Evolution of Cooperation (Section 4)

21 February 2017

Overview

- Explain extensive-form games or “game trees”.

Overview

- Explain extensive-form games or “game trees”.
- Illustration: withdrawal of later cooperation favors cooperation in repeated games.

Overview

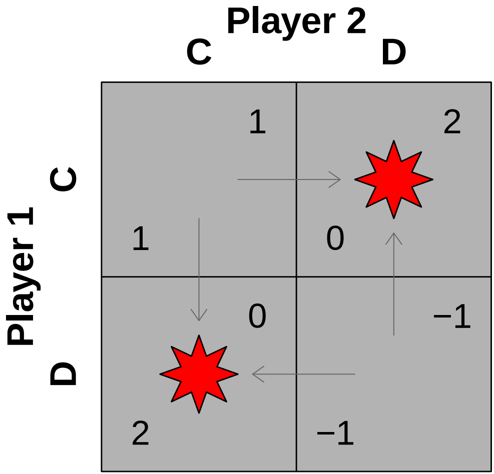
- Explain extensive-form games or “game trees”.
- Illustration: withdrawal of later cooperation favors cooperation in repeated games.
- Illustration: cooperative reputations are rewarded and favor cooperation when cost of information is low.

Overview

- Explain extensive-form games or “game trees”.
- Illustration: withdrawal of later cooperation favors cooperation in repeated games.
- Illustration: cooperative reputations are rewarded and favor cooperation when cost of information is low.
- Example: using costly signaling to communicate an unobserved trait.

