Evolution of Cooperation (Section 2)

6 February 2017

Review of Basic Games

- Two strategies and two players. The players **simultaneously** decide which strategy to select.
- The **best response** is the strategy which produces the most favorable outcome for a player, *taking the other player's strategy as given*.
- A set of (pure) strategies (A_1, A_2) is a **Nash Equilibrium** if player 1 is playing a best response against player 2, and player 2 is playing a best response against player 1!
- $U_1(A_1, A_2) > U_1(B_1, A_2)$ **and** $U_2(A_1, A_2) > U_2(A_1, B_2)$
- In a NE, neither player has an incentive to switch strategies!

Review of Basic Games

Social Dilemma

Player 1 \ Player 2	Cooperate	Defect
Cooperate	2, 2	1, 4
Defect	4, 1	3, 3

Anti−coordination

Player 1 \ Player 2	Cooperate	Defect
Cooperate	3, 3	2, 4
Defect	4, 2	1, 1

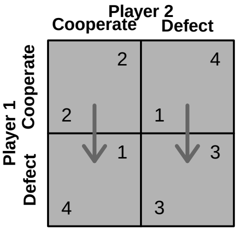
Coordination

Player 1 \ Player 2	Cooperate	Defect
Cooperate	4, 4	1, 2
Defect	2, 1	3, 3

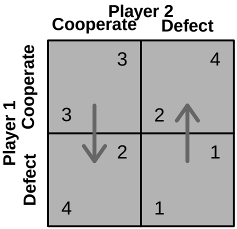
Review of Basic Games

Player 1's best responses to Player 2's strategies (vertical arrows):

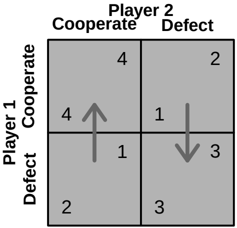
Social Dilemma



Anti−coordination



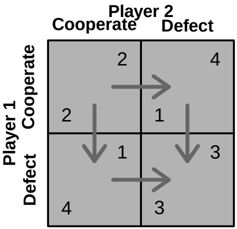
Coordination



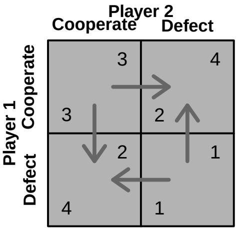
Review of Basic Games

Player 2's best responses to Player 1's strategies (horizontal arrows):

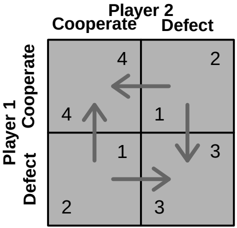
Social Dilemma



Anti−coordination



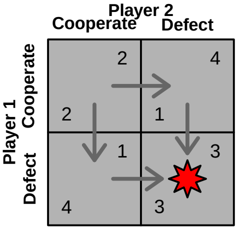
Coordination



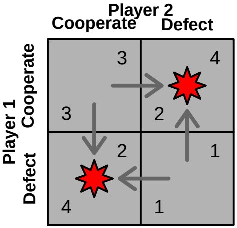
Review of Basic Games

NE: P1 and P2 play their best responses. We are stuck here!

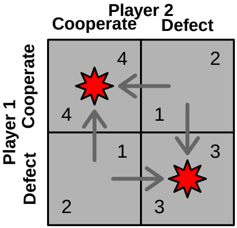
Social Dilemma



Anti−coordination



Coordination



Social Preferences

- "Homo-economicus": players are "selfish" and predicted behavior simply a function of individual payoffs.
- But even in one-shot prisoner's dilemmas, rate of cooperation is typically 40-60%.
- Why? If players have **social preferences**, behavior also depends on what happens to others and why these things happen.
- Can be "other-regarding"–my evaluation of some state depends on how you are doing.
- Can be "process-regarding"–my evaluation of some state depends on how it came about.
- Examples: altruism, fairness, reciprocity and inequity averson.

Social Preferences (Altruism)

Social Dilemma

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	$b-c$, $b-c$	$-c$, b
	Defect	b, $-c$	0, 0

Social Preferences (Altruism)

Let $b = 2$, $c = 1$, $\kappa \in (0, 1)$ "altruism" parameter.

