Null-First Discrete Angular Kinematics

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Abstract

"We are very much aware that we are exploring unconventional ideas and that there may be some basic flaw in our whole approach which we have been too stupid to see."

Sidney Coleman and Erick Weinberg (1973)

Newton's gravitational constant encodes holography directly: $G = \ell_P^2/\hbar = c^3/\hbar$ states that $\hbar$ of action creates both one Planck area of boundary *and* the corresponding bulk volume at rate c^3. These being equal means bulk spacetime carries no independent degrees of freedom.

Taking null geometry as fundamental, we show that photons are not propagating particles but vertices where null relations begin and end. Spacetime emerges as the reconstructed geometry of these connections. The primitive invariant is action, with $E = S/T$ defined only once a timescale becomes available.

This dissolves the cosmological constant problem. Quantum field theory's estimate $\rho_{\text{QFT}} \sim \hbar c/\ell_P^4$ counts bulk modes, violating holography by $(R_\Lambda/\ell_P)^2 \approx 10^{122}$—the number of horizon pixels. Counting holographically: $N_{\text{pix}} = 4\pi R_\Lambda^2/\ell_P^2$ pixels each carrying $\hbar/4$ (from black hole entropy), rendered over thermal time $\beta_\Lambda = 2\pi/H_\Lambda$, yields

$$\rho_\Lambda = \frac{3H_\Lambda^2 c^2}{8\pi G} \tag{1}$$

exactly, with no free parameters. Dark energy is not stored in spacetime. It is the cost of projecting spacetime from its null substrate. The 10^{122} discrepancy was a counting error.

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Part I

Conceptual Foundations

1 Core Thesis and Scope

This paper advances a single kinematical claim: null structure is fundamental, while spacetime geometry is emergent.

The results developed here focus on kinematics rather than dynamics. We do not derive the Einstein field equations or propose a new law of motion. However, because null geometry is inherently kinematic, we can do more with this geometry than would naturally be expected using only kinematics. Instead, we show that when null geometry is treated exactly as special relativity says, with a length-less and timeless existence, several central features of gravitational physics follow from geometric consistency alone. These include the inverse square scaling of forces, the area law for horizon entropy, and the observed magnitude of the cosmological constant.

Long standing puzzles in gravity and cosmology suggest that spatial volume and energy density may not be the correct primitive quantities in regimes dominated by null structure. Examples include Newtonian gravity [1, 2], black hole entropy [3, 4], and the cosmological constant problem [5, 6]. In each case, the relevant physics is controlled by null boundaries rather than bulk volumes.

In null geometry, intrinsic temporal and radial separation collapse. Proper time vanishes along null relations, and no intrinsic length can be assigned in the direction of propagation [7, 8]. What remains well defined is angular and projective structure. The space of physically distinct null directions at a point forms a celestial sphere, and this angular data survives exactly where spacetime intervals degenerate [9, 10, 11].

The central organizing principle of this paper is that null geometry carries angular but not volumetric information. As a result, area rather than volume provides the natural measure, and action rather than energy provides the natural invariant. Energy emerges only after a temporal scale becomes available through timelike reconstruction.

This perspective is consistent with earlier ideas about emergent spacetime and holography [12, 13, 14], but differs in emphasis. Here, emergence is grounded directly in the geometry of null relations rather than in thermodynamic or entropic postulates.

Unlike conventional frameworks where the metric defines causal structure, we reverse this dependency: the causal structure determines the emergent metric structure. As such, traditional tools like Penrose diagrams or line elements play no foundational role here.

2 Why This Matters

Before developing the technical framework, it is worth stating clearly what changes when null geometry is taken as fundamental.

2.1 Photons Don't Move—We Are Passed Between Them

In the conventional picture, photons are particles emitted at one event, traveling through spacetime, and absorbed at another. This description requires an already-existing spacetime manifold through which the photon propagates. But this is backwards.

Photons do not move through space. They *are* space—or more precisely, they are the null relations that constitute spacetime's geometric substrate. When a charged particle at event A interacts electromagnetically with a particle at event B, the photon is not an object traveling from A to B. It is the null connection linking A and B directly, with zero proper time and undefined spatial separation along the connection itself.

What we perceive as motion is the timelike worldline of a massive particle being handed from one null vertex to the next. Between vertices, the particle exists as an edge in the causal network—a reconstructed timelike interval built from the angular matching of null directions. The sensation of continuous motion through space is the macroscopic limit of discrete vertex-to-vertex transitions, just as the sensation of smooth geometry is the macroscopic limit of discrete area quanta.