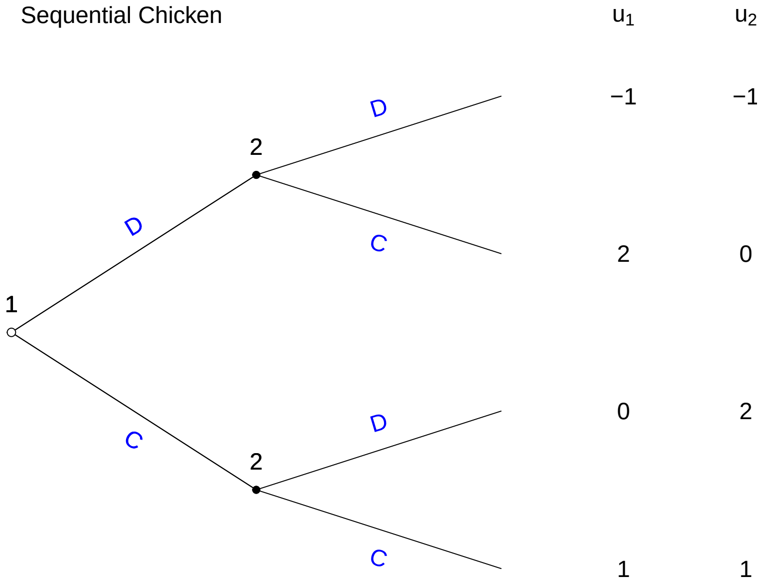
Extensive-Form Games

Chicken



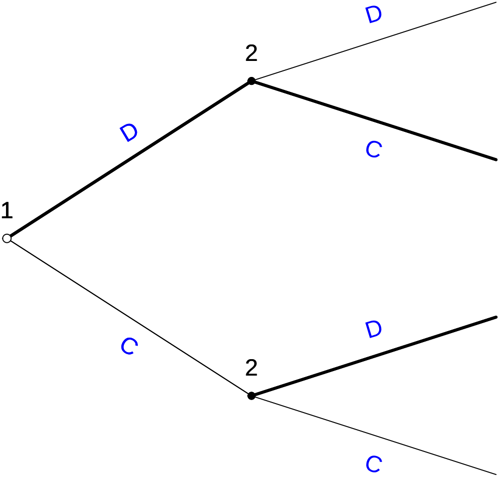
Extensive-Form Games

Sequential Chicken



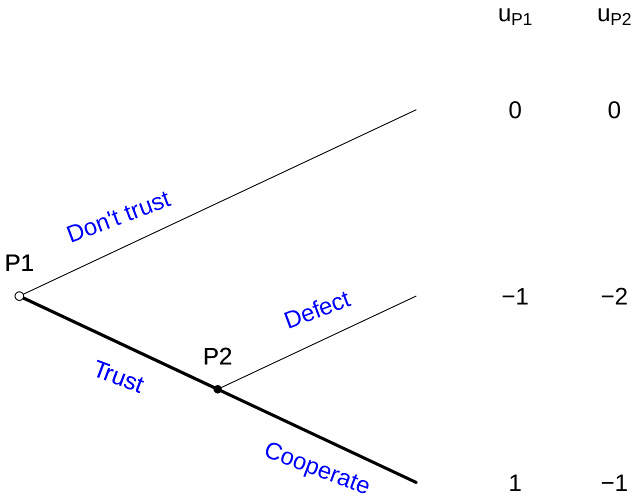
Extensive-Form Games

Sequential Chicken



u_1	u_2
−1	−1
2	0
0	2
1	1

Extensive-Form Games



Retaliation in repeated exchange game

	TFT	$ALLD$
TFT	R/ρ,R/ρ	$S + \frac{(1-\rho)P}{\rho}$,$T + \frac{(1-\rho)P}{\rho}$
$ALLD$	$T + \frac{(1-\rho)P}{\rho}$,$S + \frac{(1-\rho)P}{\rho}$	P/ρ,P/ρ

- Suppose ρ is **termination** probability (note inversion from w!).

Retaliation in repeated exchange game

	TFT	$ALLD$
TFT	$R/\rho, R/\rho$	$S + \frac{(1-\rho)P}{\rho}, T + \frac{(1-\rho)P}{\rho}$
$ALLD$	$T + \frac{(1-\rho)P}{\rho}, S + \frac{(1-\rho)P}{\rho}$	$P/\rho, P/\rho$

- Suppose ρ is **termination** probability (note inversion from w!).
- E.g. two TFT meet, cooperate on first round and continue for expected total duration of $1/\rho$ rounds.

Retaliation in repeated exchange game

	TFT	*ALLD*
TFT	$R/\rho, R/\rho$	$S + \frac{(1-\rho)P}{\rho}, T + \frac{(1-\rho)P}{\rho}$
ALLD	$T + \frac{(1-\rho)P}{\rho}, S + \frac{(1-\rho)P}{\rho}$	$P/\rho, P/\rho$

- Suppose ρ is **termination** probability (note inversion from w!).
- E.g. two TFT meet, cooperate on first round and continue for expected total duration of $1/\rho$ rounds.
- E.g. TFT meets ALLD, receives S in first round, then P until game terminates. So multiply by prob. of 2nd round, $(1-\rho)$, and expected number of rounds at beginning of any period, $1/\rho$.

Retaliation in repeated exchange game

	TFT	$ALLD$
TFT	$R/\rho, R/\rho$	$S + \frac{(1-\rho)P}{\rho}, T + \frac{(1-\rho)P}{\rho}$
$ALLD$	$T + \frac{(1-\rho)P}{\rho}, S + \frac{(1-\rho)P}{\rho}$	$P/\rho, P/\rho$

- Suppose τ is fraction playing TFT

Retaliation in repeated exchange game

	TFT	*ALLD*
TFT	$R/\rho, R/\rho$	$S + \frac{(1-\rho)P}{\rho}, T + \frac{(1-\rho)P}{\rho}$
ALLD	$T + \frac{(1-\rho)P}{\rho}, S + \frac{(1-\rho)P}{\rho}$	$P/\rho, P/\rho$

- Suppose τ is fraction playing TFT
- $\pi_T = \tau \frac{R}{\rho} + (1-\tau)\left[S + \frac{(1-\rho)P}{\rho}\right]$

Retaliation in repeated exchange game

	TFT	$ALLD$
TFT	R/ρ,R/ρ	$S+\frac{(1-\rho)P}{\rho}$,$T+\frac{(1-\rho)P}{\rho}$
$ALLD$	$T+\frac{(1-\rho)P}{\rho}$,$S+\frac{(1-\rho)P}{\rho}$	P/ρ,P/ρ