	Selfish person	Altruistic person
$U(C, C)$	$b - c = 1$	$b - c + \kappa(b - c) = 1 + \kappa$
$U(D, C)$	$b = 2$	$b - \kappa c = 2 - \kappa$
$U(C, D)$	$-c = -1$	$-c + \kappa b = 2\kappa - 1$
$U(D, D)$	0	0

Cooperative NE?

$$1 + \kappa > 2 - \kappa \implies \kappa > 1/2$$

Social Preferences (Altruism)

Let $b = 2$, $c = 1$. Suppose $\kappa = 3/4$ (high altruism):

	Selfish person	Altruistic person
$U(C, C)$	$b - c = 1$	$b - c + \kappa(b - c) = 1 + \kappa = 1.75$
$U(D, C)$	$b = 2$	$b - \kappa c = 2 - \kappa = 1.25$
$U(C, D)$	$-c = -1$	$-c + \kappa b = 2\kappa - 1 = 0.5$
$U(D, D)$	0	0

Social Preferences (Altruism)

Let $b = 2$, $c = 1$. Suppose $\kappa = 3/4$ (high altruism):

Social Dilemma?

Player 2

Player 1	Cooperate	Defect
Cooperate	1, 1	−1, 2
Defect	2, −1	0, 0

Social Dilemma?

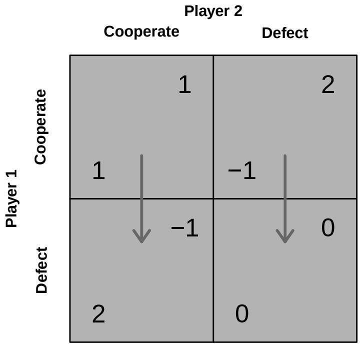
Player 2

Player 1	Cooperate	Defect
Cooperate	1.75, 1.75	−1, 1.25
Defect	1.25, −1	0, 0

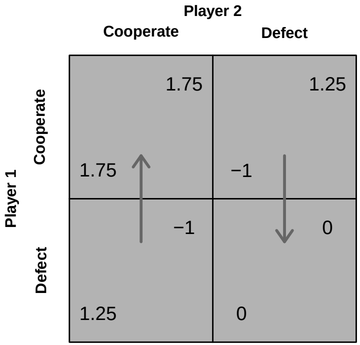
Social Preferences (Altruism)

Let $b = 2$, $c = 1$. Suppose $\kappa = 3/4$ (high altruism):

Social Dilemma?



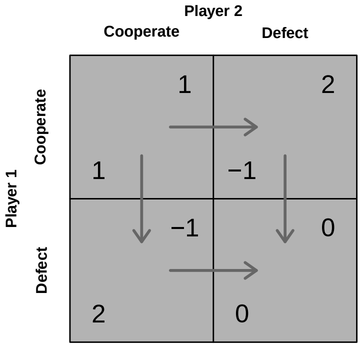
Social Dilemma?



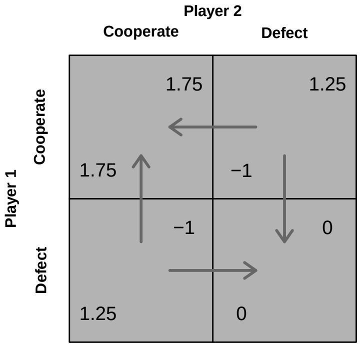
Social Preferences (Altruism)

Let $b = 2$, $c = 1$. Suppose $\kappa = 3/4$ (high altruism):

Social Dilemma?



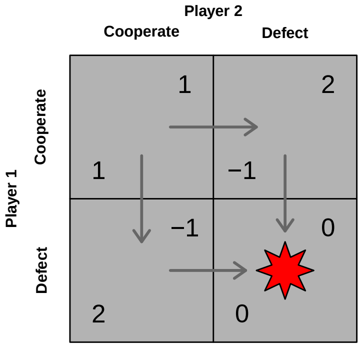
Social Dilemma?



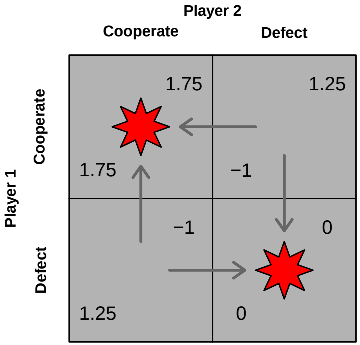
Social Preferences (Altruism)

Let $b = 2$, $c = 1$. Suppose $\kappa = 3/4$ (high altruism):

Social dilemma :−(



Coordination game!



