

Midterm 3 Content

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Bases for Polynomials

Let P_3 be the set of polynomials of degree 3 or less. Explain which of the following are bases for this set.

a) $\{ (1+x), (x^2-x-1), x^3 \}$

b) $\{ 3, (5+x^3) \}$

c) $\{ x^2+9x+3, 5x, 6x^3, 12x+3x^2, 20 \}$

d) $\{ 6x, 5x, 20x^2, 30x^3 \}$

e) $\{ 29, x^2+1, x-4, 20x^3 \}$

Def. (Basis). A subset $\beta \subseteq H$ is a **basis** for H if the following hold:

1. β is linearly independent (has no redundant information)
2. $\text{span}(\beta) = H$ (has sufficient information)

Thm. (Basis Theorem). Suppose V is a p dimensional vector space.

1. A linearly independent set of p vectors is a basis.
2. A spanning set of p vectors is a basis.

Coordinate Systems

Find the coordinates of $\begin{bmatrix} -2 \\ 1 \\ 2 \\ -20 \end{bmatrix}$ in the basis given by the previous question.

Eigenvectors and Eigenvalues

Let A be given by the matrix

$$A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 1 & 1 \end{bmatrix}$$

- a) What is the characteristic polynomial of A ?

b) What are the eigenvalues and eigenvectors of A ?

c) Is this matrix diagonalizable?

Orthogonality

Consider the following statements.

1. Suppose $\vec{u}, \vec{v}, \vec{w} \in \mathbb{R}^n$ are vectors. If \vec{u} and \vec{v} are orthogonal, then \vec{u} and \vec{w} are orthogonal.
2. For any $\vec{v} \in \mathbb{R}^n$ and any $c \in \mathbb{R}$, we have $\|c \cdot \vec{v}\| = c \cdot \|\vec{v}\|$
3. Consider $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 4 & 1 & -3 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 10 \\ -7 \\ 1 \end{bmatrix}$. Then, \vec{v} is orthogonal to any vector in the column space of A .

Which of the following is true?

- a) All of these are false.
- b) Statement 1 is true, and 2 and 3 are false.
- c) Statement 2 is true, and 1 and 3 are false.
- d) Statement 3 is true, and 1 and 2 are false.
- e) Statements 1 and 2 are true, and 3 is false.
- f) All of these are true.

Gram Schmidt

Let $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \\ 2 & 2 \end{bmatrix}$. Find an orthonormal basis for $\text{Col}(A)$.

Orthogonal Projections

Let W be the subspace of \mathbb{R}^4 spanned by the orthogonal set

$$\vec{v}_1 = \begin{bmatrix} 1 \\ -1 \\ 2 \\ 0 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

a) Find a basis for W^\perp , the orthogonal complement of W .

b) Let $\vec{y} = \begin{bmatrix} 3 \\ 0 \\ 5 \\ 2 \end{bmatrix}$. Find $\hat{y} = \text{proj}_W \vec{y}$, the projection of \vec{y} onto W .

c) Find a vector $\vec{z} \in W^\perp$ such that $\vec{y} = \hat{y} + \vec{z}$.

Fundamental Subspaces

Consider the matrix $A = \begin{bmatrix} 2 & 2 & 4 & 0 \\ 0 & 0 & -1 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$

a) What is the RREF of A ?

b) What is a basis for the columnspace of A ?

c) What is a basis for the nullspace of A ?

True/False Questions

(5.1) T/F: A matrix A is invertible if and only if 0 is an eigenvalue of A .

(6.1) T/F: If $\|u\|^2 + \|v\|^2 = \|u + v\|^2$, then u and v are orthogonal.

(6.2) T/F: Every linearly independent set is orthogonal.

(6.2) T/F: Every orthogonal set is linearly independent.

(4.4) T/F: A nonzero vector is linearly dependent.

(4.5) T/F: A vector space is infinite dimensional if it is spanned by a set of infinitely many vectors.

Good luck on the test!