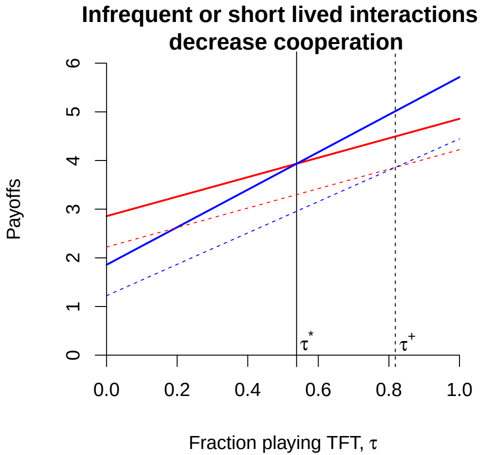
- Suppose τ is fraction playing TFT
- $\pi_T = \tau\frac{R}{\rho} + (1-\tau)\left[S + \frac{(1-\rho)P}{\rho}\right]$
- $\pi_D = \tau\left[T + \frac{(1-\rho)P}{\rho}\right] + (1-\tau)\frac{P}{\rho}$

Retaliation in repeated exchange game

	TFT	*ALLD*
TFT	$R/\rho, R/\rho$	$S + \frac{(1-\rho)P}{\rho}, T + \frac{(1-\rho)P}{\rho}$
ALLD	$T + \frac{(1-\rho)P}{\rho}, S + \frac{(1-\rho)P}{\rho}$	$P/\rho, P/\rho$

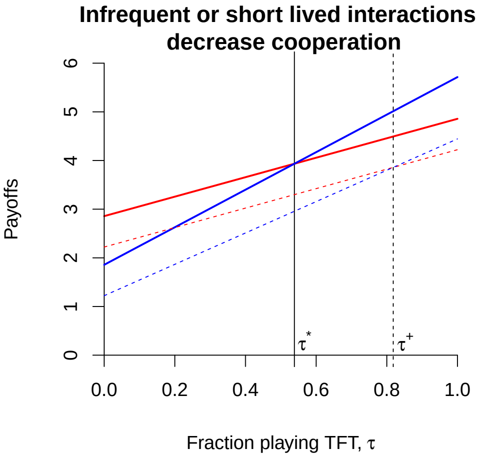
- Suppose τ is fraction playing TFT
- $\pi_T = \tau \frac{R}{\rho} + (1-\tau)\left[S + \frac{(1-\rho)P}{\rho}\right]$
- $\pi_D = \tau \left[T + \frac{(1-\rho)P}{\rho}\right] + (1-\tau)\frac{P}{\rho}$
- Unstable equilibrium at $\tau^* = \frac{P-S}{2P-T-S+(R-P)/\rho}$

Retaliation in repeated exchange game



- Increasing ρ reduces expected cost of future retaliation for a defector.

Retaliation in repeated exchange game



- Increasing ρ reduces expected cost of future retaliation for a defector.
- Diminishes basin of attraction of cooperative equlibrium ($\tau = 1$) by shifting unstable equilibrium to τ^+.

Reputation in exchange game w/ inspection

	Inspect	*Defect*
Inspect	$R-\delta, R-\delta$	$P-\delta, P$
Defect	$P, P-\delta$	P, P

- Inspectors: inspect and respond to a cooperative partner by cooperating and defector by defecting.

Reputation in exchange game w/ inspection

	Inspect	*Defect*
Inspect	$R-\delta, R-\delta$	$P-\delta, P$
Defect	$P, P-\delta$	P, P

- Inspectors: inspect and respond to a cooperative partner by cooperating and defector by defecting.
- Suppose $\delta > 0$ is "inspection cost" to examine partner's reputation. Fraction α are Inspectors.

Reputation in exchange game w/ inspection

	Inspect	*Defect*
Inspect	$R-\delta, R-\delta$	$P-\delta, P$
Defect	$P, P-\delta$	P, P

- Inspectors: inspect and respond to a cooperative partner by cooperating and defector by defecting.
- Suppose $\delta > 0$ is "inspection cost" to examine partner's reputation. Fraction α are Inspectors.
- $\pi_I = \alpha(R-\delta) + (1-\alpha)(P-\delta)$

Reputation in exchange game w/ inspection

	Inspect	*Defect*
Inspect	$R-\delta, R-\delta$	$P-\delta, P$
Defect	$P, P-\delta$	P, P