Social Preferences (Inequity aversion)

- P1's utility: $U_1 = \pi_1 - \delta_1 \max(\pi_2 - \pi_1, 0) - \alpha_1 \max(\pi_1 - \pi_2, 0)$
- δ_1: how much P1 dislikes disadvantageous ($\pi_2 - \pi_1 > 0$) differences.
- $\alpha_1 \in (0, 1)$: how much P1 dislikes advantageous ($\pi_1 - \pi_2 > 0$) differences.
- If $\alpha_1 = 1$ then P1 cares only about P2's payoffs if they fall short of her own.
- If $\delta_1 > 1$, P1 is very adverse to disadvantageous differences.
- P2's utility: $U_2 = \pi_2 - \delta_2 \max(\pi_1 - \pi_2, 0) - \alpha_2 \max(\pi_2 - \pi_1, 0)$
- When $\delta_1 \neq \delta_2$ and/or $\alpha_1 \neq \alpha_2$, things get more complicated.

Social Preferences (Inequity aversion)

Suppose α, δ are the same for both players. Again consider social dilemma setup with $c = 1, b = 2$.

	Selfish person	Inequity averse person
$U(C, C)$	$b - c = 1$	$b - c = 1$
$U(D, C)$	$b = 2$	$b - \alpha[b - (-c)] = 2 - 3\alpha$
$U(C, D)$	$-c = -1$	$-c - \delta[b - (-c)] = -1 - 3\delta$
$U(D, D)$	0	0

-Cooperative NE?

$$1 > 2 - 3\alpha \implies \alpha > 1/3$$

Social Preferences (Inequity aversion)

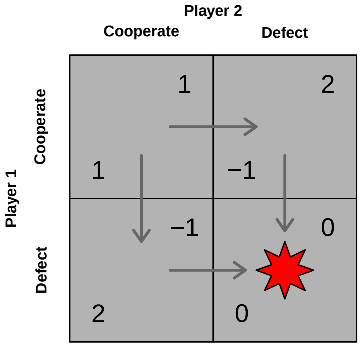
Let $b = 2$, $c = 1$. Suppose $\alpha = 3/4$, $\delta = 2$:

	Selfish person	Inequity averse person
$U(C, C)$	$b - c = 1$	$b - c = 1$
$U(D, C)$	$b = 2$	$b - \alpha[b - (-c)] = 2 - 3\alpha = -0.25$
$U(C, D)$	$-c = -1$	$-c - \delta[b - (-c)] = -1 - 3\delta = -7$
$U(D, D)$	0	0

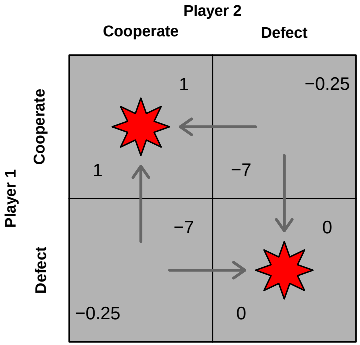
Social Preferences (Inequity aversion)

Let $b = 2$, $c = 1$. Suppose $\alpha = 3/4$, $\delta = 2$:

Social dilemma :−(



Coordination game!

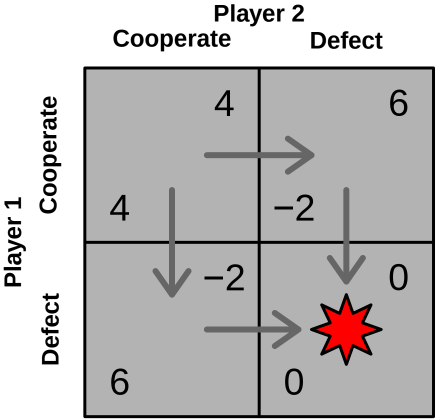


Evolutionary game theory

Key concepts:

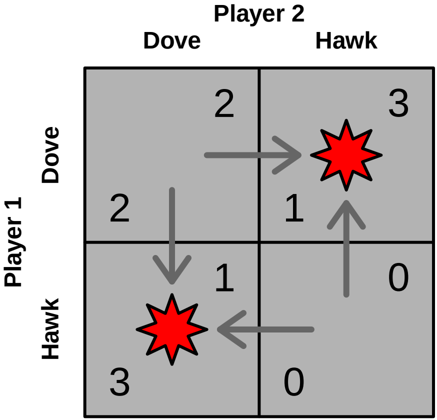
- Nash Equilibrium (NE) is a **static concept**.
- When NE is not unique, dynamics matter!
- Evolutionary game theory lets *populations* evolve.
- Fraction x of the population plays "strategy 1" and fraction $1 - x$ plays "strategy 2".
- With multiple NE (or none!), evolutionary game theory tells us how the population evolves.
- An **Evolutionary Stable Strategy (ESS)** is a strategy which, if adopted by a population, cannot be invaded by a mutant strategy that is initially rare.
- Every NE is an ESS!
- Strategy A **risk dominates** strategy B if $\pi_A > \pi_B$ at $x = 1/2$.

