Homework assignment 2:
Suggested due date: Friday, February 12 2015 at 03:30pm

1. Prove that $f(n) = 10n^4 + 2n^2 + 3$ is *O(n^4),* provide the appropriate C and k constants.
2. Prove that $f(n) = 2n^2 - n \log n + 3\log n$ is *O(n^2)*, provide the appropriate C and k constants.
3. Prove that $f(n) = 2n^4 \log n^4 - n^2 + 3\log n$ is *O(n^4 log n),* provide the appropriate C and k constants.
4. Prove or disprove

   $f(n) = 3n^2 + 2n + 1$

   $g(n) = 5n^3 - n + 3$

   :

   a. O(n^2)
   b. O(n^3)
   c. Ω(n)
   d. Θ(n^3)
   e. ω(n)
   f. o(n^2)

   Provide the appropriate C and k constants if possible.

5. What is the growth of the below functions:

   5.1. $f(n) = 2n^4 \log n^4 + n^{4.0001} - 3\log n$

   5.2. $f(n) = 3n^3 \log(n^4 - n^2) + 100000$

   5.3. $f(n) = \log^{100} n^{50} + n$

   5.4. $f(n) = n^4 \log^3 n + 4$

   5.5. $f(n) = 10000n \log n^7 + 3\log n + 1000\sqrt{n}$

   5.6. $f(n) = \sqrt[10]{n} + 10^{10} \log^{100} n + 8$

   5.7. $f(n) = \sqrt{\sqrt{n}} + 9\log n$

6. Prove that $(n+5)^{100} = \theta(n^{100})$
7. Discuss the growth of the below functions (Show the work)
   7.1. $f(n) = (\log n)^{\log n}$
   7.2. $f(n) = 2^{\sqrt{2\log n}}$
   7.3. $f(n) = (\sqrt{2})^{\log n}$
   7.4. $f(n) = n^{1/\log n}$
8. Prove transitivity of big-O: if *f(n) = O(g(n))*, and *g(n) = O(h(n))*, then *f(n) = O(h(n))*.
9. Prove that f(n)=O(g(n)) iff g(n)= Ω(f(n)).
10. Compare the growth of $f(n) = n$ and $g(n) = n^{1+\sin n}$.
11. Compare the growth of $f(n) = \sqrt{n}$ and $g(n) = n\sin(n)$.
12. Compare the growth of $f(n) = n$ and $g(n) = n\sin(n)$.
13. Prove or disprove: 2^{n+1}=O(2^n).
14. Prove or disprove: 2^{2n}=O(2^n).
15. Prove that if $lim \longrightarrow \infty \frac{f(n)}{g(n)} = C$, for some constant C>0, then f(n)= Θ(g(n)).

    Hint: $lim \longrightarrow \infty \frac{f(n)}{g(n)} = C$ means that for every ϵ>0, there exists k>=0 such that, for all n>=k, $|\frac{f(n)}{g(n)} - C| < \varepsilon$

16. Suppose ***g(n) ≥ 1*** for all *n*, and that *f(n) ≤ g(n) + L*, for some constant *L* and all *n*. Prove that *f(n) = O(g(n))*.
17. Prove or disprove: if *f(n) = O(g(n))* and ***f(n) ≥ 1*** and ***log(g(n)) ≥ 1*** for sufficiently large n, then *log(f(n)) = O(log(g(n)))*.
18. Show that *log(n!) = Θ(n log n)*.

19. Prove that $n! = o(n^n)$.

20. Prove that $n! = \omega(2^n)$.

21. Which one of the below functions grows faster? Explain.

$$f(n) = 2^{2^n}, \quad g(n) = n!$$

22. Provide a closed-form expression for the asymptotic growth of $n + n/2 + n/3 + \cdots + 1$

23. Use the integral theorem to calculate the growth of $1 + 2^k + 3^k + \ldots + n^k$

Extra Credit Question: ☺

24. Prove or disprove: if $f(n) = O(g(n))$, then $2^{f(n)} = O(2^{g(n)})$.