Null-First Discrete Angular Kinematics

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Abstract

"We are very much aware that we are exploring unconventional ideas and that there may be some basic flaw in our whole approach which we have been too stupid to see."

Sidney Coleman and Erick Weinberg (1973)

Newton's gravitational constant encodes holography directly: $G = \ell_P^2/\hbar = c^3/\hbar$ states that $\hbar$ of action creates both one Planck area of boundary *and* the corresponding bulk volume at rate c^3. These being equal means bulk spacetime carries no independent degrees of freedom.

Taking null geometry as fundamental, we show that photons are not propagating particles but vertices where null relations begin and end. Spacetime emerges as the reconstructed geometry of these connections. The primitive invariant is action, with $E = S/T$ defined only once a timescale becomes available.

This dissolves the cosmological constant problem. Quantum field theory's estimate $\rho_{\text{QFT}} \sim \hbar c/\ell_P^4$ counts bulk modes, violating holography by $(R_\Lambda/\ell_P)^2 \approx 10^{122}$—the number of horizon pixels. Counting holographically: $N_{\text{pix}} = 4\pi R_\Lambda^2/\ell_P^2$ pixels each carrying $\hbar/4$ (from black hole entropy), rendered over thermal time $\beta_\Lambda = 2\pi/H_\Lambda$, yields

$$\rho_\Lambda = \frac{3H_\Lambda^2 c^2}{8\pi G} \tag{1}$$

exactly, with no free parameters. Dark energy is not stored in spacetime. It is the cost of projecting spacetime from its null substrate. The 10^{122} discrepancy was a counting error.

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Part I

Conceptual Foundations

1 Core Thesis and Scope

This paper advances a single kinematical claim: null structure is fundamental, while spacetime geometry is emergent.

The results developed here focus on kinematics rather than dynamics. We do not derive the Einstein field equations or propose a new law of motion. However, because null geometry is inherently kinematic, we can do more with this geometry than would naturally be expected using only kinematics. Instead, we show that when null geometry is treated exactly as special relativity says, with a length-less and timeless existence, several central features of gravitational physics follow from geometric consistency alone. These include the inverse square scaling of forces, the area law for horizon entropy, and the observed magnitude of the cosmological constant.

Long standing puzzles in gravity and cosmology suggest that spatial volume and energy density may not be the correct primitive quantities in regimes dominated by null structure. Examples include Newtonian gravity [1, 2], black hole entropy [3, 4], and the cosmological constant problem [5, 6]. In each case, the relevant physics is controlled by null boundaries rather than bulk volumes.

In null geometry, intrinsic temporal and radial separation collapse. Proper time vanishes along null relations, and no intrinsic length can be assigned in the direction of propagation [7, 8]. What remains well defined is angular and projective structure. The space of physically distinct null directions at a point forms a celestial sphere, and this angular data survives exactly where spacetime intervals degenerate [9, 10, 11].

The central organizing principle of this paper is that null geometry carries angular but not volumetric information. As a result, area rather than volume provides the natural measure, and action rather than energy provides the natural invariant. Energy emerges only after a temporal scale becomes available through timelike reconstruction.

This perspective is consistent with earlier ideas about emergent spacetime and holography [12, 13, 14], but differs in emphasis. Here, emergence is grounded directly in the geometry of null relations rather than in thermodynamic or entropic postulates.

Unlike conventional frameworks where the metric defines causal structure, we reverse this dependency: the causal structure determines the emergent metric structure. As such, traditional tools like Penrose diagrams or line elements play no foundational role here.

2 Why This Matters

Before developing the technical framework, it is worth stating clearly what changes when null geometry is taken as fundamental.

2.1 Photons Don't Move—We Are Passed Between Them

In the conventional picture, photons are particles emitted at one event, traveling through spacetime, and absorbed at another. This description requires an already-existing spacetime manifold through which the photon propagates. But this is backwards.

Photons do not move through space. They *are* space—or more precisely, they are the null relations that constitute spacetime's geometric substrate. When a charged particle at event A interacts electromagnetically with a particle at event B, the photon is not an object traveling from A to B. It is the null connection linking A and B directly, with zero proper time and undefined spatial separation along the connection itself.

What we perceive as motion is the timelike worldline of a massive particle being handed from one null vertex to the next. Between vertices, the particle exists as an edge in the causal network—a reconstructed timelike interval built from the angular matching of null directions. The sensation of continuous motion through space is the macroscopic limit of discrete vertex-to-vertex transitions, just as the sensation of smooth geometry is the macroscopic limit of discrete area quanta.

This is not a metaphor. It is a strict consequence of taking null geometry seriously. Along a null relation, $d\tau = 0$ identically, and intrinsic distance in the propagation direction is undefined [8]. The null relation has no internal structure—no ticking clock, no meter stick, no evolving degrees of freedom. It is a pure geometric connection, instantaneous in its own context, linking events that would otherwise be causally disconnected.

Timelike observers reconstructing this geometry assign coordinates, measure wavelengths, and calculate frequencies. But these are descriptions *of* the reconstruction, not properties *of* the null relation itself. The photon, intrinsically, is timeless and extensionless. Spacetime emerges when collections of such relations close consistently into volumetric elements.

2.2 Energy is Not Stored—It is the Cost of Projection

In conventional quantum field theory, energy density is treated as a property of spacetime itself. The vacuum is assigned an energy per unit volume, fields carry energy, and particles represent localized energy excitations. The cosmological constant problem arises because summing these contributions naively gives $\rho \sim \hbar c/\ell_P^4$, wildly exceeding observation.

But if spacetime is emergent rather than fundamental, this entire framing is wrong. Energy cannot be stored in spacetime because spacetime is what energy *creates*. More precisely: $\hbar$ is the action cost required to project one Planck-scale pixel from the null substrate into reconstructed timelike geometry.

Consider the identity $G = \hbar/c^3$ in natural units where $\ell_P = 1$. Rearranging:

$$\hbar = c^3 G. \tag{2}$$

The right side has dimensions of volume per time (the rate at which three null directions build volume) times the gravitational coupling. The meaning: $\hbar$ is the action quantum associated with generating one unit of spacetime volume per unit time. It is the projection cost.

Every Planck volume in reconstructed spacetime requires $\hbar$ of action to exist. This is not energy sitting *in* the volume—it is the ongoing energetic cost of *maintaining* the volume. When we measure dark energy density ρ_Λ, we are not detecting energy stored in empty space. We are measuring the projection cost per unit volume, rendered into energy units using the natural timescale of the cosmological horizon.

This reinterpretation has immediate consequences:

- **No vacuum energy problem:** QFT calculates the energy of fields in an already-projected spacetime. But projection cost is not the same as field energy. The two are categorically distinct.
- **Holographic scaling is mandatory:** If bulk volume is projected from boundary data, then the action cost must scale with boundary area, not bulk volume. This is not an added principle—it follows from the projection architecture.
- **The cosmological constant is kinematical:** ρ_Λ is not a dynamical parameter to be explained by field theory. It is the conversion factor between action (intrinsic to null boundaries) and energy density (emergent in timelike bulk), fixed by horizon geometry and thermal timescales.

2.3 G is Not a Coupling—It is an Equivalence Statement

Newton's constant is usually introduced as the strength of gravitational interaction. But the form $G = \ell_P^2 c^3/\hbar$ suggests a different interpretation.

Write it two ways:

$$G = \frac{\ell_P^2}{\hbar} \quad \longrightarrow \quad \text{“}\hbar\text{ of action per Planck area of boundary”} \tag{3}$$

$$G = \frac{c^3}{\hbar} \quad \longrightarrow \quad \text{“}\hbar\text{ of action per }c^3\text{ volumetric projection rate”} \tag{4}$$

These being equal is the statement:

$$\boxed{\text{Bulk volume and boundary area have identical action budgets.}} \tag{5}$$

This is holography, encoded in the fundamental constants since Newton. We have been staring at the proof that spacetime is boundary-determined for over three centuries.

The factor c^3 has a clean geometric meaning. Volume is built from three independent null directions. If each advances at c, then $dV/dt \sim c^3$. The identity $G = c^3/\hbar$ means the action required to sustain this volumetric growth rate equals the action to populate the enclosing boundary. There are no independent bulk modes.

2.4 Scope and Falsifiability

This paper is deliberately limited to kinematics. We do not derive Einstein's equations, specify a Lagrangian, or propose modified dynamics. The claim is narrower and more specific: *if* null structure is fundamental and spacetime is emergent, *then* several major scaling relations in gravitational physics follow from geometric consistency alone, including the normalization of the cosmological constant.

The framework is falsifiable. It predicts:

- Dark energy equation of state is exactly $w = -1$ (pure cosmological constant, no evolving scalar field)
- Horizon entropy universally obeys $S = A/(4\ell_P^2)$ with no corrections at leading order
- Holographic scaling $N_{\text{dof}} \propto A/\ell_P^2$ applies to all causal diamonds
- Deviations from $\rho_\Lambda = 3H_\Lambda^2/(8\pi G)$ would falsify the projection-cost interpretation

If future observations establish $w \neq -1$ or find violations of area scaling for entropy, the framework fails. Until then, it offers a resolution to the cosmological constant problem that requires no fine-tuning, no new fields, and no modification of known physics—only a reordering of what is taken as ontologically prior.

3 The Null-First Ontology

A common objection to null-centered formulations is that one cannot Lorentz boost to a photon's rest frame. Lorentz transformations become singular as $v \to c$, and no inertial frame exists in which a photon is at rest. Questions about "what a photon experiences" are therefore dismissed as ill-posed or meaningless.

We take the opposite view. The singularity of the Lorentz boost at $v = c$ is not a barrier to understanding massless particles. It is a complete description of what they are.

Consider what special relativity actually predicts at $v = c$. Time dilation becomes infinite: $\gamma = (1 - v^2/c^2)^{-1/2} \to \infty$. No proper time elapses along a null worldline, $d\tau = 0$. Length contraction is total: spatial extension vanishes in the direction of motion. The entire trajectory from emission to absorption, which may span billions of light-years from a timelike perspective, collapses to zero interval from the null perspective.

This is not an approximation or a limiting case. This is what massless particles *are*. They exist in a regime where the fundamental machinery of timelike physics does not operate. There are no clocks to tick, no rulers to measure with, no temporal parameter to define evolution. From the perspective of the null trajectory itself, if we may use such language, existence is eternal and static. Infinite time dilation means the "lifetime" of a photon is infinite relative to its own null parameter. Total length contraction means it occupies no extended region. There is no "before" or "after" along a null worldline because there is no timelike separation.

But this regime is not structureless. While all metric distances collapse, angular relationships do not. The null limit preserves directions, polar coordinates, and spherical angular data [15]. A photon carries no information about radial scale, but it carries complete information about orientation: the angles (θ, ϕ, ψ) specifying direction and polarization, helicity h, frequency ω (which appears as an angular phase rate), and time itself reinterpreted as an angle conjugate to energy.

Everything except radius survives. This is precisely the data massless particles carry. Photons have polarization states and helicity but no rest mass. Gluons carry color charge and spin but no proper time. Gravitons encode spin-2 angular structure but propagate on null geodesics. All of these are angular properties. The observational fact that massless particles carry exactly the information that survives the null limit is not a coincidence. It is direct evidence that the null limit describes their actual ontology.

Nothing can boost into the null regime because the null regime is the projector that renders the boosts and just like a projector, a projection cannot become the projector. The boosts presuppose the existence of clocks, rulers, and temporal evolution. These do not exist at $v = c$. The mathematical singularity marks a genuine physical boundary between categorically distinct regimes [16]. The null surface is not "almost reachable" by increasing velocity. It is fundamentally different in kind.

This difference has precise mathematical structure. Timelike physics is governed by the Poincaré group, which includes translations, rotations, and boosts. Null physics is governed by the conformal boundary structure, which includes BMS transformations and asymptotic symmetries [17]. These are not the same symmetry group. The singularity of the boost is the boundary between these regimes.

Consider the implications. If massless particles inhabit a regime with infinite time dilation and zero proper time, they are, in a precise sense, eternal. They do not evolve. They do not change. They simply are. This is the defining characteristic of a vertex in a causal graph: a point-like event with no internal duration. Massive particles, by contrast, trace out worldlines with nonzero proper time. They evolve, they age, they have internal dynamics. These are edges connecting vertices.

This suggests a reversal of the usual picture. Massless particles are not objects moving through spacetime. They are causal vertices, the timeless events that define spacetime's structure. Massive particles are timelike edges connecting these vertices. When a photon is "emitted" or "absorbed," no entity enters or exits the null regime. A timelike worldline simply intersects a null relation at a vertex. What propagates through spacetime is the massive system, tracing an edge from one vertex to the next.

The null relations themselves constitute the causal structure. This is not circular. We are not deriving "null" from a pre-existing metric. We are stating that causal connectedness is primitive, and null relations are the mathematical expression of this connectedness. The light cone structure, which defines what can influence what, is built from null surfaces. Timelike and spacelike separations are derived concepts, defined relative to this null structure.

This applies to any particle propagating at c because it is massless. The ontology does not depend on spin, charge, or gauge group. It depends only on the kinematic fact $v = c$, which is equivalent to $m = 0$. Photons, gluons, and gravitons are all null relations. They all correspond to causal vertices. The fact that gravitons *are* metric perturbations, the dynamical degrees of freedom of spacetime itself, strengthens this picture. If gravitons are null relations and gravitons constitute spacetime dynamics, then null structure is not embedded in spacetime. It *is* spacetime at the fundamental level.

The standard picture, in which massless particles propagate between emission and absorption, is not wrong. It is a timelike reconstruction. From the perspective of a massive observer, photons appear to travel and we track their progress with an affine parameter. But this appearance is produced by projecting the vertex structure onto a timelike worldline. What we observe as a photon "traveling" is the interval between two vertices along our worldline. The photon itself, the null relation, experiences no travel because it experiences no time. A key distinction we make is, the affine parameter is not a property of the photon, but a property of timelike observers.

This viewpoint aligns with Carrollian limits of relativity and null boundary formulations [16, 15, 9]. It treats null geometry as a regime with its own intrinsic structure rather than as a degenerate limit of timelike physics.

4 The Geometric Identity of Gravity

The null-first ontology recasts Newton's constant G not as a dynamical coupling between energy and curvature, but as a *geometric identity* relating surface capacity to volumetric null expansion. In this framework, G encodes how angular (boundary) degrees of freedom regulate the growth of spacetime volume reconstructed from null structure.

Consider the Planck relation

$$G = \frac{\ell_P^2 c^3}{\hbar}, \tag{6}$$

where ℓ_P is the Planck length. This expression is not merely dimensional. Each factor has a precise geometric meaning in vertex-frame kinematics.