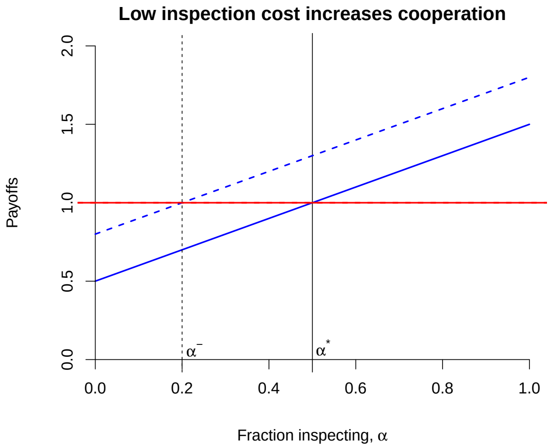
- Inspectors: inspect and respond to a cooperative partner by cooperating and defector by defecting.
- Suppose $\delta > 0$ is "inspection cost" to examine partner's reputation. Fraction α are Inspectors.
- $\pi_I = \alpha(R-\delta) + (1-\alpha)(P-\delta)$
- $\pi_D = P$

Reputation in exchange game w/ inspection

	Inspect	*Defect*
Inspect	$R-\delta, R-\delta$	$P-\delta, P$
Defect	$P, P-\delta$	P, P

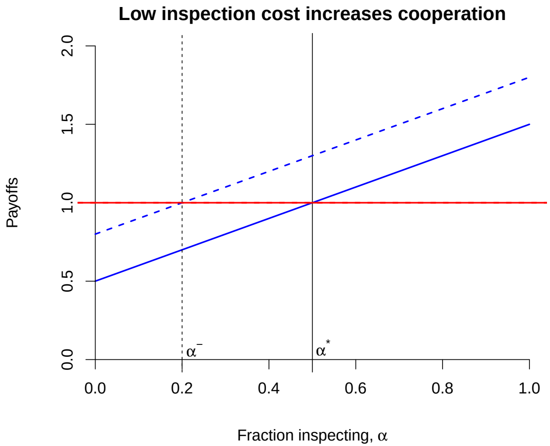
- Inspectors: inspect and respond to a cooperative partner by cooperating and defector by defecting.
- Suppose $\delta > 0$ is "inspection cost" to examine partner's reputation. Fraction α are Inspectors.
- $\pi_I = \alpha(R-\delta) + (1-\alpha)(P-\delta)$
- $\pi_D = P$
- Unstable equilibrium at $\alpha^* = \frac{\delta}{R-P}$

Reputation in exchange game w/ inspection



- Decreasing δ makes it less costly to know your partner's type.

Reputation in exchange game w/ inspection



- Decreasing δ makes it less costly to know your partner's type.
- Increases basin of attraction of cooperative equilibrium ($\alpha = 1$) by shifting unstable equilibrium to α^{-}.

Signaling

- There are two players – often called a “sender” and a “receiver”

Signaling

- There are two players – often called a “sender” and a “receiver”
- The sender can be one of several *types*.

Signaling

- There are two players – often called a "sender" and a "receiver"
- The sender can be one of several *types*.
- The sender's type is **private information**: it is known to her, but not the receiver.

Signaling

- There are two players – often called a “sender” and a “receiver”
- The sender can be one of several *types*.
- The sender’s type is **private information**: it is known to her, but not the receiver.
- The receiver has a single type, so her payoff function is common knowledge.

Signaling

- There are two players – often called a “sender” and a “receiver”
- The sender can be one of several *types*.
- The sender’s type is **private information**: it is known to her, but not the receiver.
- The receiver has a single type, so her payoff function is common knowledge.

1. **Pooling equilibria**: All the types of Player 1 choose the same action, thus revealing nothing to Player 2.

Signaling

- There are two players – often called a "sender" and a "receiver"
- The sender can be one of several *types*.
- The sender's type is **private information**: it is known to her, but not the receiver.
- The receiver has a single type, so her payoff function is common knowledge.

1. **Pooling equilibria**: All the types of Player 1 choose the same action, thus revealing nothing to Player 2.
2. **Separating equilibria**: Each type of Player 1 chooses a different action, thus revealing his type in equilibrium to Player 2.

