Evolution of Cooperation (Section 5)

28 February 2017

Overview

- Part I: review of material on structured interactions (assortment)

Overview

- Part I: review of material on structured interactions (assortment)
- Part II: work through last problem set together?

Overview

- Part I: review of material on structured interactions (assortment)
- Part II: work through last problem set together?
- Part II: group discussion of readings w/ activity?

Structured interactions: assortment example

- Assume individuals are either defectors or cooperators in single-period PD.

Structured interactions: assortment example

- Assume individuals are either defectors or cooperators in single-period PD.
- They periodically update their type in response to relative success of strategies (C and D)

Structured interactions: assortment example

- Assume individuals are either defectors or cooperators in single-period PD.
- They periodically update their type in response to relative success of strategies (C and D)
- Example: suppose people live in villages that are homogenous by type and some fraction α of their interactions take place **within the village**.

Structured interactions: assortment example

- Assume individuals are either defectors or cooperators in single-period PD.
- They periodically update their type in response to relative success of strategies (C and D)
- Example: suppose people live in villages that are homogenous by type and some fraction α of their interactions take place **within the village**.
- The remaining $1 - \alpha$ interactions take place in a city where types are well mixed.

Structured interactions: assortment example

- Assume individuals are either defectors or cooperators in single-period PD.
- They periodically update their type in response to relative success of strategies (C and D)
- Example: suppose people live in villages that are homogenous by type and some fraction α of their interactions take place **within the village**.
- The remaining $1 - \alpha$ interactions take place in a city where types are well mixed.
- If the fraction of cooperators is p, probability that cooperator pairs with a fellow cooperator is: not p, but $\alpha + (1 - \alpha)p$!

Structured interactions: assortment example

- Assume individuals are either defectors or cooperators in single-period PD.
- They periodically update their type in response to relative success of strategies (C and D)
- Example: suppose people live in villages that are homogenous by type and some fraction α of their interactions take place **within the village**.
- The remaining $1 - \alpha$ interactions take place in a city where types are well mixed.
- If the fraction of cooperators is p, probability that cooperator pairs with a fellow cooperator is: not p, but $\alpha + (1 - \alpha)p$!
- Probability that defector pairs with a defector is: $\alpha + (1 - \alpha)(1 - p)$!

Structured interactions: assortment example

- Assume individuals are either defectors or cooperators in single-period PD.
- They periodically update their type in response to relative success of strategies (C and D)
- Example: suppose people live in villages that are homogenous by type and some fraction α of their interactions take place **within the village**.
- The remaining $1-\alpha$ interactions take place in a city where types are well mixed.
- If the fraction of cooperators is p, probability that cooperator pairs with a fellow cooperator is: not p, but $\alpha + (1-\alpha)p$!
- Probability that defector pairs with a defector is: $\alpha + (1-\alpha)(1-p)$!
- **Note**: $\alpha = 1$ is extreme segmentation; $\alpha = 0$ is random pairing!

Structured interactions: assortment example

	C	D
C	R,R	S,T
D	T,S	P,P

- $\pi_C = \alpha R + (1-\alpha)[pR + (1-p)S]$

Structured interactions: assortment example

	C	D
C	R,R	S,T
D	T,S	P,P

- $\pi_C = \alpha R + (1-\alpha)[pR + (1-p)S]$
- $\pi_D = \alpha P + (1-\alpha)[pT + (1-p)P]$

Structured interactions: assortment example

	C	D
C	R,R	S,T
D	T,S	P,P

- $\pi_C = \alpha R + (1-\alpha)[pR + (1-p)S]$
- $\pi_D = \alpha P + (1-\alpha)[pT + (1-p)P]$
- Equilibrium level of cooperation: $p^* = \frac{\alpha(S-R)+P-S}{(1-\alpha)(R-S-T+P)}$

Structured interactions: assortment example

	C	D
C	R,R	S,T
D	T,S	P,P

- $\pi_C = \alpha R + (1-\alpha)[pR + (1-p)S]$
- $\pi_D = \alpha P + (1-\alpha)[pT + (1-p)P]$
- Equilibrium level of cooperation: $p^* = \frac{\alpha(S-R)+P-S}{(1-\alpha)(R-S-T+P)}$
- May be stable or unstable, depending on payoff matrix!

Structured interactions: assortment example

	C	D
C	R,R	S,T
D	T,S	P,P

- $\pi_C = \alpha R + (1-\alpha)[pR + (1-p)S]$
- $\pi_D = \alpha P + (1-\alpha)[pT + (1-p)P]$
- Equilibrium level of cooperation: $p^* = \frac{\alpha(S-R)+P-S}{(1-\alpha)(R-S-T+P)}$
- May be stable or unstable, depending on payoff matrix!
- If unstable, p^* is boundary between basin of attraction at $p = 1$ and $p = 0$.

