

## Evolution of Cooperation (Section 2)

6 February 2017

# Review of Basic Games

- ▶ Two strategies and two players. The players **simultaneously** decide which strategy to select.
- ▶ The **best response** is the strategy which produces the most favorable outcome for a player, *taking the other player's strategy as given*.
- ▶ A set of (pure) strategies  $(A_1, A_2)$  is a **Nash Equilibrium** if player 1 is playing a best response against player 2, and player 2 is playing a best response against player 1!
- ▶  $U_1(A_1, A_2) > U_1(B_1, A_2)$  **and**  $U_2(A_1, A_2) > U_2(A_1, B_2)$
- ▶ In a NE, neither player has an incentive to switch strategies!

# Review of Basic Games

## Social Dilemma

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	2, 2	4, 1
	Defect	1, 4	3, 3

## Anti-coordination

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	3, 3	4, 2
	Defect	2, 4	1, 1

## Coordination

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	4, 4	2, 1
	Defect	1, 2	3, 3

# Review of Basic Games

Player 1's best responses to Player 2's strategies (vertical arrows):

## Social Dilemma

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	2, 2	1, 4
	Defect	4, 1	3, 3

Diagram illustrating the Social Dilemma game. The matrix shows payoffs for Player 1 (rows) and Player 2 (columns). Vertical arrows indicate Player 1's best responses: from (Cooperate, Cooperate) to (Defect, Cooperate) and from (Cooperate, Defect) to (Defect, Defect).

## Anti-coordination

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	3, 3	2, 4
	Defect	4, 2	1, 1

Diagram illustrating the Anti-coordination game. The matrix shows payoffs for Player 1 (rows) and Player 2 (columns). Vertical arrows indicate Player 1's best responses: from (Cooperate, Cooperate) to (Defect, Cooperate) and from (Cooperate, Defect) to (Defect, Defect).

## Coordination

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	4, 4	1, 2
	Defect	2, 1	3, 3

Diagram illustrating the Coordination game. The matrix shows payoffs for Player 1 (rows) and Player 2 (columns). Vertical arrows indicate Player 1's best responses: from (Cooperate, Cooperate) to (Cooperate, Cooperate) and from (Defect, Defect) to (Defect, Defect).

# Review of Basic Games

Player 2's best responses to Player 1's strategies (horizontal arrows):

## Social Dilemma

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	2, 2 →	1, 4 ↓
	Defect	4, 1 ↓	3, 3 →

## Anti-coordination

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	3, 3 →	2, 4 ↑
	Defect	4, 2 ↓	1, 1 ←

## Coordination

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	4, 4 ↑	1, 2 ↓
	Defect	2, 1 →	3, 3 ↓

# Review of Basic Games

NE: P1 and P2 play their best responses. We are stuck here!

## Social Dilemma

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	2, 2	1, 4
	Defect	4, 1	3, 3

A 2x2 payoff matrix for a Social Dilemma. The rows represent Player 1's strategies (Cooperate, Defect) and the columns represent Player 2's strategies (Cooperate, Defect). The payoffs are (Player 1, Player 2). In the (Cooperate, Defect) and (Defect, Defect) cells, there is a red star indicating a Nash Equilibrium. Arrows point from the (Cooperate, Cooperate) cell to the (Cooperate, Defect) cell and from the (Defect, Cooperate) cell to the (Defect, Defect) cell.

## Anti-coordination

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	3, 3	2, 4
	Defect	4, 2	1, 1

A 2x2 payoff matrix for an Anti-coordination game. The rows represent Player 1's strategies (Cooperate, Defect) and the columns represent Player 2's strategies (Cooperate, Defect). The payoffs are (Player 1, Player 2). In the (Cooperate, Defect) and (Defect, Cooperate) cells, there is a red star indicating a Nash Equilibrium. Arrows point from the (Cooperate, Cooperate) cell to the (Cooperate, Defect) cell and from the (Defect, Cooperate) cell to the (Defect, Defect) cell.

## Coordination

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	4, 4	1, 2
	Defect	2, 1	3, 3

A 2x2 payoff matrix for a Coordination game. The rows represent Player 1's strategies (Cooperate, Defect) and the columns represent Player 2's strategies (Cooperate, Defect). The payoffs are (Player 1, Player 2). In the (Cooperate, Cooperate) and (Defect, Defect) cells, there is a red star indicating a Nash Equilibrium. Arrows point from the (Cooperate, Defect) cell to the (Cooperate, Cooperate) cell and from the (Defect, Cooperate) cell to the (Defect, Defect) cell.