The quantity ℓ_P^2 represents a *minimal surface capacity*: the smallest area element capable of supporting an independent null vertex-to-vertex constraint. It is not a volumetric voxel but an elementary *face* through which null entropy must pass.

The factor c^3 represents the characteristic rate at which null generators populate configuration space. In flat spacetime, the number of admissible null configurations grows as

$$N(\lambda) \sim \lambda^3, \tag{7}$$

reflecting the cubic growth of the null cone. The appearance of c^3 thus encodes the *volumetric expansion rate of null directions*, not a speed through space.

Finally, $\hbar$ is the action required to complete a full vertex-to-vertex cycle. As shown in Section 14, this cycle resolves four independent entropy degrees of freedom, each contributing $\hbar/4$.

Equation (6) therefore expresses the identity

$$G \sim \frac{\text{surface capacity}}{\text{action per null cycle}} \times (\text{null expansion rate}), \tag{8}$$

linking area, action, and volumetric growth in a single geometric relation.

This identity formalizes the holographic principle locally. Surface capacity scales as r^2, while null-generated volume scales as r^3. Newton's constant fixes the conversion between these scalings, ensuring that volumetric reconstruction is regulated by boundary data.

From this perspective, gravity is not a force but a bookkeeping constraint: curvature measures deviations from the flat-space $N \sim \lambda^3$ relation. Regions with fewer admissible future vertex connections than flat null kinematics predict are interpreted as positively curved; regions with more are negatively curved.

Standard quantum field theory counts degrees of freedom volumetrically, leading to the well-known overcounting that manifests as the cosmological constant problem. The null-first construction counts degrees of freedom by surface capacity, avoiding this mismatch. This distinction will be developed further in Part III.

5 Action Primacy, Energy Reconstruction, and the Null Sector

Energy is defined only in the presence of time. In the null sector, where $d\tau = 0$ identically, energy cannot be a primitive quantity. Action, by contrast, remains well defined. It appears in phase relations, boundary terms, and variational principles independently of any clock.

Principle 1 (Action primacy)**.** *In null kinematics, total action S is the fundamental invariant. Energy emerges only after reconstruction into a timelike (edge-frame) description, where a temporal scale T becomes available:*

$$E = \frac{S}{T}.$$

This is a change of representation, not a dynamical assumption.

This distinction is essential. Energy is not stored in space. All forms of energy—kinetic, potential, rest mass, and vacuum energy—represent the cost of *rendering null structure into spacetime*. Energy measures how rapidly action is converted into timelike observables.

5.1 Temporal reconstruction

For a bounded causal region, a time parameter arises only after reconstruction into an edge frame. In asymptotically de Sitter spacetimes, two natural scales appear.

The geometric scale H_Λ^{-1} is associated with the horizon radius. However, when converting action into a stationary energy, the relevant scale is the thermal period

$$\beta_\Lambda = \frac{2\pi}{H_\Lambda}, \tag{9}$$

required for Euclidean regularity of the static patch [18]. This period fixes the correct normalization between action stored on the horizon and the energy measured by timelike observers.

5.2 Energy as projection rate

Let N denote the entropy count: the number of resolved null constraints. The null-sector action is

$$S = N\hbar. \tag{10}$$

After reconstruction into a timelike frame with coordinate time t, energy is defined as the rate of action resolution:

$$E := \frac{dS}{dt} = \hbar\frac{dN}{dt}. \tag{11}$$

Definition 1 (Resolution rate)**.** *The resolution rate is*

$$\Gamma := \frac{dN}{dt}. \tag{12}$$

Thus

$$E = \hbar\Gamma. \tag{13}$$

Energy measures how quickly null constraints are rendered into timelike physics. There is no intrinsic energy flow in the null sector; all energetic notions arise from differences in resolution rate between vertices after reconstruction.

5.3 Newton's constant and surface action

Newton's constant fixes the conversion between action and surface capacity:

$$\ell_P^2 = \frac{\hbar G}{c^3}. \tag{14}$$

In the vertex-frame picture, ℓ_P^2 is interpreted as the minimal area element capable of supporting one independent surface constraint.

Lemma 1 (Action per entropy unit)**.** *Each independent entropy resolution contributes*

$$\Delta S = \frac{\hbar}{4}. \tag{15}$$

A complete vertex-to-vertex cycle resolves four such units, yielding total action $\hbar$. The quarter-action scale $\hbar/4$ is therefore the fundamental granularity of null-phase space. The Heisenberg bound $\hbar/2$ arises when two conjugate resolutions are compared; the full Planck constant $\hbar$ arises only after a complete cycle is specified.

5.4 The null sector and entropy

In the vertex frame, no intrinsic time parameter exists. Energy, frequency, and dynamics are therefore undefined.

What exists is constraint count.

Definition 2 (Entropy count)**.** *Let N denote the number of resolved null constraints at a vertex. The total null-sector action is*

$$S = N\hbar. \tag{16}$$

Only entropy and action bookkeeping exist in the null sector. All energetic and dynamical quantities arise only after reconstruction into timelike frames.

6 The Vertex Frame: A Null Thought Experiment

6.1 Two Classes of Frames

There exist two distinct and non-interchangeable notions of reference frame.

Definition 3 (Vertex–vertex frame)**.** *A vertex frame is defined by comparisons between null vertices. It encodes angular relations on the celestial sphere and internal phase labels. It admits no metric, no proper time, no spatial translations, and no notion of velocity.*

Definition 4 (Face–face (inertial) frame)**.** *A face frame is defined by comparisons between timelike faces. It supports metric structure, proper time, spatial displacement, momentum, and dynamical evolution.*

Lorentz transformations act only between face–face frames. No Lorentz transformation maps a face frame to a vertex frame.

6.2 The Thought Experiment

The standard statement that one cannot boost to a photon rest frame is correct, but it addresses a different question than the one relevant at null structure. It assumes *a priori* that all frames must be related by dynamical boosts acting on timelike worldlines.

We instead define the null frame by a thought experiment that involves no boosting: *assume one simply is a photon*. We ask what relational data exists in that situation.

As the velocity of a timelike observer approaches c, the Lorentz factor diverges,

$$\gamma(v) = \frac{1}{\sqrt{1 - v^2/c^2}} \xrightarrow{v \to c} \infty, \tag{17}$$

and the proper time increment vanishes,

$$d\tau = \frac{dt}{\gamma(v)} \xrightarrow{v \to c} 0. \tag{18}$$

These divergences arise because the boost attempts to compare *faces* whose separation is measured by a timelike metric. The divergence signals that the metric structure used to define the comparison is becoming degenerate.

In the null limit, this degeneration is exact. The induced metric along a null generator satisfies

$$ds^2 = 0, \tag{19}$$

so neither temporal nor radial separation is defined. All events along the null trajectory are coincident in the only admissible sense: there exists no invariant quantity by which they can be separated.

However, angular relations between vertices remain well-defined, as do internal labels such as helicity and phase. These quantities live entirely on the null vertex structure and require no propagation, signaling, or temporal evolution to be compared.

Because vertex–vertex comparisons do not invoke a metric, they do not involve Lorentz factors, proper time, or spatial displacement. The quantities that diverge in face–face comparisons simply do not appear. The divergence is not resolved; it is rendered irrelevant by a change in the class of comparison.

This has a direct physical consequence: there is no space in which motion could be defined. Momentum proportional to c is not a property of particles; it is a property of wave propagation in reconstructed spacetime. Waves are the mechanism by which spacetime relations are established between vertices. Particles themselves correspond to the vertices—static resolution points that define the geometric structure from which spacetime is later rendered.

The standard statement that a photon has no rest frame refers to the impossibility of defining a rest frame for a null *wave* under Lorentz boosts. The photon, understood as a vertex rather than a propagating face, does not lack a rest frame; rather, it admits a *perfect* one: a frame in which no time elapses, no distance is traversed, and no dynamical comparison is required.

Boosting is a dynamical operation acting on time-evolving face frames. Since the vertex frame contains no time parameter, it cannot be reached by any boost. This does not invalidate the vertex frame; it confirms that static relational structure is ontologically prior to dynamical spacetime description, just as static variables exist prior to program execution.

7 Superposition as Causal Ignorance and the Role of Virtual Quanta

In this framework, quantum superposition is not a statement about the ontology of the null substrate. It is a statement about epistemic incompleteness during the rendering of a causal graph when spacelike information is unavailable. At any given stage of rendering, the past causal structure is fixed, but the future causal vertex has not yet been determined. To preserve global consistency, the rendering algorithm must retain all causally admissible candidate vertices until sufficient information exists to commit one of them.

7.1 Candidate vertices and causal incompleteness

Let $\mathcal{G}_{\text{past}}$ denote the realized causal graph consisting of committed vertices. At a rendering frontier Σ, the next vertex is not uniquely determined because spacelike-separated constraints have not yet been resolved. Instead, there exists a set of candidate vertices,

$$\mathcal{V}_{\text{cand}}(\Sigma) = \{v_i \mid v_i \text{ is locally compatible with } \mathcal{G}_{\text{past}}\}. \tag{20}$$

Each v_i represents a possible null-consistent continuation of the causal graph. These vertices are not yet part of the realized spacetime. They are elements of the unresolved future. Discarding any v_i prematurely risks constructing a globally inconsistent spacetime.

Quantum superposition is the representation of this set $\mathcal{V}_{\text{cand}}$ together with weights encoding partial information. It reflects ignorance of which vertex will ultimately be committed, not indeterminacy of the null substrate itself.

7.2 Virtual quanta as unresolved causal links

In standard quantum field theory, interactions are represented by internal lines in Feynman diagrams. These internal lines correspond to virtual quanta and are integrated over all four-momenta. In the present framework, such objects admit a precise causal interpretation.

Virtual quanta correspond to candidate causal links connecting unresolved future vertices. They are not realized vertices in the causal past, but neither are they unphysical. They encode hypothetical causal connections whose consistency has not yet been decided. Their apparent off-shell character reflects the fact that they have not yet been constrained to lie on null generators of the realized graph.

Formally, off-shell propagation indicates that the vertex compatibility conditions have not yet been enforced globally.

7.3 The path integral as a causal search algorithm

The Feynman path integral provides a natural mathematical representation of this causal search. The transition amplitude between boundary configurations ϕ_i and ϕ_f is given by

$$\langle \phi_f | \phi_i \rangle = \int \mathcal{D}\phi \, \exp\left(\frac{i}{\hbar} S[\phi] \right), \tag{21}$$

where the integral ranges over all field configurations consistent with the boundary data.

In this framework, the integral is interpreted as a sum over all causally admissible completions of the unresolved future graph. Each field configuration ϕ corresponds to a particular assignment of candidate vertices and causal links. The action $S[\phi]$ acts as a consistency functional, assigning phase weights to different completions.

Virtual quanta appear as intermediate contributions within this sum. They are not physical excitations propagating through spacetime. They are components of the algorithm that explores which causal completions remain viable.

7.4 Interference as constraint propagation

Interference arises when different candidate completions impose incompatible constraints. In the path integral, such incompatibilities manifest as phase cancellations. From the causal perspective, destructive interference eliminates globally inconsistent vertex assignments, while constructive interference reinforces assignments compatible with all known constraints.

The wave character of quantum phenomena thus reflects the structure of the constraint space. It is not evidence of oscillation at the vertex level, but of the geometry of ignorance over unresolved causal possibilities.

7.5 Collapse as commitment under information completion

Collapse occurs when sufficient information becomes available to uniquely select a consistent vertex from $\mathcal{V}_{\mathrm{cand}}$. At that moment,

$$\mathcal{V}_{\mathrm{cand}} \longrightarrow \{v_*\}, \tag{22}$$

and the selected vertex v_* is committed to the causal past. Faces are closed, action is paid, and the remaining candidates are discarded.

No physical discontinuity occurs. The collapse reflects an update of the causal graph once ignorance has been resolved. Virtual quanta disappear because their role as hypothetical causal links is no longer required.

7.6 Relation to standard quantum field theory

All standard predictions of quantum field theory are recovered. External legs of Feynman diagrams correspond to committed vertices and therefore satisfy on-shell conditions. Internal lines correspond to candidate causal links and are therefore integrated over off-shell configurations.

Energy, momentum, and locality are enforced only at committed vertices, not along unresolved causal links. The apparent nonlocality and indeterminism of quantum theory arise from attempting to describe this causal search using fully rendered spacetime variables.

7.7 Summary

Quantum superposition is the bookkeeping of causal ignorance required to render a globally consistent spacetime under incomplete information. Virtual quanta are elements of the unresolved future of the causal graph. The path integral is the algorithm that explores all admissible completions. Collapse is the act of committing a vertex once consistency is assured.

> **Quantum mechanics does not describe fluctuating reality. It describes the search for a causally consistent one.**

8 Relation to Causal Set Theory

This work should be understood as a constructive geometric realization of *causal set theory*, rather than as a proposal orthogonal to it.

Causal set theory posits that the fundamental structure of the universe is a discrete set of events endowed with a partial order encoding causal relations, from which continuum spacetime emerges only as an approximation [19, 20, 21]. In this view, causal structure is primary and spacetime geometry is derivative. The present paper adopts this premise in full.

Readers who regard the causal set program itself as implausible may reasonably find the framework developed here unconvincing by construction. Conversely, for readers who take seriously the possibility that spacetime is emergent from causal structure, this work aims to address a more specific and longstanding question within that program: *what are the fundamental causal events, and what minimal structure must they possess in order to reproduce known physics?*

Causal events as null vertices

Standard causal set constructions typically treat events as structureless elements, often idealized as arising from a Poisson sprinkling into a continuum spacetime. While this abstraction is powerful, it leaves open how quantum-mechanical and geometric information are encoded at the event level.

In this paper, we propose that the causal events of a causal set admit a concrete physical interpretation as *null vertices* associated with massless excitations (e.g. photons). This identification is motivated by the observation that in the null limit ($d\tau = 0$), conventional spacetime intervals degenerate, while certain data remain well-defined:

- angular information on the celestial sphere (S^2),
- phase and helicity,
- an invariant quantum of action.

Accordingly, each causal event is taken to carry minimal internal structure: directional (angular) data and phase/helicity labels. This additional structure is not introduced ad hoc, but arises as the information that survives when spacetime localization becomes ill-defined. In this sense, the proposal may be viewed as refining, rather than replacing, the causal set notion of an event.

8.1 Position within the causal set program

The contribution of this work is therefore not the introduction of causal discreteness or emergent spacetime per se, both of which are well-established ideas in the literature, but rather:

- a geometric derivation of the minimal event structure consistent with null physics,
- an explicit construction showing how such events assemble into causal sets with local geometric meaning,
- and demonstrations that this structure reproduces known physical scales and phenomena, including horizon entropy and the observed cosmological constant scale.

From this perspective, the framework may be read as a specific realization of the causal set hypothesis in which the nature of the fundamental events is identified and constrained by null geometry. Whether this realization is ultimately correct is an empirical and mathematical question, but it should be evaluated on the same footing as other constructive developments within the causal set program.