Structured interactions: assortment example

- $p^* = \frac{\alpha(S-R)+P-S}{(1-\alpha)(R-S-T+P)}$

Structured interactions: assortment example

- $p^* = \frac{\alpha(S-R)+P-S}{(1-\alpha)(R-S-T+P)}$
- For stable (interior) equilibrium, denominator must be < 0, requiring numerator to also be negative for $p > 0$.

Structured interactions: assortment example

- $p^* = \frac{\alpha(S-R)+P-S}{(1-\alpha)(R-S-T+P)}$
- For stable (interior) equilibrium, denominator must be < 0, requiring numerator to also be negative for $p > 0$.
- That is, we need: $(1-\alpha)(T-P) > (1-\alpha)(R-S)$.

Structured interactions: assortment example

- $p^* = \frac{\alpha(S-R)+P-S}{(1-\alpha)(R-S-T+P)}$
- For stable (interior) equilibrium, denominator must be < 0, requiring numerator to also be negative for $p > 0$.
- That is, we need: $(1-\alpha)(T-P) > (1-\alpha)(R-S)$.
- Stability happens when reward from defection on a cooperator $(T-R)$ is bigger than the penalty of cooperating against a defector $(P-S)$!

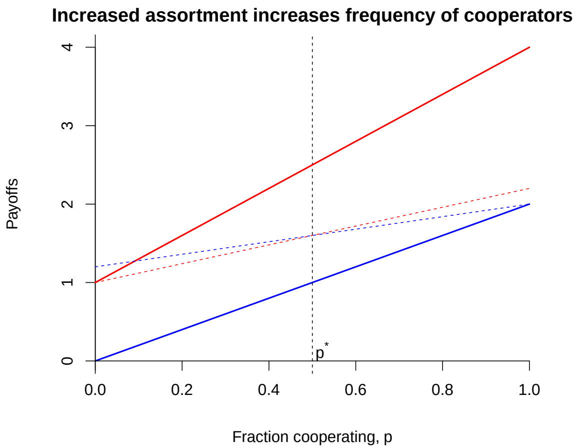
Structured interactions: assortment example

- $p^* = \frac{\alpha(S-R)+P-S}{(1-\alpha)(R-S-T+P)}$
- For stable (interior) equilibrium, denominator must be < 0, requiring numerator to also be negative for $p > 0$.
- That is, we need: $(1-\alpha)(T-P) > (1-\alpha)(R-S)$.
- Stability happens when reward from defection on a cooperator $(T-R)$ is bigger than the penalty of cooperating against a defector $(P-S)$!
- If p^* is stable, then increasing assortment will increase frequency of cooperators.

Structured interactions: assortment example

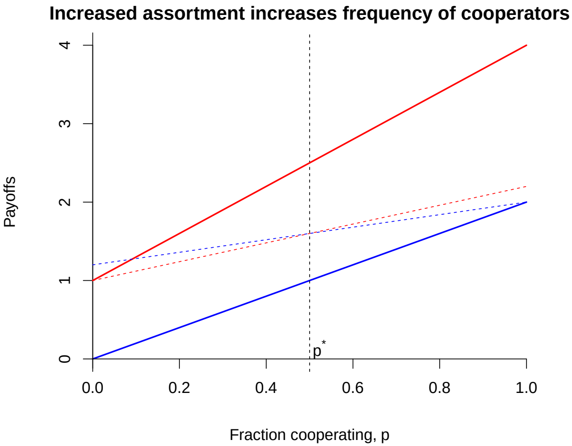
- $p^* = \frac{\alpha(S-R)+P-S}{(1-\alpha)(R-S-T+P)}$
- For stable (interior) equilibrium, denominator must be < 0, requiring numerator to also be negative for $p > 0$.
- That is, we need: $(1-\alpha)(T-P) > (1-\alpha)(R-S)$.
- Stability happens when reward from defection on a cooperator $(T-R)$ is bigger than the penalty of cooperating against a defector $(P-S)$!
- If p^* is stable, then increasing assortment will increase frequency of cooperators.
- If p^* is unstable, increasing assortment will increase cooperative basin of attraction.