Signaling

Recipe for signaling game:

1. Nature chooses a type for Player 1 that Player 2 does not know, but cares about.

Signaling

Recipe for signaling game:

1. Nature chooses a type for Player 1 that Player 2 does not know, but cares about.
2. Player 1 has atleast as many actions as there are types, and each action imposes a different cost on each type.

Signaling

Recipe for signaling game:

1. Nature chooses a type for Player 1 that Player 2 does not know, but cares about.
2. Player 1 has atleast as many actions as there are types, and each action imposes a different cost on each type.
3. Player 1 chooses an action first, and Player 2 then responds after observing Player 1's choice.

Signaling

Recipe for signaling game:

1. Nature chooses a type for Player 1 that Player 2 does not know, but cares about.
2. Player 1 has atleast as many actions as there are types, and each action imposes a different cost on each type.
3. Player 1 chooses an action first, and Player 2 then responds after observing Player 1's choice.
4. Given Player 2's belief about Player 1's strategy, Player 2 updates their belief after observing Player 1's choice. Player 2 then makes their choice as a best response to the updated beliefs.

Example: entry deterrence in elections

- In US Congressional (House and Senate) elections, incumbents tend to raise far more than necessary to finance campaigns.

Example: entry deterrence in elections

- In US Congressional (House and Senate) elections, incumbents tend to raise far more than necessary to finance campaigns.
- In 2016 Senate races, average incumbent raised $8.4m while average challenger raised only $1.8m.

Example: entry deterrence in elections

- In US Congressional (House and Senate) elections, incumbents tend to raise far more than necessary to finance campaigns.
- In 2016 Senate races, average incumbent raised $8.4m while average challenger raised only $1.8m.
- Incumbent re-election rates are usually well above 80%.

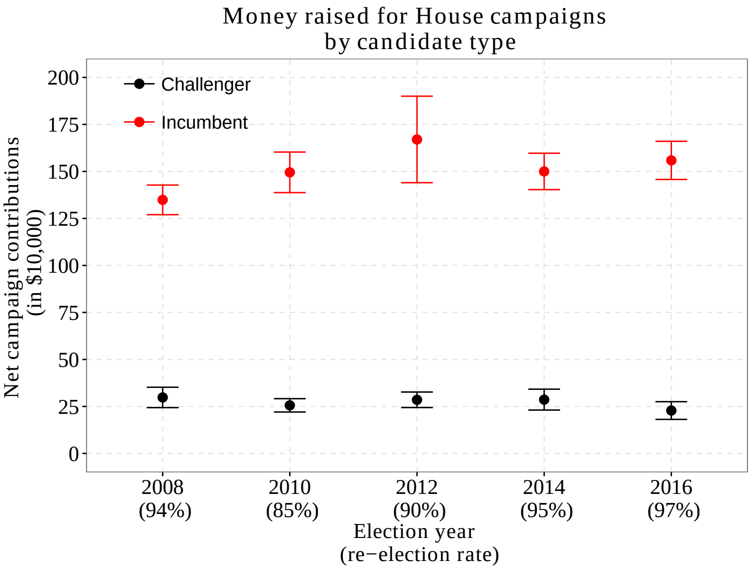
Example: entry deterrence in elections

- In US Congressional (House and Senate) elections, incumbents tend to raise far more than necessary to finance campaigns.
- In 2016 Senate races, average incumbent raised \$8.4m while average challenger raised only \$1.8m.
- Incumbent re-election rates are usually well above 80%.
- Q: Why do incumbent politicians work so hard to raise more campaign money than necessary to finance their campaigns?