9 Summary of the Foundations

The starting point of this framework is a literal reading of special relativity. Not as an approximation, not as a model with a limited regime of validity, but as an exact statement about null transport. The mathematics is unambiguous. For massless motion the interval along the generator is

$$ds^2 = -g_{ab}dx^a dx^b = 0, \tag{23}$$

and the proper time along that transport is

$$d\tau = 0. \tag{24}$$

These are not asymptotic tendencies. They are exact properties of null kinematics. This is not an interpretive choice but a direct consequence of the theory, confirmed by more than a century of experiment.

If this is accepted, then the rest follows necessarily. A trajectory with $d\tau = 0$ admits no intrinsic clock. Without a clock there is no ticking, no evolution parameter, and no internal notion of before or after. Dynamics, in the usual sense, presupposes a time parameter. When proper time is not defined, "evolution along the null" is not a physical process experienced by the null structure; it is a bookkeeping relation imposed by timelike observers.

When metric structure collapses in the null direction, what survives is not arbitrary. Directional information remains well defined. Angular relations on the celestial sphere, polarization, helicity, and relative phase survive the null limit. In the language developed in Part 10, the natural kinematical object is a *vertex frame*: a null-local structure with degenerate intrinsic geometry, carrying a preferred null direction and a Riemannian two-geometry transverse to it. The null regime does not preserve lengths and durations; it preserves *angles and comparative phase structure*.

Action is the only invariant quantity that remains meaningful when time disappears. Energy is defined as a rate,

$$E = \frac{dS}{dt}, \tag{25}$$

and therefore cannot be primitive in a context where dt is not an intrinsic parameter. Action, by contrast, appears in phase factors, variational principles, and quantization conditions independent of any clock. The quantum $\hbar$ must therefore be an action quantum. It cannot be an energy quantum at the null level because energy has not yet been defined.

The appearance of a quarter-scale, $\hbar/4$, is not mysterious and it is not a claim about a three-dimensional cell. It is an *operational* consequence of the vertex-frame degrees of freedom required to specify and compare null relations. In the vertex-frame kinematics developed in Section 14, a complete null connection is specified by a minimal four-degree data package (direction on $\mathbb{S}^2$, twist on $U(1)$, phase reference, and one comparison/closure condition). One unit of resolved constraint corresponds to one entropy increment, and the minimal consistent assignment is

$$\frac{\Delta S}{\Delta N} = \frac{\hbar}{4}. \tag{26}$$

A full vertex-to-vertex cycle resolves four such constraints, giving total action $\hbar$ per completed cycle. This same quarter-factor then reappears whenever physics reduces to counting unresolved null compatibility constraints on a two-surface (horizons, light sheets, and local holographic bottlenecks), because those are all statements about the same vertex-frame bookkeeping.

Vertex frames and edge frames are categorically distinct. One cannot "boost into" a vertex frame, because Lorentz boosts presuppose clocks, rulers, and a timelike parameterization. The divergence of the Lorentz factor as $v \to c$ does not indicate a physical breakdown. It indicates that edge-frame comparisons—which are defined by metric intervals and proper-time ratios—are being applied outside their domain of definition. Edge frames compare metrics and proper time. Vertex frames compare angular data and constraint counts. The singularity of γ marks the boundary between these types of description.

Energy emerges as a projection cost only after a timelike scale exists. In this framework, spacetime is not a pre-existing arena; it is reconstructed from a network of mutually compatible vertex-to-vertex null relations. What is fundamental is the action/phase bookkeeping that labels admissible connections. Only after an edge-frame clock exists to supply a time scale T can one convert a total action budget into a rate:

$$E = \frac{S}{T}. \tag{27}$$

On this view, the cosmological constant is not "energy stored in space." It is a baseline action/constraint throughput cost associated with maintaining a consistent reconstruction under local holographic bottlenecks. Quantum field theory computations of vacuum energy density are performed *within* an already reconstructed spacetime and therefore address a different question than the null-first accounting of reconstruction cost.

Entropy bookkeeping is primitive. Energy bookkeeping is reconstructed. What exists at the base level is counting: admissible configurations, constraints, and action increments. Energy appears only when this accounting is mapped onto a timelike worldline equipped with a clock. This is why horizon thermodynamics works: it operates at the level where entropy and surface capacity are still the native currency of the description.

None of this requires new physics. It requires taking seriously what special relativity already says about null kinematics. To avoid this picture one must assert that something changes at $v = c$, that new physics intervenes to soften the exact consequences of $d\tau = 0$ and null interval degeneracy. There is no experimental evidence for such an intervention. Every observation of massless excitations is consistent with null kinematics as stated.

The alternative is to treat the null regime as merely a calculational limit that is never realized, or to declare questions about null structure meaningless. That position is not supported by the mathematics. The mathematics states that null transport has $ds^2 = 0$ and $d\tau = 0$ exactly. Either the implications of that statement are accepted at the kinematical level, or new physics must be proposed to modify it.

Until such physics is demonstrated, this framework is not optional. It is what consistency demands.

Part II

Geometry of the Null

10 Vertex Frames and Null Geometry

We introduce the vertex frame as the natural rest frame of a null observer. This is not a limit or approximation but a distinct kinematical structure in which proper time vanishes identically and all geometric information is encoded in angular data on the celestial sphere. The vertex frame clarifies why area rather than volume is the fundamental counting measure and why holographic constraints appear locally rather than asymptotically.

10.1 Vertex frames versus edge frames

A timelike observer traverses a worldline parameterized by proper time $\tau > 0$. The observer compares events separated by timelike intervals, measuring distances via the metric. We refer to this as an *edge frame*, emphasizing that the observer moves along edges of the spacetime graph.

A null generator, by contrast, has $\tau = 0$ identically. There is no proper-time parameterization. Instead, null transport relates discrete vertices without a timelike worldline between them. We refer to the kinematical structure at such a vertex as a *vertex frame*.

Formally, consider a null vector k^a satisfying

$$k^a k_a = 0, \qquad k^a \nabla_a k^b = 0. \tag{28}$$

Null transport along the integral curve of k^a has

$$ds^2 = -g_{ab} dx^a dx^b = 0 \tag{29}$$

identically. Proper time is not defined for null transport. Edge-frame clocks attempting to measure elapsed time along the null generator assign divergent dilation relative to the null affine parameter λ:

$$\frac{d\tau_{\text{edge}}}{d\lambda} \to \infty. \tag{30}$$

Proper time never ticks in a vertex frame. The affine parameter λ is not intrinsic to the null observer but is a construction imposed by timelike observers to track what we will identify as *partial entropy* along the generator.

10.2 Intrinsic geometry of the vertex frame

The intrinsic geometry of a null hypersurface $\mathcal{N}$ is degenerate. Let k^a be the null generator. The induced metric on $\mathcal{N}$ has signature $(0, +, +)$ in coordinates adapted to the generator. There is a preferred null direction but no intrinsic notion of distance along that direction.

Transverse to k^a, the geometry is Riemannian. Let S denote a spatial cross-section of $\mathcal{N}$ transverse to the null flow. This is a two-sphere in spherically symmetric cases or a more general two-surface in the general case. The induced metric on S is positive-definite:

$$h_{AB}\, dy^A dy^B, \qquad A, B = 1, 2, \tag{31}$$

where y^A are coordinates on S.

All geometric information intrinsic to the vertex frame is encoded in h_{AB} and the null generator k^a. This is the standard Carrollian (or null boundary) geometric structure carried by null hypersurfaces [22, 23]. The vertex frame has angular structure but no radial metric.

10.3 Entropy as the vertex frame evolution parameter

Because proper time vanishes, we cannot use τ as an evolution parameter. The affine parameter λ is available, but it is extrinsic: it depends on a choice of normalization and reflects the perspective of an external timelike observer.

We introduce the entropy count N as the intrinsic evolution parameter for vertex frames. Let $N(\lambda)$ denote the number of unresolved null configurations compatible with the vertex-to-vertex connection up to affine parameter λ. We interpret N as a constraint count: it measures how many distinct null paths are admissible given the boundary data.

Define the signed entropy parameter

$$u = \begin{cases} +N & \text{(future-directed vertex reconstruction),} \\ -N & \text{(past-directed vertex reconstruction).} \end{cases} \tag{32}$$

Increasing u corresponds to entropy growth: more configurations become admissible as the vertex frame extends into the future. Decreasing u corresponds to entropy reduction: fewer configurations remain consistent as the vertex frame is traced backward.

The relationship between N and λ encodes the rate at which configurations proliferate. In Minkowski space, with the standard affine normalization and the identification $r \propto \lambda$, the number of admissible configurations scales cubically:

$$N(\lambda) \sim \lambda^3 \quad \text{(flat-space benchmark),} \tag{33}$$

reflecting the volumetric growth of the null cone. Deviations from this cubic scaling signal spacetime curvature.

10.4 The celestial sphere and angular coordinates

Each vertex frame has an associated celestial sphere $\mathbb{S}^2$ of null directions. A point on the celestial sphere is specified by two angles (θ, ϕ) in standard spherical coordinates.

However, vertex frame kinematics requires three angles, not two. The third angle ψ encodes the phase twist or helicity: it parameterizes the $U(1)$ freedom associated with rotations of the transverse complex structure.

Thus, each vertex frame configuration is labeled by

$$(\theta, \phi, \psi) \in \mathbb{S}^2 \times U(1). \tag{34}$$

These are not arbitrary coordinates. They arise from the spinorial structure of null directions, as we will show. For now, we note:

- θ, ϕ specify the null direction on the celestial sphere,
- ψ specifies the twist of the phase reference frame relative to that direction.

The angle ψ is directly related to the massless little group of the Lorentz group. For a massless particle with helicity h, a rotation of the transverse frame by $\psi \to \psi + \alpha$ induces the transformation

$$|\text{state}\rangle \to e^{ih\alpha}|\text{state}\rangle. \tag{35}$$

This connection will be made precise in Section 15.3.

The celestial sphere $\mathbb{S}^2$ is the natural phase space for vertex frame kinematics. Null directions approaching the antipodal point of a given vertex correspond to the spacelike boundary of that vertex's causal access.

10.5 Vertex-to-vertex comparison as primary

A null excitation is not an object propagating through spacetime. It is a *constraint relation* between two vertices:

$$p_- \in \mathcal{V}_{\text{past}}, \qquad p_+ \in \mathcal{V}_{\text{future}}, \tag{36}$$

where $\mathcal{V}_{\text{past}}$ and $\mathcal{V}_{\text{future}}$ denote sets of admissible past and future vertices relative to a given event.

The excitation is admissible if and only if the null generators through p_- and p_+ are mutually consistent. Consistency requires:

1. Angular data $(\theta_-, \phi_-, \psi_-)$ at p_- and $(\theta_+, \phi_+, \psi_+)$ at p_+ satisfy geometric compatibility conditions,

2. Entropy budgets N_- and N_+ satisfy conservation along the null path,
3. The intervening spacetime geometry (if already specified) admits a null geodesic connecting the two vertices.

Critically, this is a *constraint search*, not a creation algorithm. Both p_- and p_+ are already admissible events. The vertex frame kinematics determines which pairs can be connected by null relations. No information propagates backward in time. No future boundary condition influences past events dynamically. The search is over already-existing configurations.

This is directly analogous to the role of advanced Green's functions or adjoint states in quantum field theory. They appear in inner products and constraint satisfaction but do not represent causal signals.

11 Entropy as the Null Evolution Parameter and the Entropy Resolution Rate

A central feature of null-first kinematics is that intrinsic time does not exist at the level of null structure. Along a null relation, proper time vanishes identically, $d\tau = 0$, and no clock can be defined. As a result, there is no independent temporal parameter with respect to which evolution could be described.

What does exist is bookkeeping: the resolution of null compatibility constraints. Each resolved constraint corresponds to a definite increment of entropy and action. This observation leads to a key structural identification.

11.1 Identity of null evolution and entropy

In the vertex frame, the only admissible notion of evolution is the accumulation of resolved null constraints. We therefore identify the intrinsic null evolution parameter u with entropy itself:

$$\boxed{du \equiv dS.} \tag{37}$$

This is not a choice of clock, nor an appeal to thermodynamic analogy. It is a statement of identity. There is no other quantity available in the null sector that can parametrize change.

All rates defined after reconstruction into a timelike (edge-frame) description — including energy, expansion, and entropy flow — are projections of this intrinsic entropy evolution into a frame equipped with clocks and rulers.

11.2 Minimal action per entropy increment

As shown in the analysis of vertex-frame degrees of freedom, a complete vertex-to-vertex null closure requires the specification and resolution of four independent constraints: two angular directions on the celestial sphere, a transverse twist (helicity), and one closure/comparison condition. Operationally, these four resolutions form a minimal consistent cycle.

Each independent resolution contributes an action increment

$$\Delta S_{\text{action}} = \frac{\hbar}{4}, \tag{38}$$

so that a full null cycle carries total action $\hbar$. The quarter-action scale $\hbar/4$ is therefore not arbitrary and does not arise from quantization of a three-dimensional cell. It follows from the minimal structure required to define and compare null relations.

11.3 Surface capacity and entropy support

Null constraints are supported on two-dimensional surfaces. The fundamental surface capacity capable of supporting one independent null constraint is the Planck area ℓ_P^2. This identification is enforced by the equivalence encoded in Newton's constant,

$$\ell_P^2 = \frac{\hbar G}{c^3}, \tag{39}$$

which equates surface capacity with volumetric null expansion regulated by action. Entropy in the null sector is therefore intrinsically an area-counting quantity, consistent with the Bekenstein–Hawking entropy law.

11.4 Entropy-defined clock and the role of H_Λ

Horizons correspond to entropy saturation surfaces: all admissible null constraints passing from past to future must pass through a finite surface capacity. The present cosmological horizon therefore encodes the current entropy state of the universe.

The observed Hubble scale H_Λ is not an independent dynamical parameter in this framework. It is the timelike imprint of the present entropy resolution rate of the null sector. The associated thermal (Euclidean) period,

$$\beta_\Lambda = \frac{2\pi}{H_\Lambda}, \tag{40}$$

is thus the entropy-defined clock appropriate for converting null action into timelike energy. This choice is fixed by horizon regularity and does not represent an external assumption.

11.5 Entropy resolution rate

Combining the minimal action per entropy increment, the fundamental surface capacity, and the entropy-defined clock yields a unique conversion factor between null bookkeeping and timelike rates. The entropy resolution rate is

$$\boxed{\Gamma_S = \frac{dS}{dt} = \frac{\hbar/4}{\ell_P^2 \cdot \beta_\Lambda}.} \tag{41}$$

This relation provides the bridge between:

- vertex-frame kinematics (entropy and action),
- horizon thermodynamics (area-supported entropy),
- and quantum mechanics (the appearance of $\hbar$ as action).

