Def. (Limit). A sequence a_n converges to a limit L if, for every $\varepsilon > 0$, there exists $N \in \mathbb{N}$, such that if $n > N$, then $|a_n - L| < \varepsilon$

Proof Template:

Claim. lim ( ) =

Proof. Let $\varepsilon > 0$ be arbitrary.

Choose $N =$

Then, for any $n > N$, we have

$$|a_n - L| = \qquad\qquad\qquad\qquad = \varepsilon$$

□

Example Proof:

Claim. $\lim\left(\underline{\tfrac{1}{n}}\right) = \underline{0}$

Proof. Let $\varepsilon > 0$ be arbitrary.

Choose $N = \underline{\frac{1}{\varepsilon}}$ (this is well-defined since $\varepsilon \neq 0$)

Then, for any $n > N$, we have

$$|a_n - L| = \underline{\left|\frac{1}{n} - 0\right| = \frac{1}{n} < \frac{1}{N}} = \varepsilon$$

□