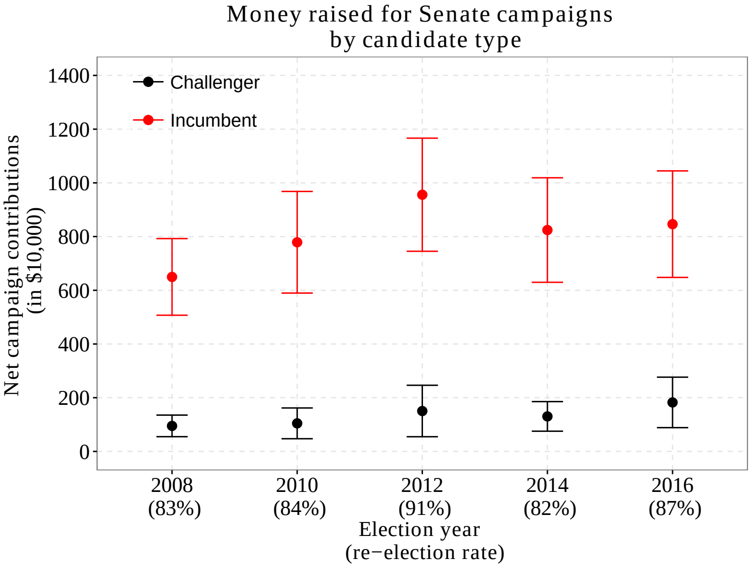
Example: entry deterrence in elections

- In US Congressional (House and Senate) elections, incumbents tend to raise far more than necessary to finance campaigns.
- In 2016 Senate races, average incumbent raised $8.4m while average challenger raised only $1.8m.
- Incumbent re-election rates are usually well above 80%.
- Q: Why do incumbent politicians work so hard to raise more campaign money than necessary to finance their campaigns?
- A: Fundraising is a costly signal of incumbent's strength.

Example: entry deterrence in elections



Example: entry deterrence in elections



Example: entry deterrence in elections

Setup for game:

- Suppose both the Challenger (C) and the Incumbent (I) receive 1 unit of utility from elected office.

Example: entry deterrence in elections

Setup for game:

- Suppose both the Challenger (C) and the Incumbent (I) receive 1 unit of utility from elected office.
- "Nature" decides whether incumbent is either Strong (S) or Weak (W).

Example: entry deterrence in elections

Setup for game:

- Suppose both the Challenger (C) and the Incumbent (I) receive 1 unit of utility from elected office.
- "Nature" decides whether incumbent is either Strong (S) or Weak (W).
- Probability C wins against W is π_w, against S is π_s.

Example: entry deterrence in elections

Setup for game:

- Suppose both the Challenger (C) and the Incumbent (I) receive 1 unit of utility from elected office.
- "Nature" decides whether incumbent is either Strong (S) or Weak (W).
- Probability C wins against W is π_w, against S is π_s.
- k is C's cost of running. Assume $\pi_w > k > \pi_s$.

Example: entry deterrence in elections

Setup for game:

- Suppose both the Challenger (C) and the Incumbent (I) receive 1 unit of utility from elected office.
- "Nature" decides whether incumbent is either Strong (S) or Weak (W).
- Probability C wins against W is π_w, against S is π_s.
- k is C's cost of running. Assume $\pi_w > k > \pi_s$.
- If $k > \pi_w$ then C never enters. If $k < \pi_s$ then C always enters.

Example: entry deterrence in elections

Setup for game:

- Suppose both the Challenger (C) and the Incumbent (I) receive 1 unit of utility from elected office.
- "Nature" decides whether incumbent is either Strong (S) or Weak (W).
- Probability C wins against W is π_w, against S is π_s.
- k is C's cost of running. Assume $\pi_w > k > \pi_s$.
- If $k > \pi_w$ then C never enters. If $k < \pi_s$ then C always enters.
- Types S and W incur different costs: $c_s < c_w$.

Example: entry deterrence in elections

Setup for game:

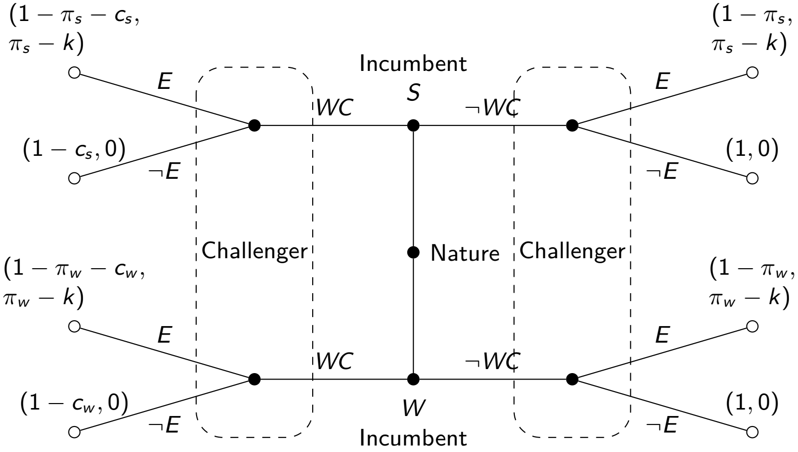
- Suppose both the Challenger (C) and the Incumbent (I) receive 1 unit of utility from elected office.
- "Nature" decides whether incumbent is either Strong (S) or Weak (W).
- Probability C wins against W is π_w, against S is π_s.
- k is C's cost of running. Assume $\pi_w > k > \pi_s$.
- If $k > \pi_w$ then C never enters. If $k < \pi_s$ then C always enters.
- Types S and W incur different costs: $c_s < c_w$.
- Incumbent decides to build a war chest WC or not $\neg WC$.