# Social Preferences

- ▶ “Homo-economicus”: players are “selfish” and predicted behavior simply a function of individual payoffs.
- ▶ But even in one-shot prisoner’s dilemmas, rate of cooperation is typically 40-60%.
- ▶ Why? If players have **social preferences**, behavior also depends on what happens to others and why these things happen.
- ▶ Can be “other-regarding”—my evaluation of some state depends on how you are doing.
- ▶ Can be “process-regarding”—my evaluation of some state depends on how it came about.
- ▶ Examples: altruism, fairness, reciprocity and inequity aversion.

# Social Preferences (Altruism)

## Social Dilemma

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	$b-c$ $b-c$	$b$ $-c$
	Defect	$-c$ $b$	$0$ $0$



## Social Preferences (Altruism)

Let  $b = 2$ ,  $c = 1$ ,  $\kappa \in (0, 1)$  “altruism” parameter.

	Selfish person	Altruistic person
$U(C, C)$	$b - c = 1$	$b - c + \kappa(b - c) = 1 + \kappa$
$U(D, C)$	$b = 2$	$b - \kappa c = 2 - \kappa$
$U(C, D)$	$-c = -1$	$-c + \kappa b = 2\kappa - 1$
$U(D, D)$	0	0

Cooperative NE?

$$1 + \kappa > 2 - \kappa \implies \kappa > 1/2$$

## Social Preferences (Altruism)

Let  $b = 2$ ,  $c = 1$ . Suppose  $\kappa = 3/4$  (high altruism):

	Selfish person	Altruistic person
$U(C, C)$	$b - c = 1$	$b - c + \kappa(b - c) = 1 + \kappa = 1.75$
$U(D, C)$	$b = 2$	$b - \kappa c = 2 - \kappa = 1.25$
$U(C, D)$	$-c = -1$	$-c + \kappa b = 2\kappa - 1 = 0.5$
$U(D, D)$	0	0

# Social Preferences (Altruism)

Let  $b = 2$ ,  $c = 1$ . Suppose  $\kappa = 3/4$  (high altruism):

**Social Dilemma?**

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	1, 1	2, -1
	Defect	2, -1	0, 0

**Social Dilemma?**

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	1.75, 1.75	1.25, -1
	Defect	1.25, -1	0, 0

# Social Preferences (Altruism)

Let  $b = 2$ ,  $c = 1$ . Suppose  $\kappa = 3/4$  (high altruism):

**Social Dilemma?**

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	1 ↓ 1	2 ↓ -1
	Defect	-1 ↓ 2	0 ↓ 0

**Social Dilemma?**

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	1.75 ↑ 1.75	1.25 ↓ -1
	Defect	-1 ↑ 1.25	0 ↓ 0

# Social Preferences (Altruism)

Let  $b = 2$ ,  $c = 1$ . Suppose  $\kappa = 3/4$  (high altruism):

**Social Dilemma?**

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	1 → 1 ↓ -1	2 → -1 ↓ 0
	Defect	2 → -1 ↓ 0	0 → -1 ↓ 0

**Social Dilemma?**

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	1.75 ← 1.75 ↑ -1	1.25 ← -1 ↓ 0
	Defect	1.25 → -1 ↓ 0	0 → -1 ↓ 0

# Social Preferences (Altruism)

Let  $b = 2$ ,  $c = 1$ . Suppose  $\kappa = 3/4$  (high altruism):

**Social dilemma :-**

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	1 → 1	2 → -1
	Defect	-1 ↓ 2	0 ↓ 0

Diagram illustrating a social dilemma. Player 1's strategies are Cooperate and Defect. Player 2's strategies are Cooperate and Defect. Payoffs are shown in the cells. A red star is placed in the (Defect, Defect) cell, indicating a high payoff for Player 2 (0) and a low payoff for Player 1 (0).

**Coordination game!**

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	1.75 ← 1.75	1.25 ← -1
	Defect	-1 ↑ 1.25	0 ↓ 0

Diagram illustrating a coordination game. Player 1's strategies are Cooperate and Defect. Player 2's strategies are Cooperate and Defect. Payoffs are shown in the cells. A red star is placed in the (Cooperate, Cooperate) cell, indicating a high payoff for both players (1.75).