Energy, defined only after timelike reconstruction as $E = dS_{\text{action}}/dt$, is therefore a measure of entropy throughput rather than energy stored in space. The cosmological constant arises as the stationary entropy resolution rate of the present horizon, not as vacuum energy in the bulk.

This identity is central: it fixes the normalization of dark energy, explains the origin of the $\hbar/4$ factor, and unifies null kinematics, holography, and quantum action without introducing new dynamical assumptions.

12 Phase Encoding and the Complex Plane

Phase is not an intrinsic property of a single vertex. It is defined relationally through comparison between a forward vertex structure and an adjoint (past-directed) reference. We show how the complex plane encodes this comparison and how the twist angle ψ determines helicity.

12.1 Forward and adjoint vertex structures

Let p be a vertex in the null graph. Associated to p is a forward null structure, which we denote by a spinor $\xi^\alpha(p)$. This spinor encodes the null direction along which the vertex extends into the future.

Phase, however, requires a reference. We introduce an adjoint structure, represented by the conjugate spinor $\bar{\xi}^{\dot\alpha}(p)$. This adjoint structure does not represent a physical null ray propagating into the past. Instead, it encodes the comparison frame needed to define phase, frequency, and helicity.

The relationship between forward and adjoint is analogous to the pairing of kets and bras in quantum mechanics:

$$|\psi\rangle \quad \text{(forward)} \qquad \langle\psi| \quad \text{(adjoint)}. \tag{42}$$

Neither alone is observable. Physical quantities emerge from pairing the two.

12.2 The complex plane and the past node search

The complex plane at vertex p tracks the backward-in-time directed vector to the past causal node. Let p_- be a candidate past vertex. The phase at p relative to p_- is determined by the $SL(2,\mathbb{C})$-invariant pairing between spinors at the two vertices.

Formally, introduce the antisymmetric spinor $\epsilon_{\alpha\beta}$ with $\epsilon_{12} = 1$. The phase Φ satisfies

$$\Phi(p \,|\, p_-) = \arg\left(\epsilon^{\alpha\beta}\, \xi_\alpha(p)\, \xi_\beta(p_-)\right), \tag{43}$$

where $\xi_\alpha(p)$ is the unprimed spinor at vertex p and $\xi_\beta(p_-)$ is the unprimed spinor from the past node p_-. The contraction $\epsilon^{\alpha\beta}\xi_\alpha\xi_\beta$ is the standard invariant scalar product on spinor space.

This means the phase depends on *which past causal node is detected.* Different choices of p_- yield different phase values. There is no absolute phase, only relative phase between vertex connections.

The search over past nodes $p_- \in \mathcal{V}_{\text{past}}$ is a constraint satisfaction problem. The vertex frame at p searches for all p_- consistent with:

- The null direction (θ_-, ϕ_-) aligning with the generator reaching p,
- The entropy budget N_- being compatible with $N(p)$,
- The phase twist ψ_- yielding a consistent helicity winding.

This search is algorithmic, not dynamical. It evaluates compatibility over already-admissible configurations. No causal influence flows from p to p_-.

12.3 The twist angle and phase shift

The twist angle ψ parameterizes rotations of the adjoint spinor frame on the transverse two-sphere. A rotation by ψ induces a phase shift

$$\Phi \to \Phi + \psi. \tag{44}$$

This is the geometric origin of helicity. Helicity measures the winding of the phase around the null direction. It is not an independent quantum number but a manifestation of the $U(1)$ freedom in the adjoint comparison structure.

Under a 2π rotation of the transverse frame,

$$\psi \to \psi + 2\pi, \tag{45}$$

the phase shifts by π:

$$\Phi \to \Phi + \pi. \tag{46}$$

This is the spinorial 2π sign flip. A full 4π rotation is required to return the phase to its original value. This periodicity structure is not imposed but follows from the geometry of null directions and their spinorial representation.

12.4 Phase slope and entropy resolution

Let $\Phi(\lambda)$ denote the phase along a null generator parameterized by affine parameter λ. The physically meaningful quantity is the phase gradient or slope:

$$k = \frac{d\Phi}{d\lambda}. \tag{47}$$

A change in slope Δk corresponds operationally to resolving new information about the vertex-to-vertex connection. We postulate the correspondence

$$\Delta N \longleftrightarrow \text{independent slope resolution.} \tag{48}$$

That is, each independent change in the phase gradient maps to a discrete entropy increment. This is not a dynamical law but a kinematical counting rule: resolving one constraint (one slope change) costs one unit of entropy.

The slope is not intrinsic to the vertex frame. It depends on the choice of past node p_-:

$$k(p \,|\, p_-) = \frac{d}{d\lambda} \arg\left(\bar{\xi}^{\dot\alpha}(p(\lambda))\, \xi_{\dot\alpha}(p_-)\right). \tag{49}$$

Different past nodes yield different slopes. The search over $p_- \in \mathcal{V}_{\text{past}}$ simultaneously searches over admissible slope configurations.

13 Spacetime Emergence from Vertex Superposition

Spacetime is not a pre-existing arena. It emerges from the network of compatible vertex-to-vertex connections. We show how volumetric structure arises from entropy distribution, how the c^3 growth law reflects the expansion of configuration space, and how the path integral formulation is the natural expression of the vertex frame constraint search.

13.1 Spacetime from vertex constraint networks

A single vertex has no spacetime. It has only a celestial sphere of null directions $(\theta, \phi, \psi) \in \mathbb{S}^2 \times U(1)$ and an entropy count N.

Spacetime requires at least two vertices. Given vertices p_- and p_+, a spacetime event is defined by the intersection of their null generators:

$$\ell^a_-(p_-) \cap \ell^a_+(p_+) = x^a. \tag{50}$$

This intersection exists if and only if the angular data and entropy budgets are mutually consistent. Not all pairs of vertices define spacetime events. The admissible pairs form a constraint network.

From the perspective of a single vertex p, spacetime extends outward as a superposition over all future and past vertices compatible with the boundary data at p. This is a search algorithm:

1. Given (θ, ϕ, ψ, N) at p,
2. Search over $p_- \in \mathcal{V}_{\text{past}}$ satisfying angular and entropy compatibility,
3. Search over $p_+ \in \mathcal{V}_{\text{future}}$ satisfying angular and entropy compatibility,
4. Each compatible triple (p_-, p, p_+) contributes one admissible null path.

The set of all such paths defines the spacetime accessible from p. This is not creation. It is search over configurations that are already admissible based on the constraint structure.

13.2 Entropy distribution and volumetric growth

Consider a vertex p with entropy count $N(p)$. The null cone extending from p sweeps out a region in spacetime. In flat space, the volume of this region grows as

$$V(\lambda) \sim \lambda^3. \tag{51}$$

From equation (33), this implies

$$V(N) \sim N^3/N^3_{\max}, \tag{52}$$

where $N_{\max}$ is a normalization constant related to the maximum constraint capacity.

The c^3 scaling law states that volume grows cubically with entropy. This is not a geometric accident. It reflects the fact that each unit of entropy at the source vertex seeds an independent null direction. The configuration space expands at rate

$$\frac{dV}{dN} \sim N^2 \sim r^2, \tag{53}$$

where $r \sim N^{1/3}$ is an effective radial parameter.

The surface area at fixed N scales as $N^{2/3} \sim r^2$, while the volume interior scales as $N \sim r^3$. This is the origin of the holographic mismatch: volumetric entropy storage grows faster than surface throughput.

13.3 The present vertex as holographic bottleneck

Let p be the present vertex. Past vertices converge entropy into p from a volumetric region:

$$N_{\text{past}} \sim r_-^3. \tag{54}$$

Future vertices diverge entropy from p into a volumetric region:

$$N_{\text{future}} \sim r_+^3. \tag{55}$$

The present vertex itself has surface capacity

$$N_{\text{surface}}(p) \sim r^2. \tag{56}$$

This creates an information bottleneck. All entropy flow from past to future must pass through the present vertex surface. Conservation of constraint count requires

$$\sum_{p_- \in \text{past}} N(p_-) \;\longrightarrow\; N_{\text{surface}}(p) \;\longrightarrow\; \sum_{p_+ \in \text{future}} N(p_+). \tag{57}$$

The bottleneck is local. It occurs at every vertex where past and future null cones intersect. This is *local holography*, distinct from asymptotic holography at null infinity.

The mismatch between volumetric entropy (r^3) and surface capacity (r^2) is the origin of the holographic bound. Entropy cannot be stored arbitrarily in the bulk. It must respect the throughput constraint at vertex surfaces.

13.4 Curvature as deviation from c^3 scaling

In curved spacetime, the relation (33) is modified. The Raychaudhuri equation governs the evolution of the null cone expansion θ:

$$\frac{d\theta}{d\lambda} = -\frac{1}{2}\theta^2 - \sigma_{ab}\sigma^{ab} + \omega_{ab}\omega^{ab} - R_{ab}\ell^a\ell^b, \tag{58}$$

where σ_{ab} is the shear, ω_{ab} is the twist, and R_{ab} is the Ricci tensor [24, 8].

The expansion θ measures the rate of change of the null cone cross-sectional area:

$$\theta = \frac{1}{A}\frac{dA}{d\lambda}. \tag{59}$$

In vertex frame language, $A \sim N^{2/3}$, so

$$\theta \sim \frac{1}{N}\frac{dN}{d\lambda}. \tag{60}$$

Deviations from the flat-space relation $N \sim \lambda^3$ signal spacetime curvature through the Ricci focusing term $R_{ab}\ell^a\ell^b$ in (58).

In vertex-to-vertex language, curvature is detected by observing that the entropy budget does not distribute as c^3. Vertices in a curved region have fewer (or more) compatible future connections than flat-space kinematics would predict.

13.5 The path integral as constraint search

The vertex frame search over admissible null connections is identical in structure to the Feynman path integral formulation of quantum mechanics [25]. This is not an analogy. The path integral *is* the mathematical expression of the vertex frame constraint search.

Consider a vertex-to-vertex connection from p_- (emission) to p_+ (absorption). The amplitude for this connection is

$$\mathcal{A}(p_+|p_-) = \sum_{\text{paths}} e^{iS[\text{path}]/\hbar}, \tag{61}$$

where the sum is over all admissible null paths connecting p_- to p_+, and $S[\text{path}]$ is the action along each path.

In vertex frame language, this becomes a sum over compatible angular data and entropy configurations:

$$\mathcal{A}(p_+|p_-) = \sum_{(\theta,\phi,\psi,N)} \exp\left(i\,\Phi[\theta,\phi,\psi,N]\right), \tag{62}$$

where $\Phi[\theta,\phi,\psi,N]$ is the phase accumulated along the vertex-to-vertex connection with specified angular data (θ,ϕ,ψ) and entropy count N.

The phase Φ is related to the action by

$$\Phi = \frac{S}{\hbar}. \tag{63}$$

Each term in the sum (62) corresponds to one admissible configuration satisfying:

- Angular compatibility: the null direction (θ_-,ϕ_-) at p_- aligns with the generator reaching p_+,
- Entropy conservation: $N_- + \Delta N = N_+$, where ΔN is the entropy change along the path,
- Phase consistency: the twist ψ yields a continuous helicity winding.

Configurations violating these constraints contribute zero amplitude. The sum over all admissible configurations is the vertex frame constraint search.

In the continuum limit, the sum becomes a path integral:

$$\mathcal{A}(p_+|p_-) = \int \mathcal{D}[\text{path}]\, e^{iS[\text{path}]/\hbar}, \tag{64}$$

where the measure $\mathcal{D}[\text{path}]$ weights each path by its entropy contribution.

The classical limit corresponds to configurations where the phase is stationary:

$$\delta\Phi = 0 \quad \Longleftrightarrow \quad \delta S = 0. \tag{65}$$

This is the principle of least action. In vertex frame language, it states that the dominant contribution comes from vertex-to-vertex connections where the entropy distribution extremizes the total constraint count.

The path integral formulation makes explicit that:

1. The amplitude is a *sum* over configurations, not a single trajectory,
2. Each configuration is already admissible (satisfies constraints),
3. The search is over vertex connections, not continuous worldlines,
4. Classical trajectories emerge as stationary points of the entropy-weighted sum.

This clarifies that the vertex frame search is not creating events or propagating information backward in time. It is evaluating which of the already-existing vertex connections are mutually compatible, exactly as the path integral sums over already-defined paths.

The connection to standard quantum mechanics is direct [26, 25]. The vertex frame formalism provides a geometric origin for the path integral structure: it is the natural way to implement constraint search over null configurations.

13.6 Superposition and interference

The path integral formulation reveals why quantum superposition is unavoidable in vertex frame kinematics. Given vertices p_- and p_+, there are generically multiple admissible null paths connecting them. Each path contributes a phase $e^{i\Phi}$ to the total amplitude.

These phases interfere constructively or destructively depending on the relative phase differences:

$$|\mathcal{A}|^2 = \left|\sum_{\text{paths}} e^{i\Phi_{\text{path}}}\right|^2 = \sum_{\text{paths}} 1 + 2\sum_{j<k}\cos(\Phi_j - \Phi_k). \tag{66}$$

The cross terms $\cos(\Phi_j - \Phi_k)$ are the interference contributions. They vanish when averaged over many configurations unless the phases are correlated, which occurs when the entropy budgets N_j and N_k are nearly equal.

In vertex frame language, interference arises because:

- Multiple vertex connections can satisfy the same boundary data,
- Each connection accumulates a different phase depending on its angular path and entropy distribution,
- The total amplitude requires summing all compatible connections,
- Observable probabilities $|\mathcal{A}|^2$ include interference between paths.

This is not a separate quantum postulate. It is the inevitable consequence of searching over multiple admissible vertex configurations. The vertex frame structure *requires* superposition because constraint satisfaction is generally not unique.

Classical trajectories emerge in the limit where one path dominates, which occurs when:

$$S_{\text{classical}} \gg \hbar, \tag{67}$$

so that small variations in the path cause large phase differences, leading to destructive interference for all non-stationary configurations.

In vertex frame terms, this corresponds to entropy budgets $N \gg 1$, where the number of admissible configurations is large and the stationary entropy distribution is sharply peaked.

14 Action Quantization and the Four-Degree Structure

We derive the fundamental action quantum $\hbar/4$ from vertex frame comparison and build up to the full quantum $\hbar$ as the cost of a complete vertex-to-vertex cycle.

14.1 Vertex frame comparison and minimal action

A *frame* is an operational linearization of the phase around a given vertex. It requires specifying the slope $k = d\Phi/d\lambda$ relative to a chosen reference.

