and right Riemann sums of  $f$  over  $[a, b]$  with  $n$  subdivisions.

- (a) Explain why  $R_n - L_n = (f(b) - f(a)) \cdot \frac{b-a}{n}$ .
- (b) Suppose further that  $f$  is increasing on  $[a, b]$ . Explain why, in this case, the error between either  $L_n$  or  $R_n$  and  $\int_a^b f(x) dx$  is at most  $(f(b) - f(a)) \cdot \frac{b-a}{n}$ .
- (c) Find an example of a function where the error between  $L_n$  and  $\int_a^b f(x) dx$  is greater than  $(f(b) - f(a)) \cdot \frac{b-a}{n}$ .

*Problem 2.* Suppose that

$$A = \int_0^{2\pi} \sin^2 t \, dt \quad \text{and} \quad B = \int_0^{2\pi} \cos^2 t \, dt.$$

Without computing any antiderivatives, show that  $A + B = 2\pi$  and  $A = B$ .

*Problem 3.* Evaluate the following integrals using the Fundamental Theorem of Calculus and basic properties of integrals:

- (a)  $\int_{-1}^2 (x^2 - 3x) dx$
- (b)  $\int_1^2 \frac{2}{x^3} dx$
- (c)  $\int_0^{\pi/4} \sec^2 \theta \, d\theta$
- (d)  $\int_0^{\pi/2} (x - \sin x) dx$

*Problem 4.* A horizontal cylindrical tank has cross-sectional area  $A(x) = 4(6x - x^2)$  m<sup>2</sup> at height  $x$  meters above the bottom when  $x \leq 3$ .

- (a) The volume  $V$  of the tank between heights  $a$  and  $b$  is  $\int_a^b A(x) dx$ . Find the volume between heights 2 m and 3 m.
- (b) Suppose that oil is being pumped into the tank at a rate of 50 L/min. Using the chain rule, at how many meters per minute is the height of the oil in the tank changing, expressed in terms of  $x$ , when the height is at  $x$  meters? (Note that there are 1,000 liters in a cubic meter; you will need this fact in order for your units to work out properly.)
- (c) How long does it take to fill the tank to 3 m when you start from a fill level of 2 m?