## Social Preferences (Inequity aversion)

- ▶ P1's utility:  $U_1 = \pi_1 - \delta_1 \max(\pi_2 - \pi_1, 0) - \alpha_1 \max(\pi_1 - \pi_2, 0)$
- ▶  $\delta_1$ : how much P1 dislikes disadvantageous ( $\pi_2 - \pi_1 > 0$ ) differences.
- ▶  $\alpha_1 \in (0, 1)$ : how much P1 dislikes advantageous ( $\pi_1 - \pi_2 > 0$ ) differences.
- ▶ If  $\alpha_1 = 1$  then P1 cares only about P2's payoffs if they fall short of her own.
- ▶ If  $\delta_1 > 1$ , P1 is very adverse to disadvantageous differences.
- ▶ P2's utility:  $U_2 = \pi_2 - \delta_2 \max(\pi_1 - \pi_2, 0) - \alpha_2 \max(\pi_2 - \pi_1, 0)$
- ▶ When  $\delta_1 \neq \delta_2$  and/or  $\alpha_1 \neq \alpha_2$ , things get more complicated.

## Social Preferences (Inequity aversion)

Suppose  $\alpha, \delta$  are the same for both players. Again consider social dilemma setup with  $c = 1, b = 2$ .

	Selfish person	Inequity averse person
$U(C, C)$	$b - c = 1$	$b - c = 1$
$U(D, C)$	$b = 2$	$b - \alpha[b - (-c)] = 2 - 3\alpha$
$U(C, D)$	$-c = -1$	$-c - \delta[b - (-c)] = -1 - 3\delta$
$U(D, D)$	0	0

-Cooperative NE?

$$1 > 2 - 3\alpha \implies \alpha > 1/3$$



## Social Preferences (Inequity aversion)

Let  $b = 2$ ,  $c = 1$ . Suppose  $\alpha = 3/4$ ,  $\delta = 2$ :

	Selfish person	Inequity averse person
$U(C, C)$	$b - c = 1$	$b - c = 1$
$U(D, C)$	$b = 2$	$b - \alpha[b - (-c)] = 2 - 3\alpha = -0.25$
$U(C, D)$	$-c = -1$	$-c - \delta[b - (-c)] = -1 - 3\delta = -7$
$U(D, D)$	0	0

# Social Preferences (Inequity aversion)

Let  $b = 2$ ,  $c = 1$ . Suppose  $\alpha = 3/4$ ,  $\delta = 2$ :

**Social dilemma :-**

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	<div>1</div> <div>1</div> <div>→</div> <div>2</div>	<div>-1</div> <div>↓</div> <div>0</div>
	Defect	<div>-1</div> <div>↓</div> <div>2</div>	<div>0</div> <div>→</div> <div>0</div> <div></div>

**Coordination game!**

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	<div>1</div> <div>1</div> <div>←</div> <div>-0.25</div> <div></div>	<div>-0.25</div> <div>↓</div> <div>-7</div>
	Defect	<div>-7</div> <div>↑</div> <div>-0.25</div>	<div>0</div> <div>→</div> <div>0</div> <div></div>

# Evolutionary game theory

Key concepts:

- ▶ Nash Equilibrium (NE) is a **static concept**.
- ▶ When NE is not unique, dynamics matter!
- ▶ Evolutionary game theory lets *populations* evolve.
- ▶ Fraction  $x$  of the population plays “strategy 1” and fraction  $1 - x$  plays “strategy 2”.
- ▶ With multiple NE (or none!), evolutionary game theory tells us how the population evolves.
- ▶ An **Evolutionary Stable Strategy (ESS)** is a strategy which, if adopted by a population, cannot be invaded by a mutant strategy that is initially rare.
- ▶ Every NE is an ESS!
- ▶ Strategy A **risk dominates** strategy B if  $\pi_A > \pi_B$  at  $x = 1/2$ .