Comparing two vertex frames requires resolving two conjugate slope changes. We postulate that the minimal action for such a comparison is

$$\Delta S_{\min} = \frac{\hbar}{2}. \tag{68}$$

This postulate reproduces the standard Heisenberg uncertainty bound upon rendering vertex frame data into edge-frame observables. Under the edge-frame mapping, slope resolution Δk maps to momentum uncertainty Δp and affine parameter variation $\Delta\lambda$ maps to position uncertainty Δx, yielding $\Delta x \cdot \Delta p \geq \hbar/2$.

In vertex frame language, each slope resolution corresponds to one entropy unit:

$$\Delta N = 1 \quad \longleftrightarrow \quad \text{one independent slope resolution.} \tag{69}$$

The action cost per entropy resolution is therefore

$$\frac{\Delta S}{\Delta N} = \frac{\hbar}{4}. \tag{70}$$

This is the fundamental action-entropy quantum. Comparing two frames requires two entropy resolutions (one for each slope), giving

$$\Delta S_{\min} = 2 \times \frac{\hbar}{4} = \frac{\hbar}{2}, \tag{71}$$

consistent with (68).

14.2 Four degrees of freedom per vertex cycle

A complete vertex-to-vertex connection requires specifying four independent degrees of freedom:

1. The phase Φ at the emission vertex,
2. The phase slope $k = d\Phi/d\lambda$,
3. The twist angle ψ (transverse orientation/helicity),
4. The comparison between emission and absorption vertex frames.

Each degree of freedom contributes one entropy unit:

$$\Delta N_{\text{total}} = 4. \tag{72}$$

The total action per complete cycle is therefore

$$\Delta S_{\text{cycle}} = 4 \times \frac{\hbar}{4} = \hbar. \tag{73}$$

When this vertex frame action is mapped to a timelike clock with proper time τ, the frequency is defined as the phase advance per unit clock time:

$$\omega = \frac{d\Phi}{d\tau}. \tag{74}$$

The energy associated with the vertex-to-vertex connection is then

$$E = \hbar\omega. \tag{75}$$

This is not a postulate. It is the rendering of vertex frame action into the edge frame language of a timelike observer. The factor of $\hbar$ arises because a complete vertex frame cycle requires four entropy resolutions, each costing $\hbar/4$.

14.3 The factor of four and operational constraints

The appearance of four degrees of freedom is not arbitrary. It arises from the spinorial structure of null directions and the necessity of pairing forward and adjoint vertex structures.

In spinorial language, a null direction is encoded by a two-component spinor ξ^α. Specifying the phase reference requires the conjugate spinor $\bar{\xi}^{\dot{\alpha}}$. Together, these have four real degrees of freedom (or two complex degrees of freedom).

However, null directions are projective: $\xi^\alpha \sim \lambda \xi^\alpha$ for any nonzero $\lambda \in \mathbb{C}$. This removes one complex degree of freedom, leaving one complex degree (two real). The twist ψ adds one more real degree, and the frame comparison adds the fourth.

The four-degree structure is thus a consequence of:

- Two real parameters for the null direction (θ, ϕ) on $\mathbb{S}^2$,
- One real parameter for the twist ψ on $U(1)$,
- One real parameter for the phase Φ (or equivalently, the comparison between emission and absorption).

This is not multiple time dimensions. It is not solid angle. It is the minimal set of operational constraints required to define a vertex-to-vertex null connection.

15 Spinorial Encoding and the 4π Periodicity

We make precise the role of spinors in vertex frame kinematics. The three-angle structure (θ, ϕ, ψ) is shown to coordinatize the spinorial double cover of the Lorentz group, and the 4π periodicity is explained as the geometric origin of quantum phase.

15.1 Null directions as spinor bilinears

In four-dimensional Lorentzian spacetime, any null vector k^a can be written as a bilinear of Weyl spinors [27]:

$$k^a = \sigma^a_{\ \alpha\dot{\alpha}}\, \xi^\alpha \bar{\xi}^{\dot{\alpha}}, \tag{76}$$

where σ^a are the Pauli matrices extended to spacetime and ξ^α is a two-component complex spinor.

The spinor ξ^α is defined up to phase:

$$\xi^\alpha \sim e^{i\chi} \xi^\alpha. \tag{77}$$

This $U(1)$ freedom is precisely the twist angle ψ in vertex frame language.

The spin group $\mathrm{Spin}(3,1)$ double-covers the Lorentz group $\mathrm{SO}(3,1)$. As a result, a rotation by 2π in physical space corresponds to a -1 multiplication in spinor space:

$$\xi^\alpha \to -\xi^\alpha. \tag{78}$$

The null vector k^a constructed from (76) is invariant under this sign flip:

$$k^a(\xi) = k^a(-\xi). \tag{79}$$

Only after a 4π rotation does the spinor return to itself.

15.2 The three-angle parameterization as spinor coordinates

The three angles (θ, ϕ, ψ) introduced in (34) are not independent of spinorial structure. They are coordinates on the space of spinors modulo the stabilizer.

Explicitly, the spinor ξ^α can be parameterized as

$$\xi^\alpha = \begin{pmatrix} \cos(\theta/2)\, e^{i(\phi+\psi)/2} \\ \sin(\theta/2)\, e^{i(\phi-\psi)/2} \end{pmatrix}. \tag{80}$$

Under this parameterization:

- $\theta \in [0, \pi]$ and $\phi \in [0, 2\pi)$ coordinatize the celestial sphere $\mathbb{S}^2$,

- $\psi \in [0, 4\pi)$ coordinatizes the $U(1)$ twist (note the 4π range).

The ranges reflect the double-covering. A 2π rotation in ψ yields

$$\psi \to \psi + 2\pi \quad \Longrightarrow \quad \xi^\alpha \to -\xi^\alpha, \tag{81}$$

consistent with (78).

The three-angle structure is thus not imposed but arises geometrically from the spinorial encoding of null directions.

15.3 Helicity and the twist angle

Helicity measures the projection of angular momentum onto the direction of motion. For massless particles, helicity is defined by the action of the massless little group, which is $ISO(2)$ (or $U(1)$ after gauge fixing).

The twist angle ψ parameterizes this $U(1)$ little-group action. Under a rotation of the transverse spinor frame by angle $\delta\psi$, the quantum state transforms as

$$|\text{state}\rangle \to e^{ih\delta\psi}|\text{state}\rangle, \tag{82}$$

where h is the helicity quantum number.

For photons (spin-1), $h = \pm 1$, so a rotation by $\delta\psi = 2\pi$ yields

$$|\text{photon}\rangle \to e^{i(\pm 1)\cdot 2\pi}|\text{photon}\rangle = |\text{photon}\rangle, \tag{83}$$

as expected for integer-spin particles.

For fermions (spin-1/2), $h = \pm 1/2$, so a rotation by $\delta\psi = 2\pi$ yields

$$|\text{fermion}\rangle \to e^{i(\pm 1/2)\cdot 2\pi}|\text{fermion}\rangle = -|\text{fermion}\rangle, \tag{84}$$

the characteristic spinor sign flip.

The relationship between twist angle and phase is

$$\Phi \to \Phi + h \cdot \psi. \tag{85}$$

In vertex frame language, helicity is identified with the winding number of the phase around the null direction, encoded by the $U(1)$ twist parameter ψ. The half-integer structure for fermions is unavoidable because the underlying spinorial parametrization of null directions has 4π periodicity, as we now show.

15.4 Forward and adjoint spinor structures

We clarify the notation for forward and adjoint structures.

The *forward* null structure at vertex p is encoded by the unprimed spinor

$$\xi^\alpha(p). \tag{86}$$

This spinor fixes the null direction along which the vertex extends into the future.

The *adjoint* comparison structure is encoded by the primed (conjugate) spinor

$$\bar{\xi}^{\dot{\alpha}}(p). \tag{87}$$

This spinor fixes the phase reference frame needed to define Φ.

The Hermitian conjugate $\xi^\dagger$ is shorthand for the dual pairing when forming scalar products:

$$\xi^\dagger \bar{\xi} = \xi^\alpha \bar{\xi}_\alpha. \tag{88}$$

No independent object beyond $(\xi^\alpha, \bar{\xi}^{\dot{\alpha}})$ is introduced.

Rotations of the adjoint spinor correspond to rotations of the complex basis used to read phase. A relative twist between forward and adjoint manifests as a phase shift. The adjoint structure can be represented as a past-directed null construction, but it plays no dynamical role. It encodes comparison, not propagation.

15.5 Curvature and spinor parallel transport

Null geodesics are determined by the condition

$$k^b \nabla_b k^a = 0, \tag{89}$$

where $k^a = \nabla^a \Phi$ in the geometric optics limit.

In spinorial terms, curvature enters through the spin connection, which parallel transports the spinor ξ^α along the null generator. Let D_λ denote the covariant derivative along the generator parameterized by λ:

$$D_\lambda \xi^\alpha = \frac{d\xi^\alpha}{d\lambda} + \Gamma^\alpha{}_\beta \xi^\beta, \tag{90}$$

where $\Gamma^\alpha{}_\beta$ is the spin connection.

Parallel transport requires

$$D_\lambda \xi^\alpha = 0. \tag{91}$$

The continuous rotation of the local spinor frame induced by Γ produces a continuous rotation of the phase gradient. This is the geometric origin of curved null trajectories.

Importantly, null rays do not "choose" their direction discretely. The phase evolution is smooth, governed by parallel transport. In the null-first framework, the relations (76) and the parallel transport condition are not imposed as independent postulates. They are *derived* from the requirement that vertex-to-vertex constraints propagate consistently across the null substrate.

15.6 Spinorial entropy and the 4π cycle

In the spinorial picture, the entropy count N measures the number of independent spinorial constraint configurations compatible with a given vertex-to-vertex connection.

Equivalently, N counts the logarithm of the number of admissible states in the space

$$\mathcal{H}_{\text{vertex}} = \{(\Phi, k, \psi, \text{frame}) \,|\, \text{admissible}\}. \tag{92}$$

A full physical cycle of a null wave corresponds to a 4π spinorial rotation. This cycle requires specifying all four operational degrees of freedom. Only after all four are resolved does the physical state return to itself.

The action per complete cycle is

$$\Delta S_{\text{cycle}} = \hbar, \tag{93}$$

arising from four entropy resolutions at $\hbar/4$ each.

The quarter-action scale $\hbar/4$ is the fundamental granularity of vertex frame phase space. Pairing two degrees of freedom (e.g., position and momentum in the edge frame rendering) yields the Heisenberg bound $\hbar/2$. Completing all four yields the Planck constant $\hbar$.

This explains the emergence of $\hbar$ as the action per complete cycle without modifying or replacing quantum mechanics. The origin of the characteristic quantum scale is geometric, arising from the spinorial structure of vertex frames.

16 Forward and Adjoint Geometries: de Sitter and Anti-de Sitter

We distinguish the geometry of spacetime reconstruction (de Sitter-like) from the geometry of phase comparison (anti-de Sitter-like). This resolves the apparent tension between forward and backward vertex structures without invoking retrocausality.

16.1 The critical distinction

Anti-de Sitter geometry does not describe physical spacetime in this framework. It characterizes the adjoint comparison structure used to define phase, frequency, and helicity relative to forward null propagation.

Spacetime itself remains Lorentzian and causal. The dS/AdS asymmetry arises because:

- forward vertex reconstruction is entropy-increasing and exhibits de Sitter-like defocusing,
- adjoint vertex reconstruction is entropy-decreasing and exhibits anti-de Sitter-like focusing.

These are not two different spacetimes. They are two dual aspects of the same vertex frame kinematics.

16.2 Forward reconstruction: de Sitter-like

Forward reconstruction proceeds along entropy-increasing null directions. The signed parameter u from (32) is positive:

$$u = +N, \qquad \frac{du}{d\lambda} > 0. \tag{94}$$

As u increases, the number of admissible configurations grows. Null generators defocus. The effective geometry exhibits positive curvature:

$$R_{\text{eff}} > 0 \quad \text{(dS-like)}. \tag{95}$$

In de Sitter space, the expansion of null cones is governed by

$$\frac{d\theta}{d\lambda} = -\frac{1}{2}\theta^2 + \frac{\Lambda}{3}, \tag{96}$$

where $\Lambda > 0$ is the cosmological constant. The positive Λ term opposes focusing and drives expansion.

In vertex frame language, this is not a fundamental cosmological constant but the effective behavior of entropy-increasing reconstruction. Future vertices proliferate, creating volumetric expansion in configuration space.

16.3 Adjoint reconstruction: anti-de Sitter-like

Adjoint reconstruction proceeds along entropy-decreasing directions. The signed parameter is negative:

$$u = -N, \qquad \frac{du}{d\lambda} < 0. \tag{97}$$

As $|u|$ increases (moving further into the adjoint structure), the number of admissible configurations shrinks. Null generators focus. The effective geometry exhibits negative curvature:

$$R_{\text{eff}} < 0 \quad \text{(AdS-like)}. \tag{98}$$

In anti-de Sitter space, null generators reach the conformal boundary in finite affine parameter. This is a standard property of AdS geometry and does not indicate pathology.

In vertex frame language, the adjoint structure is compact. Phase lives on $U(1)$, a compact space. The entire phase cycle can be traversed in finite constraint depth. The "boundary" of the adjoint structure is reachable because phase space has finite extent.

16.4 Why AdS appears without retrocausality

The AdS geometry of the adjoint structure does not imply time reversal or backward causal influence. Consider the standard quantum mechanical inner product:

$$\langle \psi_f | \psi_i \rangle = \int \psi_f^*(x)\, \psi_i(x)\, dx. \tag{99}$$

The final state ψ_f appears conjugated (adjoint). This does not mean ψ_f propagates backward in time. It means the adjoint is needed to form the observable pairing.

Similarly, in vertex frame kinematics:

- The forward structure ξ^α encodes spacetime propagation (dS-like, entropy-increasing),
- The adjoint structure $\bar{\xi}^{\dot{\alpha}}$ encodes phase reference (AdS-like, entropy-decreasing),
- Observable physics emerges from pairing the two.

Neither alone is physical. No information propagates into the past. The search over past vertices $p_- \in \mathcal{V}_{\text{past}}$ is a constraint satisfaction problem over configurations that already exist. The adjoint structure functions purely as a comparison device.

16.5 Phase space compactness and finite affine reach

The reason AdS reaches its boundary in finite affine parameter is that phase space is compact. The twist angle ψ has range $[0, 4\pi)$. A full traversal of this range corresponds to a complete winding of the adjoint phase reference.

In AdS$_{d+1}$, the metric can be written (schematically) as

$$ds^2 = \frac{1}{\cos^2\rho}\left(-dt^2 + d\rho^2 + \sin^2\rho\, d\Omega_{d-1}^2\right), \quad \rho \in [0, \pi/2). \tag{100}$$

The boundary is at $\rho = \pi/2$, which is reached in finite t along null trajectories satisfying $dt = \pm d\rho$.