This is not a metaphor. It is a strict consequence of taking null geometry seriously. Along a null relation, $d\tau = 0$ identically, and intrinsic distance in the propagation direction is undefined [8]. The null relation has no internal structure—no ticking clock, no meter stick, no evolving degrees of freedom. It is a pure geometric connection, instantaneous in its own context, linking events that would otherwise be causally disconnected.

Timelike observers reconstructing this geometry assign coordinates, measure wavelengths, and calculate frequencies. But these are descriptions *of* the reconstruction, not properties *of* the null relation itself. The photon, intrinsically, is timeless and extensionless. Spacetime emerges when collections of such relations close consistently into volumetric elements.

2.2 Energy is Not Stored—It is the Cost of Projection

In conventional quantum field theory, energy density is treated as a property of spacetime itself. The vacuum is assigned an energy per unit volume, fields carry energy, and particles represent localized energy excitations. The cosmological constant problem arises because summing these contributions naively gives $\rho \sim \hbar c/\ell_P^4$, wildly exceeding observation.

But if spacetime is emergent rather than fundamental, this entire framing is wrong. Energy cannot be stored in spacetime because spacetime is what energy *creates*. More precisely: $\hbar$ is the action cost required to project one Planck-scale pixel from the null substrate into reconstructed timelike geometry.

Consider the identity $G = \hbar/c^3$ in natural units where $\ell_P = 1$. Rearranging:

$$\hbar = c^3 G. \tag{2}$$

The right side has dimensions of volume per time (the rate at which three null directions build volume) times the gravitational coupling. The meaning: $\hbar$ is the action quantum associated with generating one unit of spacetime volume per unit time. It is the projection cost.

Every Planck volume in reconstructed spacetime requires $\hbar$ of action to exist. This is not energy sitting *in* the volume—it is the ongoing energetic cost of *maintaining* the volume. When we measure dark energy density ρ_Λ, we are not detecting energy stored in empty space. We are measuring the projection cost per unit volume, rendered into energy units using the natural timescale of the cosmological horizon.

This reinterpretation has immediate consequences:

- **No vacuum energy problem:** QFT calculates the energy of fields in an already-projected spacetime. But projection cost is not the same as field energy. The two are categorically distinct.
- **Holographic scaling is mandatory:** If bulk volume is projected from boundary data, then the action cost must scale with boundary area, not bulk volume. This is not an added principle—it follows from the projection architecture.
- **The cosmological constant is kinematical:** ρ_Λ is not a dynamical parameter to be explained by field theory. It is the conversion factor between action (intrinsic to null boundaries) and energy density (emergent in timelike bulk), fixed by horizon geometry and thermal timescales.

2.3 G is Not a Coupling—It is an Equivalence Statement

Newton's constant is usually introduced as the strength of gravitational interaction. But the form $G = \ell_P^2 c^3/\hbar$ suggests a different interpretation.

Write it two ways:

$$G = \frac{\ell_P^2}{\hbar} \quad \longrightarrow \quad \text{“}\hbar \text{ of action per Planck area of boundary”} \tag{3}$$

$$G = \frac{c^3}{\hbar} \quad \longrightarrow \quad \text{“}\hbar \text{ of action per } c^3 \text{ volumetric projection rate”} \tag{4}$$

These being equal is the statement:

$$\boxed{\text{Bulk volume and boundary area have identical action budgets.}} \tag{5}$$

This is holography, encoded in the fundamental constants since Newton. We have been staring at the proof that spacetime is boundary-determined for over three centuries.

The factor c^3 has a clean geometric meaning. Volume is built from three independent null directions. If each advances at c, then $dV/dt \sim c^3$. The identity $G = c^3/\hbar$ means the action required to sustain this volumetric growth rate equals the action to populate the enclosing boundary. There are no independent bulk modes.

2.4 The $\hbar/4$ is Not Mysterious—It is Minimal Closure

Black hole entropy is $S_{\text{BH}} = A/(4\ell_P^2)$ in units where $k_B = 1$. The factor of $1/4$ has been regarded as requiring explanation—why not $1/2$, or $1/\pi$, or some other coefficient?

The null-first answer is geometric. A closed volumetric element requires at minimum four boundary faces (tetrahedral topology). If action is distributed over boundary pixels, and the number of faces equals the number of independent angular matching conditions, then consistency requires equal action per face. For total voxel action $\hbar$, each of four faces contributes $\hbar/4$.