## Evolutionary game theory: “dominance”

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	4, 4	-2, 6
	Defect	6, -2	0, 0

The matrix shows the following payoffs:

- Player 1 Cooperate, Player 2 Cooperate: (4, 4)
- Player 1 Cooperate, Player 2 Defect: (-2, 6)
- Player 1 Defect, Player 2 Cooperate: (6, -2)
- Player 1 Defect, Player 2 Defect: (0, 0)

- ▶ Mutual defection is the NE. This is also the ESS.
- ▶  $\pi_C = 4x - 2(1 - x) = 6x - 2$ ,  $\pi_D = 6x + 0(1 - x) = 6x$
- ▶ At  $x = 1/2$ ,  $\pi_C = 1 < \pi_D = 3$  so  $D$  is **risk dominant**
- ▶ Population will evolve to 100% defectors.

## Evolutionary game theory: “co-existence”

		Player 2	
		Dove	Hawk
Player 1	Dove	2, 2	1, 3
	Hawk	3, 1	0, 0

The matrix shows the following payoffs:

- (Dove, Dove): 2, 2
- (Dove, Hawk): 1, 3
- (Hawk, Dove): 3, 1
- (Hawk, Hawk): 0, 0

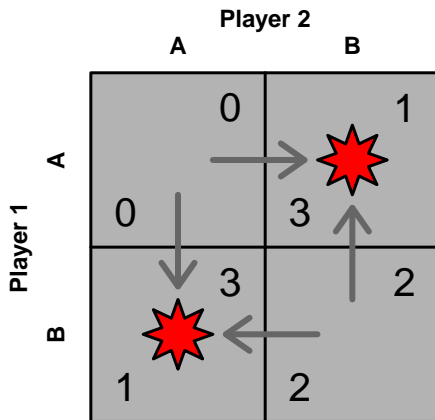
- ▶ DH and HD are both equilibria.
- ▶  $\pi_D = 2x + (1 - x) = x + 1$ ,  $\pi_H = 3x + 0(1 - x) = 3x$
- ▶ At  $x = 1/2$ ,  $\pi_D = 3/2 = \pi_H = 3/2$  so neither **risk dominant**!
- ▶ Population will evolve to stable mix of both types.

## Evolutionary game theory: “bistability”

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	1.75, 1.75	1.25, -1
	Defect	1.25, -1	0, 0

- ▶ CC and DD are both equilibria, but mixture of types unstable!
- ▶  $\pi_C = 1.75x - (1-x) = 2.75x - 1$ ,  $\pi_D = 1.25x + 0(1-x) = 1.25x$
- ▶ At  $x = 1/2$ ,  $\pi_C = 1.375 > \pi_D = 0.625$  so C is **risk dominant**.
- ▶ Population will *most likely* evolve to 100% Cooperators.

## Evolutionary Dynamics (worked example)



- ▶ AB and BA are both equilibria.
- ▶  $\pi_A = 0 \cdot x + 3(1 - x) = 3 - 3x$ ,  $\pi_B = 1x + 2(1 - x) = 2 - x$
- ▶ At  $x = 1/2$ ,  $\pi_A = 3/2 = \pi_B = 3/2$  so neither **risk dominant**.

## Evolutionary Dynamics (worked example)

- ▶ Suppose we start in period  $t$  with  $x_t = 0.1$  percent playing A and  $(1 - x_t) = 0.9$  percent playing B in population.
- ▶ If  $\pi_A > \pi_B$  then  $x_{t+1}$  increases by 0.1.
- ▶ If  $\pi_A < \pi_B$  then  $x_{t+1}$  decreases by 0.1.
- ▶ If  $\pi_A = \pi_B$  then population is at equilibrium.
- ▶ If  $x_{t^*} = 0$  or  $x_{t^*} = 1$  then we are also at equilibrium.

$x_t$	$\pi_A$	$\pi_B$	$x_{t+1}$
0.10	2.70	1.90	0.20
0.20	2.40	1.80	0.30
0.30	2.10	1.70	0.40
0.40	1.80	1.60	0.50
0.50	1.50	1.50	0.50
0.50	1.50	1.50	0.50



## Evolutionary Dynamics (worked example)

- ▶ What if we start at  $x_t = 0.9$  instead?

$x_t$	$\pi_A$	$\pi_B$	$x_{t+1}$
0.90	0.30	1.10	0.80
0.80	0.60	1.20	0.70
0.70	0.90	1.30	0.60
0.60	1.20	1.40	0.50
0.50	1.50	1.50	0.50
0.50	1.50	1.50	0.50

- ▶ Regardless of initial conditions, the population will end up with 50/50 mix.

## Evolutionary Dynamics (worked example)