In vertex frame language, we may heuristically identify the radial coordinate with the entropy angle:

$$\rho \sim \frac{N}{N_{\max}} \cdot \frac{\pi}{2}. \tag{101}$$

The adjoint structure has $N_{\max} < \infty$ (the phase cycle is finite), so the boundary is reachable in finite constraint depth. This mapping is not a derivation of AdS geometry from first principles but an operational identification between entropy-decreasing reconstruction and the compactness of phase space.

16.6 Observable physics from vertex frame pairing

Physical observables require pairing forward and adjoint vertex structures:

$$\mathcal{O} = \langle \text{adjoint} | \, \mathcal{O}_{\text{op}} \, | \text{forward} \rangle. \tag{102}$$

In vertex frame language:

$$\text{Forward vertex} : \xi^{\alpha}(p_+), \quad u = +N, \quad \text{dS-like}, \tag{103}$$

$$\text{Adjoint vertex} : \bar{\xi}^{\dot{\alpha}}(p_-), \quad u = -N, \quad \text{AdS-like}, \tag{104}$$

$$\text{Observable} : \Phi(p_+|p_-) = \arg\left(\bar{\xi}^{\dot{\alpha}}(p_+)\, \xi_{\dot{\alpha}}(p_-)\right). \tag{105}$$

The intersection of the forward (dS-like) and adjoint (AdS-like) structures produces Lorentzian physics. Neither alone is observable. The apparent dS/AdS asymmetry is a reflection of the directionality of entropy flow, not a modification of spacetime geometry.

17 Horizons, Entropy Saturation, and Phase Accessibility

We show how horizons arise as vertices where future null cone expansion saturates and how black hole entropy corresponds to unresolved vertex-to-vertex connections on the horizon surface.

17.1 Horizons as entropy saturation vertices

A horizon is characterized by a marginally trapped surface where the expansion of outgoing null geodesics vanishes. In vertex frame language, this corresponds to a vertex p_H where the future-directed null cone ceases to expand.

For a congruence of outgoing null generators with expansion θ, a marginally outer trapped surface satisfies [24, 8]

$$\theta|_{\text{horizon}} = 0. \tag{106}$$

The Raychaudhuri equation (58) governs how θ evolves. Under null energy conditions ($R_{ab}\ell^a\ell^b \geq 0$), focusing causes θ to decrease along outgoing generators. At a horizon, this focusing drives θ to zero and maintains it there.

In vertex frame language, the entropy budget from past vertices converges onto p_H, but the future entropy does not diverge volumetrically. Instead, all future vertices $p_+ \in \mathcal{V}_{\text{future}}(p_H)$ lie tangent to the horizon surface. The volumetric growth saturates:

$$r_+ \to r_H \quad \text{(constant)}, \tag{107}$$

and all entropy flow becomes tangent to the surface rather than expanding into the bulk.

The surface capacity at the horizon is

$$N_{\text{surface}}(p_H) \sim \frac{A_H}{\ell_P^2}, \tag{108}$$

where A_H is the horizon area. The volumetric budget $N_{\text{volume}}(p_H)$ does not grow beyond the horizon, so all entropy is concentrated on the surface.

This is the vertex frame origin of the Bekenstein-Hawking area law.

17.2 Partial phase accessibility and horizon entropy

At the horizon, outgoing null generators experience infinite redshift relative to external timelike observers. The affine parameter λ along the generator remains finite, but the proper time of an external observer diverges:

$$\frac{d\tau}{d\lambda} \to \infty. \tag{109}$$

In vertex frame language, this means the phase evolution along the generator is smooth, but the mapping to an external vertex frame becomes singular. Let $\Phi(\lambda)$ be the phase along a generator crossing the horizon. An external vertex at finite λ_{ext} can only access

$$\Phi_{\text{accessible}} = \Phi(\lambda)\Big|_{\lambda<\lambda_H}, \tag{110}$$

where λ_H is the affine parameter at the horizon crossing.

The phase information for $\lambda > \lambda_H$ is inaccessible to the external vertex. This inaccessible phase is recorded as boundary entropy. The number of unresolved vertex-to-vertex connections is

$$N_{\text{unresolved}} \sim \frac{A_H}{\ell_P^2}. \tag{111}$$

Black hole entropy is thus the restricted portion of the null phase ledger corresponding to vertex connections that cross the horizon but cannot be resolved by external vertex frames.

17.3 Causal diamonds and local holographic bounds

Consider a causal diamond: the intersection of the future of a vertex p_- and the past of a vertex p_+. The boundary of the causal diamond is a marginally trapped surface with area A.

The Bousso covariant entropy bound states that the entropy on any light sheet of this surface satisfies [28, 29]

$$S \le \frac{A}{4G\hbar/c^3} = \frac{A}{4\ell_P^2}. \tag{112}$$

In vertex frame language, the entropy count N on a light sheet is bounded by the number of vertex-to-vertex connections that can pass through the surface. This number is limited by the surface capacity:

$$N \le \frac{A}{\ell_P^2}. \tag{113}$$

The factor of $1/4$ in (112) is consistent with the four-degree structure of vertex frame cycles introduced in Section 14.2. Each Planck area ℓ_P^2 supports one independent vertex connection, which carries four entropy units ($\Delta N = 4$ from a complete cycle), but only one of these corresponds to an independent surface degree of freedom after accounting for compatibility constraints. In later sections we show how this quarter-factor matches the gravitational normalization from the Einstein-Hilbert action.

18 Interpretational Summary and Scope

18.1 What vertex frames determine

The vertex frame picture explains several foundational features of quantum gravity and cosmology without invoking dynamics:

- **Angular primacy**: Vertex frames have well-defined angles but no intrinsic radial metric. Area rather than volume is the natural counting measure.

- **Holographic scaling**: Every vertex is a holographic screen where volumetric entropy (r^3) compresses through surface capacity (r^2) into future expansion.
- **Action quantization**: The fundamental quantum $\hbar/4$ arises from the cost of resolving one constraint in vertex frame phase space. The Planck constant $\hbar$ is the action per complete four-degree vertex cycle.
- **Spinorial structure**: The three-angle parameterization (θ, ϕ, ψ) coordinatizes the double cover of the Lorentz group. The 4π periodicity and half-integer spin are geometric necessities.
- **Phase compactness**: The adjoint (comparison) structure is AdS-like because phase space is compact. This does not imply retrocausality or backward propagation.
- **Horizon entropy**: Black holes saturate entropy flow at their surface. The area law reflects counting unresolved vertex-to-vertex connections on the horizon.

18.2 Manifest causality and the search interpretation

Throughout this construction, causality is preserved. The vertex frame kinematics is a *constraint search*, not a creation algorithm.

Both forward and adjoint structures are purely algorithmic. They search over already-admissible vertex connections and do not create events. Each vertex-to-vertex "handshake" is re-evaluated whenever the constraint count N changes, i.e., whenever entropy is updated. No information is sent into the past. No future boundary condition exerts dynamical influence on earlier events.

The role of the adjoint structure is identical to that of advanced Green's functions in quantum field theory. In standard QFT [25, 26], the retarded Green's function propagates causally forward in time:

$$G_{\text{ret}}(x, x') = \theta(t - t')\, \langle 0|[\phi(x), \phi(x')]|0\rangle, \tag{114}$$

while the advanced Green's function formally propagates backward:

$$G_{\text{adv}}(x, x') = -\theta(t' - t)\, \langle 0|[\phi(x), \phi(x')]|0\rangle. \tag{115}$$

Advanced and retarded Green's functions are mathematical tools for constructing the Feynman propagator:

$$G_F(x, x') = \theta(t - t')G_{\text{ret}}(x, x') + \theta(t' - t)G_{\text{adv}}(x, x'). \tag{116}$$

The advanced Green's function appears in correlation functions and boundary value problems but does not represent signals traveling backward in time. It is used to impose causal boundary conditions and construct propagators, not to describe physical backward propagation.

Similarly, the adjoint vertex structure (past-directed, entropy-decreasing) is a comparison device. It appears in the phase pairing (43):

$$\Phi(p_+|p_-) = \arg\left(\bar{\xi}^{\dot{\alpha}}(p_+)\, \xi_{\dot{\alpha}}(p_-)\right), \tag{117}$$

exactly as ψ^* appears in quantum inner products:

$$\langle\psi|\phi\rangle = \int \psi^*(x)\, \phi(x)\, dx. \tag{118}$$

No causal influence flows from future to past. The search over past vertices $p_- \in \mathcal{V}_{\text{past}}$ evaluates compatibility over configurations that already exist.

Unlike Wheeler-Feynman absorber theory, this framework does not require future boundary conditions to influence dynamics. The adjoint structure enters constraint satisfaction, not causal propagation. The path integral (61) sums over admissible paths; it does not select paths based on future outcomes.

18.3 Connection to standard physics

The vertex frame formalism connects naturally to established structures:

- **Null hypersurfaces**: The degenerate geometry of vertex frames is the standard Carrollian structure of null boundaries [9, 11].
- **Spinorial geometry**: The encoding of null directions via spinor bilinears is standard differential geometry on Lorentzian manifolds [27].

- **Adjoint states**: The pairing of forward and adjoint vertex structures is the null-geometric analogue of bra-ket pairing in quantum mechanics.
- **Holographic bounds**: The Bousso covariant entropy bound emerges automatically from vertex surface capacity constraints.
- **Celestial holography**: The celestial sphere as primary phase space aligns with recent work on BMS symmetry and celestial CFT [30, 31].

We are not inventing new structure. We are explaining the origin of existing structure by taking vertex frames as kinematically primary.

18.4 What is not claimed

This part has developed the kinematics of vertex frames. Several questions remain outside its scope:

- **Dynamics**: We have not derived the Einstein field equations or explained how vertex configurations evolve. That requires a dynamical principle governing the vertex graph, addressed in a companion work.
- **Asymptotic structure**: We have not explained why the universe has its particular asymptotic entropy or expansion rate. Those questions involve boundary conditions and cosmological initial data.
- **Matter fields**: We have not addressed fermions, gauge fields beyond $U(1)$, or the standard model. The vertex frame structure suggests natural extensions (e.g., $SU(2)$ for weak interactions), but these are not developed here.

What has been established is that the vertex frame kinematical structure is internally consistent, tightly constrained by geometry, and sufficient to account for action quantization, horizon entropy, holographic scaling, and the spinorial origin of quantum phase. These are not independent postulates but consequences of taking null geometry as primary.

Part III

Dissolution of the Cosmological Constant via Action Pixelation

19 The Cosmological Constant Problem

19.1 Statement of the Discrepancy

A Planck-scale cutoff estimate for vacuum energy density in quantum field theory gives

$$\rho_{\mathrm{QFT}} \sim \frac{\hbar c}{\ell_P^4}, \tag{119}$$

while observations are consistent with a late-time de Sitter scale

$$\rho_\Lambda = \frac{3H_\Lambda^2 c^2}{8\pi G}. \tag{120}$$

Convention note. In this paper ρ denotes an *energy density*. Some cosmology conventions use ρ for mass density, in which case the corresponding relation is $\rho_{\Lambda,\text{mass}} = 3H_\Lambda^2/(8\pi G)$ and $\rho = \varepsilon/c^2$.

The ratio is of order 10^{122} [5, 32]. The standard presentation treats this as a fine-tuning crisis.

19.2 The Category Error

In the null-first picture, the mismatch is better understood as a category error with two parts.

First, the QFT estimate counts bulk volumetric modes as if they were fundamental. Gravitating systems obey holographic scaling, where independent degrees of freedom scale with boundary area rather than bulk volume [13, 14, 29].

Second, the QFT estimate treats energy as primitive in a regime controlled by null structure. In null kinematics, energy is not intrinsic because it requires a time scale. What is intrinsic is action, which is defined without clocks and appears directly in quantum phases and boundary terms.

The resolution in this paper is therefore not a modification of gravity. It is a change in what is being counted. We count horizon pixels and their action, then render to energy using the physically correct de Sitter time scale.

20 From Horizon Pixels to Total Action

20.1 De Sitter Horizon Geometry

For an asymptotically de Sitter universe, the cosmological horizon radius is

$$R_\Lambda = \frac{c}{H_\Lambda}, \tag{121}$$

with horizon area

$$A_\Lambda = 4\pi R_\Lambda^2. \tag{122}$$

The number of Planck-area pixels on the horizon is

$$N_{\text{pix}} = \frac{A_\Lambda}{\ell_P^2}. \tag{123}$$

Numerically, this is of order 10^{122} for the observed late-time scale. In this framework, that number is not a pathology. It is simply the horizon pixel count.

20.2 Action Pixelation

In the vertex-frame formulation, action is discretized by the resolution of independent null compatibility constraints. As shown in Section 14, a complete vertex-to-vertex null cycle requires four independent constraint resolutions: null direction on $\mathbb{S}^2$, transverse twist on $U(1)$, phase reference, and frame comparison. Each resolved constraint contributes one entropy unit and carries a fundamental action increment

$$\Delta S_{\text{pix}} = \frac{\hbar}{4}. \tag{124}$$

This quarter-action scale is not imposed by hand. It is fixed by the operational degrees of freedom required to define and compare vertex frames, and it matches the normalization required by semiclassical horizon entropy [3, 4, 29].

A null horizon of radius R_Λ carries a two-dimensional surface capacity given by the number of Planck-area angular elements,

$$N_{\text{pix}} = \frac{A_\Lambda}{\ell_P^2} = \frac{4\pi R_\Lambda^2}{\ell_P^2}. \tag{125}$$

The total action associated with the null boundary is therefore

$$S_{\text{tot}} = \Delta S_{\text{pix}}\, N_{\text{pix}} = \frac{\hbar}{4}\,\frac{A_\Lambda}{\ell_P^2} = \frac{\pi\hbar R_\Lambda^2}{\ell_P^2}. \tag{126}$$

Using the Planck-area relation

$$\ell_P^2 = \frac{G\hbar}{c^3}, \tag{127}$$

this becomes

$$S_{\text{tot}} = \frac{\pi R_\Lambda^2 c^3}{G} = \frac{\pi c^5}{G H_\Lambda^2}. \tag{128}$$

This quantity is *action on a null boundary*. It does not represent energy stored in a bulk volume. Energy emerges only after rendering this action into an edge frame with a timelike parameter, where a rate can be defined. At the vertex-frame level, the horizon bookkeeping is purely kinematical and holographic.

21 Rendering Action into Energy

21.1 Which Timescale

Timelike observers measure energies and energy densities, so we convert action to energy via

$$E = \frac{S}{T}. \tag{129}$$

For a de Sitter horizon there are two natural time scales: the Hubble time H_Λ^{-1} and the thermal (KMS) period

$$\beta_\Lambda = \frac{2\pi}{H_\Lambda}. \tag{130}$$

The thermal period is fixed by Euclidean regularity of the static patch and equals the inverse Gibbons–Hawking temperature [18]. Since we are rendering boundary action into a stationary energy scale associated with horizon equilibrium, β_Λ is the relevant time scale.