This is not a derivation of $\hbar$ or a dynamical principle. It is a consistency condition: *given* $\hbar$ as the action quantum and *given* that minimal closure is tetrahedral, the distribution is forced. The factor of $1/4$ tracks back to the Euler characteristic of the sphere and the minimum number of faces for a closed 3D boundary.

The same $\hbar/4$ appears in black hole entropy, cosmological constant calculations, and Newtonian gravity because all are counting the same thing: action quanta on null boundaries, rendered into observable quantities through timelike reconstruction.

2.5 Scope and Falsifiability

This paper is deliberately limited to kinematics. We do not derive Einstein's equations, specify a Lagrangian, or propose modified dynamics. The claim is narrower and more specific: *if* null structure is fundamental and spacetime is emergent, *then* several major scaling relations in gravitational physics follow from geometric consistency alone, including the normalization of the cosmological constant.

The framework is falsifiable. It predicts:

- Dark energy equation of state is exactly $w = -1$ (pure cosmological constant, no evolving scalar field)
- Horizon entropy universally obeys $S = A/(4\ell_P^2)$ with no corrections at leading order
- Holographic scaling $N_{\text{dof}} \propto A/\ell_P^2$ applies to all causal diamonds
- Deviations from $\rho_\Lambda = 3H_\Lambda^2/(8\pi G)$ would falsify the projection-cost interpretation

If future observations establish $w \neq -1$ or find violations of area scaling for entropy, the framework fails. Until then, it offers a resolution to the cosmological constant problem that requires no fine-tuning, no new fields, and no modification of known physics—only a reordering of what is taken as ontologically prior.

3 The Null-First Ontology

A common objection to null-centered formulations is that one cannot Lorentz boost to a photon's rest frame. Lorentz transformations become singular as $v \to c$, and no inertial frame exists in which a photon is at rest. Questions about "what a photon experiences" are therefore dismissed as ill-posed or meaningless.

We take the opposite view. The singularity of the Lorentz boost at $v = c$ is not a barrier to understanding massless particles. It is a complete description of what they are.

Consider what special relativity actually predicts at $v = c$. Time dilation becomes infinite: $\gamma = (1 - v^2/c^2)^{-1/2} \to \infty$. No proper time elapses along a null worldline, $d\tau = 0$. Length contraction is total: spatial extension vanishes in the direction of motion. The entire trajectory from emission to absorption, which may span billions of light-years from a timelike perspective, collapses to zero interval from the null perspective.

This is not an approximation or a limiting case. This is what massless particles *are*. They exist in a regime where the fundamental machinery of timelike physics does not operate. There are no clocks to tick, no rulers to measure with, no temporal parameter to define evolution. From the perspective of the null trajectory itself, if we may use such language, existence is eternal and static. Infinite time dilation means the "lifetime" of a photon is infinite relative to its own null parameter. Total length contraction means it occupies no extended region. There is no "before" or "after" along a null worldline because there is no timelike separation.

But this regime is not structureless. While all metric distances collapse, angular relationships do not. The null limit preserves directions, polar coordinates, and spherical angular data [15]. A photon carries no information about radial scale, but it carries complete information about orientation: the angles (θ, ϕ, ψ) specifying direction and polarization, helicity h, frequency ω (which appears as an angular phase rate), and time itself reinterpreted as an angle conjugate to energy.

Everything except radius survives. This is precisely the data massless particles carry. Photons have polarization states and helicity but no rest mass. Gluons carry color charge and spin but no proper time. Gravitons encode spin-2 angular structure but propagate on null geodesics. All of these are angular properties. The observational fact that massless particles carry exactly the information that survives the null limit is not a coincidence. It is direct evidence that the null limit describes their actual ontology.

Nothing can boost into the null regime because the null regime is the projector that renders the boosts and just like a projector, a projection cannot become the projector. The boosts presuppose the existence of clocks, rulers, and temporal evolution. These do not exist at $v = c$. The mathematical singularity marks a genuine physical boundary between categorically distinct regimes [16]. The null surface is not "almost reachable" by increasing velocity. It is fundamentally different in kind.

This difference has precise mathematical structure. Timelike physics is governed by the Poincaré group, which includes translations, rotations, and boosts. Null physics is governed by the conformal boundary structure, which includes BMS transformations and asymptotic symmetries [17]. These are not the same symmetry group. The singularity of the boost is the boundary between these regimes.