Example: entry deterrence in elections

Setup for game:

- Suppose both the Challenger (C) and the Incumbent (I) receive 1 unit of utility from elected office.
- "Nature" decides whether incumbent is either Strong (S) or Weak (W).
- Probability C wins against W is π_w, against S is π_s.
- k is C's cost of running. Assume $\pi_w > k > \pi_s$.
- If $k > \pi_w$ then C never enters. If $k < \pi_s$ then C always enters.
- Types S and W incur different costs: $c_s < c_w$.
- Incumbent decides to build a war chest WC or not $\neg WC$.
- After observing whether I builds a war chest, C decides whether to enter the race E or not $\neg E$.

Example: entry deterrence in elections



Example: entry deterrence in elections

Recall: $\pi_w > k > \pi_s$ and $c_s < c_w$

		Challenger	
		E	$\neg E$
Incumbent	$WC \mid Strong$	$1 - \pi_s - c_s, \pi_s - k$	$1 - c_s, 0$
	$WC \mid Weak$	$1 - \pi_w - c_w, \pi_w - k$	$1 - c_w, 0$
	$\neg WC \mid Strong$	$1 - \pi_s, \pi_s - k$	$1,0$
	$\neg WC \mid Weak$	$1 - \pi_w, \pi_w - k$	$1,0$

- Only strong incumbent builds War Chest:

$$U_I(WC|Strong, E) > U_I(WC|Weak, E)$$
$$1 - \pi_s - c_s > 1 - \pi_w - c_w$$
$$\pi_s + c_s < \pi_w + c_w$$

Example: entry deterrence in elections

Recall: $\pi_w > k > \pi_s$ and $c_s < c_w$

		Challenger	
		E	$\neg E$
Incumbent	$WC \mid Strong$	$1 - \pi_s - c_s, \pi_s - k$	$1 - c_s, 0$
	$WC \mid Weak$	$1 - \pi_w - c_w, \pi_w - k$	$1 - c_w, 0$
	$\neg WC \mid Strong$	$1 - \pi_s, \pi_s - k$	$1, 0$
	$\neg WC \mid Weak$	$1 - \pi_w, \pi_w - k$	$1, 0$

- Only strong incumbent builds War Chest:

$$U_I(WC \mid Strong, \neg E) > U_I(WC \mid Weak, \neg E)$$

$$1 - c_s < 1 - c_w$$

$$c_s < c_w$$

Example: entry deterrence in elections

Recall: $\pi_w > k > \pi_s$ and $c_s < c_w$

		Challenger	
		E	$\neg E$
Incumbent	$WC \mid Strong$	$1-\pi_s-c_s, \pi_s-k$	$1-c_s, 0$
	$\neg WC \mid Weak$	$1-\pi_w, \pi_w-k$	$1, 0$

- Challenger does not enter if Incumbent builds war chest:

$$U_C(WC|Strong, E) < U_C(WC|Strong, \neg E)$$
$$\pi_s - k < 0$$

Example: entry deterrence in elections

Recall: $\pi_w > k > \pi_s$ and $c_s < c_w$

		Challenger	
		E	$\neg E$
Incumbent	$WC \mid Strong$	$1 - \pi_s - c_s, \pi_s - k$	$1 - c_s, 0$
	$\neg WC \mid Weak$	$1 - \pi_w, \pi_w - k$	$1, 0$

- Challenger only enters if Incumbent does *not* build war chest:

$$U_C(\neg WC | Weak, E) > U_C(\neg WC | Weak, \neg E)$$
$$\pi_w - k > 0$$