Evolutionary game theory: “dominance”



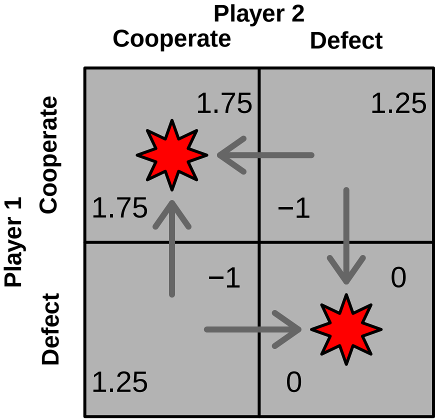
- Mutual defection is the NE. This is also the ESS.
- $\pi_C = 4x - 2(1-x) = 6x - 2$, $\pi_D = 6x + 0(1-x) = 6x$
- At $x = 1/2$, $\pi_C = 1 < \pi_D = 3$ so D is **risk dominant**
- Population will evolve to 100% defectors.

Evolutionary game theory: "co-existence"



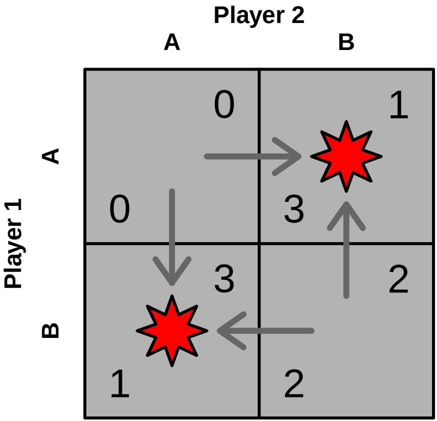
- DH and HD are both equilibria.
- $\pi_D = 2x + (1 - x) = x + 1$, $\pi_H = 3x + 0(1 - x) = 3x$
- At $x = 1/2$, $\pi_D = 3/2 = \pi_H = 3/2$ so neither **risk dominant**!
- Population will evolve to stable mix of both types.

Evolutionary game theory: “bistability”



- CC and DD are both equilibria, but mixture of types unstable!
- $\pi_C = 1.75x - (1-x) = 2.75x - 1, \pi_D = 1.25x + 0(1-x) = 1.25x$
- At $x = 1/2$, $\pi_C = 1.375 > \pi_D = 0.625$ so C is **risk dominant**.
- Population will *most likely* evolve to 100% Cooperators.

Evolutionary Dynamics (worked example)



- AB and BA are both equilibria.
- $\pi_A = 0 * x + 3(1-x) = 3 - 3x, \pi_B = 1x + 2(1-x) = 2 - x$
- At $x = 1/2$, $\pi_A = 3/2 = \pi_B = 3/2$ so neither **risk dominant**.

Evolutionary Dynamics (worked example)

- Suppose we start in period t with $x_t = 0.1$ percent playing A and $(1 - x_t) = 0.9$ percent playing B in population.
- If $\pi_A > \pi_B$ then x_{t+1} increases by 0.1.
- If $\pi_A < \pi_B$ then x_{t+1} decreases by 0.1.
- If $\pi_A = \pi_B$ then population is at equilibrium.
- If $x_{t^*} = 0$ or $x_{t^*} = 1$ then we are also at equilibrium.

x_t	π_A	π_B	x_{t+1}
0.10	2.70	1.90	0.20
0.20	2.40	1.80	0.30
0.30	2.10	1.70	0.40
0.40	1.80	1.60	0.50
0.50	1.50	1.50	0.50
0.50	1.50	1.50	0.50

Evolutionary Dynamics (worked example)

- What if we start at $x_t = 0.9$ instead?

x_t	π_A	π_B	x_{t+1}
0.90	0.30	1.10	0.80
0.80	0.60	1.20	0.70
0.70	0.90	1.30	0.60
0.60	1.20	1.40	0.50
0.50	1.50	1.50	0.50
0.50	1.50	1.50	0.50

- Regardless of initial conditions, the population will end up with 50/50 mix.

Evolutionary Dynamics (worked example)