Limit Definition of e

Kyle Sung

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$$\lim_{n\to\infty}(1+\frac{1}{n})^n = \lim_{n\to\infty} e^{\ln(1+\frac{1}{n})^n} \tag{1}$$

$$= e^{\lim_{n\to\infty} \ln(1+\frac{1}{n})^n} \tag{2}$$

$$= e^{\lim_{n\to\infty} n\ln(1+\frac{1}{n})} \tag{3}$$

$$= e^{\lim_{n\to\infty} \frac{\ln(1+\frac{1}{n})}{\frac{1}{n}}} \tag{4}$$

$$\text{By L'Hopital's Rule } = e^{\lim_{n\to\infty} \frac{\frac{-n^{-2}}{(1+\frac{1}{n})}}{-n^{-2}}} \tag{5}$$

$$= e^{\lim_{n\to\infty} \frac{1}{1+\frac{1}{n}}} \tag{6}$$

$$= e \tag{7}$$