21.2 Why the Thermal Time β_Λ Is the Natural Scale

A potential point of confusion is the appearance of a *thermal* time scale, β_Λ, in what is otherwise a geometric and kinematical construction. We emphasize that β_Λ is not introduced as a thermodynamic assumption, but arises unavoidably from the structure of null horizons and the absence of any intrinsic time scale on the null substrate.

In the null-first framework, null relations carry no proper time ($d\tau = 0$) and therefore cannot support a primitive notion of energy. The only invariant quantity that can be accumulated along null structure is *action*. Time, energy, and temperature emerge only after rendering null relations into a timelike description appropriate to massive observers.

For spacetimes with a cosmological horizon, this rendering is unique. A causal horizon enforces a finite information-access boundary, and the corresponding observer necessarily experiences a Kubo–Martin–Schwinger (KMS) state with inverse temperature

$$\beta_\Lambda = \frac{2\pi}{H_\Lambda}, \tag{131}$$

where H_Λ is the de Sitter expansion rate. This is not a choice: it is the only time scale that can be constructed from the null geometry of the horizon itself.

Crucially, β_Λ is the *thermal time* associated with horizon closure, not a microscopic cutoff. Planck time does not play this role, because the Planck scale governs *resolution* of action quanta, not the rate at which null structure must be rendered into a consistent timelike evolution. The horizon instead fixes the global pacing of action accumulation visible to the observer.

From the null perspective, the appearance of temperature is therefore a bookkeeping artifact of partial access to null relations. The inverse temperature β_Λ measures the minimal temporal interval required for the observer to resolve independent horizon-crossing events. It is the macroscopic shadow of null closure, in exactly the same sense that horizon area is the geometric shadow of action counting.

This explains why β_Λ, and not an arbitrary cosmological or Planckian timescale, governs the effective energy density associated with the cosmological constant. Once action quanta of size $\hbar$ are rendered at a rate fixed by β_Λ^{-1}, the observed vacuum energy density follows as a consistency condition, not as a dynamical input.

21.3 Spacelike Constraint and Indirect Gravitational Influence

A subtle but important point concerns the role of spacelike-separated structure in gravitational and thermodynamic phenomena. Degrees of freedom beyond a causal horizon do not locally interact with the observer and therefore do not directly gravitate with them in the usual sense: no stress–energy can be exchanged, and no local force is exerted.

However, this absence of direct coupling does not imply irrelevance. Gravity is not solely a local interaction but a global constraint on geometry. Spacelike-inaccessible degrees of freedom remain coupled to degrees of freedom that *do* interact with the observer, and through this coupling they participate indirectly in determining the consistent spacetime geometry.

In particular, horizons impose closure conditions on null structure. Although the observer cannot access the full null graph, the requirement that null relations close consistently across the horizon constrains the admissible timelike rendering of spacetime. This constraint manifests not as a force or energy flux, but as fixed geometric and thermodynamic parameters, such as surface gravity, horizon temperature, and the associated thermal time scale β_Λ.

From this perspective, spacelike-separated structure does not "gravitate on" the observer dynamically, but it does gravitate *geometrically* by fixing the global consistency conditions under which the observer's local physics must operate. The resulting thermal activity is therefore not a consequence of microscopic emission or radiation, but of enforced equilibrium arising from partial access to null structure.

This clarifies why horizon-induced thermal behavior is unavoidable in the null-first framework. Once time is imposed on a system with spacelike-excluded null relations, the observer necessarily experiences thermal activity as the timelike shadow of global geometric constraint.

21.4 Energy and Energy Density

The rendered horizon energy is

$$E_\Lambda = \frac{S_{\text{tot}}}{\beta_\Lambda} = \frac{\pi c^5}{G H_\Lambda^2} \cdot \frac{H_\Lambda}{2\pi} = \frac{c^5}{2 G H_\Lambda}. \tag{132}$$

The emergent static-patch volume enclosed by the horizon is

$$V_\Lambda = \frac{4\pi}{3} R_\Lambda^3 = \frac{4\pi c^3}{3 H_\Lambda^3}. \tag{133}$$

The corresponding energy density is

$$\rho_\Lambda = \frac{E_\Lambda}{V_\Lambda} = \frac{c^5/(2GH_\Lambda)}{4\pi c^3/(3H_\Lambda^3)} = \frac{3H_\Lambda^2 c^2}{8\pi G}. \tag{134}$$

This matches the standard Friedmann form for a Λ-dominated universe, but here it appears as an accounting identity: horizon action (area scaling) rendered into energy (thermal time) and distributed over reconstructed volume.

Theorem 1 (Natural de Sitter Scale from Pixelated Horizon Action)**.** *For an asymptotically de Sitter horizon, assigning $\hbar/4$ of action per Planck-area pixel and rendering over the thermal period $\beta_\Lambda = 2\pi/H_\Lambda$ yields*

$$\rho_\Lambda = \frac{3H_\Lambda^2 c^2}{8\pi G}. \tag{135}$$

No vacuum energy assumption is required.

21.5 Why the 2π Matters

If one uses $T = H_\Lambda^{-1}$ instead of β_Λ, the result is off by a factor of 2π. The 2π is not a fit. It is the same Euclidean regularity and KMS periodicity factor that fixes Hawking and Gibbons–Hawking temperatures in horizon thermodynamics [4, 18].

22 Why the 10^{122} Appears

The QFT estimate is internally consistent given its assumptions. The problem is that those assumptions treat Planck-scale degrees of freedom as independent and local in the bulk. Holography says otherwise [13, 14, 29].

A compact way to see the mismatch is to compare the two scales directly. The pixelated horizon result can be written as

$$\rho_\Lambda = \frac{3H_\Lambda^2 c^2}{8\pi G} \sim \frac{\hbar c}{\ell_P^2 R_\Lambda^2}, \tag{136}$$

using $\ell_P^2 = G\hbar/c^3$ and $R_\Lambda = c/H_\Lambda$. The QFT cutoff estimate scales as

$$\rho_{\text{QFT}} \sim \frac{\hbar c}{\ell_P^4}. \tag{137}$$

The ratio is therefore

$$\frac{\rho_{\text{QFT}}}{\rho_\Lambda} \sim \frac{\hbar c/\ell_P^4}{\hbar c/(\ell_P^2 R_\Lambda^2)} = \frac{R_\Lambda^2}{\ell_P^2} = \left(\frac{R_\Lambda}{\ell_P}\right)^2 \sim 10^{122}. \tag{138}$$

This is the square of the ratio between the horizon scale and the Planck scale. It is also the order of magnitude of the horizon pixel count $N_{\text{pix}} = A_\Lambda/\ell_P^2$.

In this framework, the mismatch is the numerical signature of counting bulk modes when only boundary pixels are fundamental.

23 Friedmann as Global Accounting

Equation (134) can be rewritten as

$$H_\Lambda^2 = \frac{8\pi G}{3}\rho_\Lambda. \tag{139}$$

In the present interpretation, this is a consistency condition tying together three ingredients:

- the area scaling of null boundary degrees of freedom,
- the conversion from action to energy using the de Sitter thermal period,
- the geometric relation between horizon area and enclosed volume.

The standard dynamical derivation from Einstein's equations remains valid. The point here is that the same relation also follows from boundary action bookkeeping once the null structure is treated as fundamental.

24 Expansion as the Geometric Trace of Action Accumulation

Standard cosmology interprets cosmic expansion as the dynamical response of spacetime geometry to matter and energy distributions. The null-first framework reverses this causal relationship. Rather than metric evolution driven by stress-energy, the fundamental process is the accumulation of action through sequential resolution of null relations.

We begin with the geometric interpretation of Newton's constant,

$$G = \frac{\ell_P^2}{\hbar}, \tag{140}$$

which reveals that spatial area serves as a natural bookkeeping quantity for accumulated action. In this view, action—not energy density—is the fundamental invariant.

Event Count and Action Ledger. Consider N_v as the total number of resolved null vertices in an observer's causal past. Each vertex contributes approximately one quantum of action, $s \approx \hbar$, giving a total accumulated action of

$$S = N_v \hbar. \tag{141}$$

The geometric identity encoded in G then yields

$$\Delta A = G\,\Delta S = \ell_P^2\,\Delta N_v. \tag{142}$$

This relationship suggests that cosmological expansion is not a stretching of pre-existing space, but rather the geometric consequence of adding new null relations to the causal network. The universe doesn't expand into a pre-existing vacuum; instead, the vacuum itself emerges as a holographic projection of the growing causal structure.

From this perspective, the Hubble parameter H loses its character as a dynamical driver of expansion. Instead, it quantifies the rate at which new events are resolved relative to the existing causal set, determining how rapidly new geometric area must be generated to preserve consistency.

25 Null Closure, Energy, and the Emergence of the Friedmann Scaling

This section presents a closed kinematical derivation of the cosmological density–scale relation without assuming an expansion rate as a primitive input. The only geometric datum is a finite null screen of areal radius R, carrying a finite angular resolution capacity at Planck area. All dynamical quantities emerge from consistency conditions on null resolution.

25.1 Angular Capacity of a Null Screen

Let the null screen be a compact spacelike cross section with areal radius R and area

$$A = 4\pi R^2. \tag{143}$$

Under Planck area resolution, the number of independent angular pixels is

$$N_{\rm pix} = \frac{A}{\ell_P^2} = \frac{4\pi R^2}{\ell_P^2}. \tag{144}$$

25.2 Resolution Rate and the Energy Identity

Let Γ denote the null resolution rate, defined as the number of resolved action quanta per unit reconstructed time. Resolution occurs in discrete units $\Delta S \simeq \hbar$. If $S(t)$ denotes the total resolved action in a causal region, then

$$\frac{dS}{dt} \simeq \Gamma\hbar. \tag{145}$$

Independently, by definition of action,

$$S = \int E(t)\,dt \qquad \Longrightarrow \qquad \frac{dS}{dt} = E. \tag{146}$$

Equating (145) and (146) yields

$$\Gamma = \frac{E}{\hbar}. \tag{147}$$

Thus the null resolution rate is not an independent parameter but is fixed directly by the energy content of the region.

25.3 Stationarity and Terminal Surface Capacity

Stationarity of null reconstruction requires that, over one characteristic reconstruction timescale β, the number of resolved action quanta saturates the available *exterior* capacity of the null screen associated with a vertex frame.

The null screen is a two-dimensional surface encoding angular data (θ, ϕ, ψ) and phase comparison structure. While the total number of Planck-scale angular elements on this screen is

$$N_{\mathrm{pix}} = \frac{4\pi R^2}{\ell_P^2}, \tag{148}$$

not all resolved action quanta contribute independently to the exterior causal boundary.

As established in Section 14, a complete vertex-to-vertex null cycle requires four independent constraint resolutions: null direction, twist, phase, and frame comparison. Only one of these four degrees of freedom corresponds to an independent *surface-accessible* constraint at the terminal null screen. The remaining degrees of freedom are consumed internally in enforcing consistency between past, future, and adjoint vertex structures.

As a result, the effective exterior capacity of the null screen is reduced by a factor of four. The stationarity condition therefore requires that the total number of resolved action quanta over the reconstruction interval β matches the surface-accessible constraint capacity:

$$\Gamma\,\beta = \frac{N_{\mathrm{pix}}}{4}. \tag{149}$$

Substituting (147) and (144) into (149) yields

$$\frac{E}{\hbar}\,\beta = \frac{1}{4}\,\frac{4\pi R^2}{\ell_P^2} = \frac{\pi R^2}{\ell_P^2}. \tag{150}$$

25.4 Elimination of $\hbar$

Using the Planck-area relation

$$\ell_P^2 = \frac{G\hbar}{c^3}, \tag{151}$$

equation (150) becomes

$$E\,\beta = \frac{\pi R^2 c^3}{G}. \tag{152}$$

This relation is purely kinematical. It depends only on surface capacity, constraint counting, and null geometry, and makes no reference to any expansion rate, equation of state, or dynamical field equation.

25.5 Energy Density at Fixed Closure Radius

Define the energy density ρ_E in the causal region as

$$\rho_E \equiv \frac{E}{V}, \tag{153}$$

where the associated causal volume is taken as

$$V = \frac{4\pi}{3} R^3. \tag{154}$$

Dividing (152) by (154) yields

$$\rho_E\, \beta = \frac{\pi R^2 c^3}{G} \cdot \frac{3}{4\pi R^3} = \frac{3c^3}{4GR}. \tag{155}$$

25.6 Thermal Time of the Null Screen

A null screen carries a natural thermal timescale associated with its surface gravity. For a spherical null screen of radius R, the intrinsic reconstruction timescale is

$$\beta = \frac{2\pi R}{c}. \tag{156}$$

Substituting (156) into (155) gives

$$\rho_E = \frac{3c^3}{4GR} \cdot \frac{c}{2\pi R} = \frac{3c^4}{8\pi G R^2}. \tag{157}$$

Expressed as a mass density $\rho_M = \rho_E / c^2$, this becomes

$$\rho_M = \frac{3c^2}{8\pi G R^2}. \tag{158}$$

25.7 Derived Definition of the Expansion Rate

For comparison with standard cosmological notation, one may define

$$H \equiv \frac{c}{R}. \tag{159}$$

This is not an input but a label for the null closure scale. Equation (157) then takes the familiar Friedmann form

$$\rho_E = \frac{3H^2 c^2}{8\pi G}. \tag{160}$$

25.8 Spectral Closure and the 8π Identity

The null closure scale can also be encoded spectrally. Modeling the null screen as the sphere S^2_R, the Laplace–Beltrami eigenvalues are

$$\lambda_\ell = \frac{\ell(\ell+1)}{R^2}, \tag{161}$$

so the spectral gap is

$$\lambda_1 = \frac{2}{R^2}. \tag{162}$$

Combining (162) with (144) yields the exact identity

$$\lambda_1\, \ell_P^2\, N_{\text{pix}} = 8\pi. \tag{163}$$

This relation expresses a fixed conversion between spectral rigidity of the null screen and its finite angular capacity. The appearance of the factor 8π matches the normalization appearing in the Einstein field equations, but arises here purely from kinematical closure rather than dynamical coupling.

25.9 Interpretation and Scope

The derivation above determines the functional relation between energy density, closure radius, and reconstruction timescale without assuming an expansion rate or field equations. The numerical value of the closure radius R is a boundary condition reflecting the total energy content of the causal patch. This is analogous to the role of initial data in general relativity or state selection in quantum mechanics.

The result shows that once a stationary null screen exists, finite angular capacity and action quantization force the scaling $\rho \propto 1/(GR^2)$ and fix the normalization exactly. Dynamics and evolution of the energy content are addressed separately.