Structured interactions: assortment example



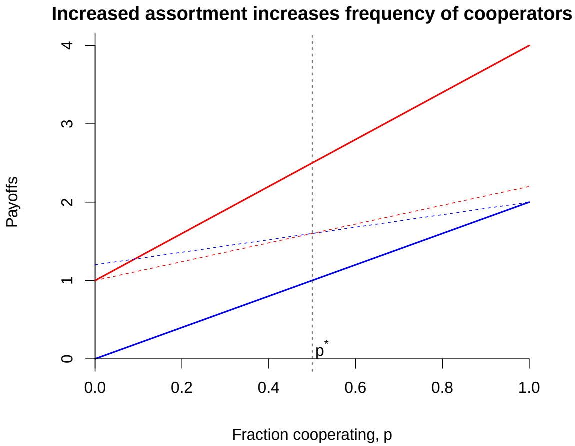
- Let $T = 4, R = 2, S = 0, P = 1$ so that $T > R > P > S$.

Structured interactions: assortment example



- Let $T = 4, R = 2, S = 0, P = 1$ so that $T > R > P > S$.
- Solid lines: $\alpha = 0$ (no assortment). Dotted lines: $\alpha = 0.6$

Structured interactions: assortment example



- Let $T = 4, R = 2, S = 0, P = 1$ so that $T > R > P > S$.
- Solid lines: $\alpha = 0$ (no assortment). Dotted lines: $\alpha = 0.6$
- $p^* = .50$ is stable equilibrium (Why?)

Structured interactions: assortment example

	C	D
C	R,R	S,T
D	T,S	P,P

- Recall C is ESS if:

Structured interactions: assortment example

	C	D
C	R,R	S, T
D	T,S	P,P

- Recall C is ESS if:

1. $\pi(C, C) > \pi(D, C)$ OR

Structured interactions: assortment example

	C	D
C	R,R	S,T
D	T,S	P,P

- Recall C is ESS if:

1. $\pi(C, C) > \pi(D, C)$ OR
2. $\pi(C, C) = \pi(D, C)$ and $\pi(C, D) > \pi(D, D)$

Structured interactions: assortment example

	C	D
C	R,R	S,T
D	T,S	P,P

- Recall C is ESS if:

1. $\pi(C,C) > \pi(D,C)$ OR
2. $\pi(C,C) = \pi(D,C)$ and $\pi(C,D) > \pi(D,D)$

- Probability D meets C: $(1-\alpha)p$

Structured interactions: assortment example

	C	D
C	R, R	S, T
D	T, S	P, P

- Recall C is ESS if:

1. $\pi(C, C) > \pi(D, C)$ OR
2. $\pi(C, C) = \pi(D, C)$ and $\pi(C, D) > \pi(D, D)$

- Probability D meets C: $(1 - \alpha)p$
- Probability C meets C: $\alpha + p(1 - \alpha)$

Structured interactions: assortment example

	C	D
C	R,R	S,T
D	T,S	P,P

- Recall C is ESS if:

1. $\pi(C, C) > \pi(D, C)$ OR
2. $\pi(C, C) = \pi(D, C)$ and $\pi(C, D) > \pi(D, D)$

- Probability D meets C: $(1 - \alpha)p$
- Probability C meets C: $\alpha + p(1 - \alpha)$
- Probability D meets D: $\alpha + (1 - \alpha)(1 - p)$

Structured interactions: assortment example

	C	D
C	R,R	S,T
D	T,S	P,P

- Recall C is ESS if:

1. $\pi(C,C) > \pi(D,C)$ OR
2. $\pi(C,C) = \pi(D,C)$ and $\pi(C,D) > \pi(D,D)$

- Probability D meets C: $(1-\alpha)p$
- Probability C meets C: $\alpha + p(1-\alpha)$
- Probability D meets D: $\alpha + (1-\alpha)(1-p)$
- $\pi(C,C) = \alpha R + p(1-\alpha)R$; $\pi(D,C) = (1-\alpha)pT$

Structured interactions: assortment example

	C	D
C	$R = 2, R = 2$	$S = 0, T = 4$
D	$T = 4, S = 0$	$P = 1, P = 1$

- $\pi(C, C) = \alpha R + p(1-\alpha)R > \pi(D, D) = (1-\alpha)pT$

Structured interactions: assortment example

	C	D
C	$R = 2, R = 2$	$S = 0, T = 4$
D	$T = 4, S = 0$	$P = 1, P = 1$

- $\pi(C, C) = \alpha R + p(1 - \alpha)R > \pi(D, D) = (1 - \alpha)pT$
- $1.2 + 0.8p > 1.6p$ at $\alpha = 0.6$

Structured interactions: assortment example

	C	D
C	$R = 2, R = 2$	$S = 0, T = 4$
D	$T = 4, S = 0$	$P = 1, P = 1$

- $\pi(C, C) = \alpha R + p(1-\alpha)R > \pi(D, D) = (1-\alpha)pT$
- $1.2 + 0.8p > 1.6p$ at $\alpha = 0.6$
- $1.6 > 0.8$ at $\alpha = 0.6,\ p^* = 0.5$