26 A Complete Derivation of the Cosmological Constant Scale

This section presents a compact, self-contained derivation of the observed cosmological constant energy density from null-first horizon bookkeeping. No dynamical field equations or vacuum mode sums are assumed.

Geometric inputs

For a de Sitter horizon with Hubble parameter H_Λ, the horizon radius is

$$R_\Lambda = \frac{c}{H_\Lambda}. \tag{164}$$

The horizon area and enclosed volume are

$$A_\Lambda = 4\pi R_\Lambda^2, \qquad V_\Lambda = \frac{4\pi}{3} R_\Lambda^3. \tag{165}$$

Horizon pixelation and action

We discretize the horizon into Planck-area pixels,

$$N_{\text{pix}} = \frac{A_\Lambda}{\ell_P^2}, \tag{166}$$

and assign a fundamental action increment $\Delta S_{\text{pix}} = \hbar/4$ per pixel. The total horizon action is therefore

$$S_\Lambda = N_{\text{pix}} \frac{\hbar}{4} = \frac{\hbar}{4} \frac{4\pi R_\Lambda^2}{\ell_P^2} = \pi\hbar \frac{R_\Lambda^2}{\ell_P^2}. \tag{167}$$

Using the Planck-area identity $\ell_P^2 = \hbar G/c^3$, this becomes

$$S_\Lambda = \pi \frac{c^3}{G} R_\Lambda^2 = \pi \frac{c^5}{G H_\Lambda^2}. \tag{168}$$

Rendering action into energy

Energy is reconstructed as action per unit time. For a stationary de Sitter horizon, the appropriate timescale is the thermal (Euclidean) period

$$\beta_\Lambda = \frac{2\pi}{H_\Lambda}. \tag{169}$$

The associated horizon energy is

$$E_\Lambda = \frac{S_\Lambda}{\beta_\Lambda} = \left(\pi \frac{c^5}{G H_\Lambda^2}\right)\left(\frac{H_\Lambda}{2\pi}\right) = \frac{c^5}{2 G H_\Lambda}. \tag{170}$$

Energy density

Dividing by the enclosed horizon volume yields the energy density

$$\varepsilon_\Lambda = \frac{E_\Lambda}{V_\Lambda} = \frac{\frac{c^5}{2GH_\Lambda}}{\frac{4\pi}{3}\left(\frac{c}{H_\Lambda}\right)^3} = \frac{3c^2 H_\Lambda^2}{8\pi G}. \tag{171}$$

Equivalently, the corresponding mass density is

$$\rho_\Lambda = \frac{\varepsilon_\Lambda}{c^2} = \frac{3H_\Lambda^2}{8\pi G}. \tag{172}$$

26.1 Remarks

This result is obtained without summing vacuum modes or introducing free parameters. The scale of the cosmological constant follows from horizon area counting, action quantization, and the stationary thermal timescale of the de Sitter horizon.

Part IV

Conclusions

27 Positioning Relative to Existing Work

It is useful to distinguish the present framework by its choice of primitive variables. In Regge-style discretizations, edge lengths are primary and curvature is encoded by deficit angles [33]. In loop-quantum-gravity programs, discrete area spectra arise from representation labels on graphs and the fundamental kinematics is formulated in terms of connection/holonomy variables and fluxes [34, 35, 36]. In causal-set programs, order and counting are taken as primitive and spacetime volume emerges from element counts [19, 37, 21]. By contrast, the null-first picture takes *null relations* and their *projective angular data* as primary, following the classical analysis of null hypersurfaces and asymptotic structure [10, 11, 9, 38].

A central organizing principle is that metric quantities (lengths, volumes, energies) appear only after closure and timelike reconstruction. In this sense, the present construction is not a discretization of an already-metric spacetime, but a kinematical ordering in which null boundary structure is specified first, and reconstructed bulk quantities are secondary descriptions constrained by boundary consistency. The comparison points below are therefore meant as points of contact, not claims of equivalence.

27.1 Null hypersurfaces, characteristic evolution, and the "metric from null data" ethos

The emphasis on null structure aligns naturally with the characteristic (null) initial value formulation of general relativity, where free data is specified on null hypersurfaces and the bulk geometry is reconstructed by propagation along null generators [11, 39]. Related null-surface formalisms also treat families of null hypersurfaces as fundamental and view the metric as derivative data constrained by integrability and consistency conditions [9]. The present work can be read as a discretized kinematical analogue of this viewpoint: null boundary relations are primitive, while the metric is a reconstructed bookkeeping device that becomes meaningful only after a timelike rendering map is defined.

27.2 Carrollian geometry and the intrinsic structure of null boundaries

A null hypersurface carries a degenerate intrinsic geometry: a preferred null direction together with an induced (rank-two) metric on spatial cross-sections. This naturally leads to *Carrollian* structures as the appropriate intrinsic kinematics on null boundaries and null infinity, where the causal structure is "frozen" along the null direction while angular geometry remains nondegenerate [40, 15, 22, 23]. In this language, the null-first postulate that "angles are primary while timelike reconstruction is secondary" can be viewed as taking the Carrollian boundary data as fundamental and regarding Lorentzian bulk geometry as emergent from consistency constraints and reconstruction. We emphasize that we do not assume a particular Carrollian connection or boundary dynamics here; we only use the structural fact that null boundaries support canonical angular data and preferred null generators.

27.3 BMS symmetry, soft structure, and celestial viewpoints

At null infinity, the asymptotic symmetry group is the Bondi–Metzner–Sachs (BMS) group, whose action organizes gravitational radiation, memory effects, and the identification of physically meaningful "cuts" of $\mathscr{I}$ [10, 11, 31]. Recent developments in celestial holography further emphasize that scattering data and radiative degrees of freedom admit a natural description on the celestial two-sphere, with conformal weights encoding energy/frequency information and spin weights encoding helicity [30, 31]. The present framework shares the same organizing emphasis: angular (celestial) data is primary, while energy and time scales appear only after a rendering map selects an appropriate timelike parameterization (e.g. via thermal/KMS periods for stationary horizons). We do not assume the full celestial

CFT program; we note the conceptual compatibility in treating the S^2 of null directions as the natural state space for kinematics.

27.4 Twistor theory as a null-primitive coordinate system

Twistor theory was introduced precisely to reformulate spacetime physics in terms of null structure: points in spacetime arise as derived objects, while twistors encode null directions together with phase/spinorial data [41, 27, 42]. In Minkowski space, projective spinors parameterize null directions, and the incidence relations encode how null rays intersect to produce spacetime events. From this perspective, "null primacy" is not an idiosyncratic choice but a well-established alternative kinematical foundation. The present work does not import the complex-analytic machinery of twistor geometry; rather, it uses a discretized action-counting substrate whose primitive observables are (i) null relations and (ii) projective angular data on an S^2. Nevertheless, the shared geometric spine is clear: both approaches treat null rays/sheets as fundamental and regard metric spacetime as reconstructed structure.

27.5 Thermodynamic and holographic gravity

The present reconstruction perspective is compatible with thermodynamic and holographic approaches to gravity [12, 6, 29]. Jacobson derives Einstein's equations from $\delta Q = T\delta S$ applied to local Rindler horizons [12], while Padmanabhan emphasizes horizon degrees of freedom and holographic equipartition in cosmology [6]. Holographic dark energy models motivate $\rho \propto 1/(GR^2)$ using an infrared cutoff scale [43]. The present framework differs in that the relevant time scale and normalization are fixed by (i) the pixelated horizon action bookkeeping and (ii) the stationary KMS/thermal period, rather than introduced phenomenologically. In particular, the de Sitter energy density scale is obtained without assigning independent vacuum energy to bulk degrees of freedom; the large QFT discrepancy is traced to a counting measure that ignores holographic constraints.

27.6 Relation to Loop Quantum Gravity, Spin Foams, and Tensor Networks

It is useful to situate the null-first framework relative to other leading approaches to quantum gravity that employ discrete structures. The intent here is not to claim equivalence, but to clarify points of contact and sharp distinctions at the level of kinematical primitives.

Loop quantum gravity (LQG). Canonical loop quantum gravity formulates quantum geometry on graphs whose edges carry holonomies of an $\mathrm{SU}(2)$ connection and whose dual faces carry flux variables. Discreteness enters through representation labels, yielding discrete spectra for area and volume operators. Geometry is therefore encoded in group-theoretic data assigned to graph elements.

The null-first framework is also graph-based, but the primitive labels differ. Vertices represent null events rather than spacetime points, and edges represent null-consistent relations rather than timelike worldlines. Each vertex carries projective angular data $(\theta, \phi, \psi) \in S^2 \times U(1)$ together with an entropy/action count. The appearance of a quarter-action unit $\hbar/4$ is not attributed to representation theory but to the minimal operational data required to specify and compare null relations. Area and action units are therefore fixed by null closure and spinorial structure rather than by choosing spin labels.

In this sense, the null-first approach is compatible with the graph-based spirit of LQG while differing in what is taken as fundamental: angular null data and action bookkeeping rather than connection and triad variables.

Spin foams. Spin foam models may be viewed as covariant histories of spin networks, in which a sum over labeled two-complexes provides a path-integral formulation of quantum geometry. The null-first framework admits a closely related but conceptually distinct structure.

Here, the path integral is interpreted as a sum over globally admissible completions of a partially specified null-vertex constraint network. Virtual quanta correspond to unresolved causal links, and superposition reflects causal incompleteness rather than fluctuating geometry. The sum is therefore not over bulk simplicial geometries, but over null-consistent vertex-to-vertex compatibility assignments. This plays a role analogous to a spin-foam sum, while remaining rooted in null kinematics rather than discretized bulk metrics.

Tensor networks and holographic constructions. Tensor network approaches to quantum gravity and holography emphasize that bulk geometry can emerge from the pattern of connections among boundary degrees of freedom. In such models, geometry is often interpreted as an effective description of entanglement structure.

The null-first framework shares the core architectural idea that bulk structure is secondary to boundary capacity. Spacetime volume emerges from the closure of null constraints subject to local holographic bottlenecks, with surface capacity scaling as area and volumetric growth scaling cubically. However, the present approach does not begin with entanglement as a primitive. Instead, the bottleneck structure follows directly from null geometry and action counting, with entanglement appearing only after timelike reconstruction.

Summary of the comparison. Loop quantum gravity, spin foams, and tensor networks all employ discrete structures to tame gravitational degrees of freedom. The null-first framework differs one level earlier in the hierarchy: it identifies null relations and their angular/phase structure as the primitive data, with metric geometry, energy, and dynamics arising only after constraint closure and timelike rendering. The points of contact outlined above indicate that the framework is not isolated from existing approaches, but occupies a complementary position focused on null kinematics rather than bulk discretization.

27.7 Summary of the distinguishing choice

The distinguishing choice is thus not a new discretization of metric geometry, but a re-ordering of kinematics: null boundary structure and projective angular data are taken as primitive; metric quantities, energy, and timelike evolution are reconstructed secondary descriptions constrained by closure and consistency. This ordering places the framework in direct contact with the characteristic/null-surface tradition, Carrollian boundary kinematics, BMS/celestial structures, and the twistor program, while remaining operationally minimal in the present (kinematical) paper.

28 Closing Remarks

This paper argued for a simple but strict reordering of what is taken as fundamental in spacetime physics. Null structure is primary. Timelike spacetime, energy, volume, and dynamics are reconstructed descriptions that become available only after null relations are organized into consistent geometric closures.

The core move was kinematical rather than dynamical. We did not assume Einstein's equations, vacuum energy, or a microscopic field theory in the bulk. Instead, we asked what follows if null geometry is treated as an ontological boundary condition, with angular data, action, and causal connectivity as the primitive ingredients. From that starting point, several otherwise disconnected results line up with little additional input.

First, null geometry supports angular but not metric information. This singles out the celestial sphere as the intrinsic carrier of degrees of freedom, consistent with classical analyses of null hypersurfaces and asymptotic structure [9, 11, 10]. Once this is accepted, area rather than volume becomes the natural counting measure, and holographic scaling is no longer an added principle but a direct consequence of null degeneracy, in line with earlier insights by 't Hooft [13] and Susskind [14].

Second, Newton's constant reveals its meaning when written as

$$G = \frac{\ell_P^2 c^3}{\hbar}.$$

In this form, G encodes three independent facts at once: holography, through the equivalence of bulk and boundary action; dimensionality, through the c^3 volumetric construction rate from three null directions; and universality, through the fixed action cost $\hbar$ per fundamental angular degree of freedom. This perspective aligns with and sharpens thermodynamic approaches to gravity, such as those of Jacobson [12] and Padmanabhan [6], by supplying a concrete kinematical substrate.

Most importantly, the cosmological constant ceases to be mysterious. When action, not energy, is taken as primitive, and when degrees of freedom are counted on null boundaries rather than in bulk volume, the observed value of ρ_Λ follows directly. The factor of 10^{122} is not a fine-tuning problem but the number of Planck-area angular pixels on the asymptotic de Sitter horizon. Rendering their total action into energy using the unique thermal timescale fixed by the KMS condition and Euclidean regularity [18] reproduces the Friedmann form

$$\rho_\Lambda = \frac{3H_\Lambda^2 c^2}{8\pi G}$$

with no adjustable parameters. In this sense, dark energy is not vacuum energy stored in spacetime but the baseline projection cost of maintaining spacetime itself.

This picture connects naturally to, but is distinct from, other holographic and emergent gravity proposals. Unlike phenomenological holographic dark energy models [43], no infrared cutoff or fitted coefficient is introduced. Unlike causal set approaches with sequential growth [20], temporal ordering is not taken as primitive at the null level but arises only after timelike reconstruction. The framework also complements loop and spin network ideas by fixing the area quantum directly through geometric closure rather than representation labels.

The scope of this paper has been deliberately limited. We have not derived the full Einstein field equations, nor explained why the universe has its particular asymptotic entropy or expansion rate. Those questions belong to dynamics, not kinematics, and are addressed in a companion work where the evolution of angular phase networks is studied. What has been established here is that the null-first kinematical structure is internally consistent, tightly constrained, and already sufficient to account for several deep numerical facts about gravity and cosmology.

The broader lesson is methodological. Many of the hardest problems in fundamental physics appear insoluble when framed in terms of energy, volume, and local bulk degrees of freedom. When reframed in terms of null geometry, action, and boundary data, those same problems often collapse into bookkeeping identities. The cosmological constant problem is the clearest example, but the same perspective also clarifies inverse-square laws, horizon entropy, and the universality of gravitational coupling.

If future observations were to establish a deviation from $w = -1$ for dark energy, or a breakdown of holographic entropy bounds, this framework would be falsified. Short of that, null-first kinematics offers a coherent and economical foundation on which a full theory of spacetime dynamics can be built.

Part V

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