Consider the implications. If massless particles inhabit a regime with infinite time dilation and zero proper time, they are, in a precise sense, eternal. They do not evolve. They do not change. They simply are. This is the defining characteristic of a vertex in a causal graph: a point-like event with no internal duration. Massive particles, by contrast, trace out worldlines with nonzero proper time. They evolve, they age, they have internal dynamics. These are edges connecting vertices.

This suggests a reversal of the usual picture. Massless particles are not objects moving through spacetime. They are causal vertices, the timeless events that define spacetime's structure. Massive particles are timelike edges connecting these vertices. When a photon is "emitted" or "absorbed," no entity enters or exits the null regime. A timelike worldline simply intersects a null relation at a vertex. What propagates through spacetime is the massive system, tracing an edge from one vertex to the next.

The null relations themselves constitute the causal structure. This is not circular. We are not deriving "null" from a pre-existing metric. We are stating that causal connectedness is primitive, and null relations are the mathematical expression of this connectedness. The light cone structure, which defines what can influence what, is built from null surfaces. Timelike and spacelike separations are derived concepts, defined relative to this null structure.

This applies to any particle propagating at c because it is massless. The ontology does not depend on spin, charge, or gauge group. It depends only on the kinematic fact $v = c$, which is equivalent to $m = 0$. Photons, gluons, and gravitons are all null relations. They all correspond to causal vertices. The fact that gravitons *are* metric perturbations, the dynamical degrees of freedom of spacetime itself, strengthens this picture. If gravitons are null relations and gravitons constitute spacetime dynamics, then null structure is not embedded in spacetime. It *is* spacetime at the fundamental level.

The standard picture, in which massless particles propagate between emission and absorption, is not wrong. It is a timelike reconstruction. From the perspective of a massive observer, photons appear to travel and we track their progress with an affine parameter. But this appearance is produced by projecting the vertex structure onto a timelike worldline. What we observe as a photon "traveling" is the interval between two vertices along our worldline. The photon itself, the null relation, experiences no travel because it experiences no time. A key distinction we make is, the affine parameter is not a property of the photon, but a property of timelike observers.

This viewpoint aligns with Carrollian limits of relativity and null boundary formulations [16, 15, 9]. It treats null geometry as a regime with its own intrinsic structure rather than as a degenerate limit of timelike physics.

4 The Geometric Identity of Gravity

This null-first ontology has profound implications for the fundamental constants of nature. The gravitational constant G, typically regarded as a coupling strength between mass-energy and spacetime curvature, can be understood instead as a *geometric identity* that encodes the area-volume duality inherent in the null construction of spacetime.

Consider the standard expression for G in terms of Planck units:

$$G = \frac{\ell_P^2 c^3}{\hbar} \tag{6}$$

where ℓ_P is the Planck length. This expression is not merely a dimensional convenience—it reveals a fundamental identity. Interpret ℓ_P^2 as an elementary area, a fundamental "pixel" at the Planck scale associated with the surface area of

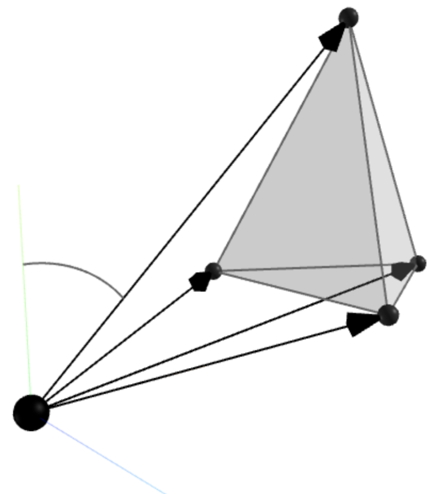

Figure 1: Tetrahedral voxel geometry emerging from null vertex structure. A null vertex at the origin (black sphere) projects along four null directions (black vectors) to define the corners of a minimal tetrahedral voxel. Gray edges show the emergent tetrahedron, while the gray arc indicates the angular coordinate φ (azimuthal) parametrizing one of the null directions.

a null relation. Interpret c^3 as a volumetric propagation rate—the rate at which null surfaces sweep out causal volume. Then G expresses a precise relationship between area and volume mediated by the quantum of action $\hbar$:

$$G = \frac{\text{area}}{\hbar} = \frac{\text{volumetric rate}}{\hbar} \tag{7}$$

This identity states that the quantum action cost to "create" or "account for" a Planck-scale voxel is identical whether calculated from its boundary area or from the volumetric rate at which null surfaces generate spacetime. Area and volume are dual aspects of the same quantum geometric process.

This duality is not an arbitrary mathematical coincidence but reflects the null-first ontology: the fundamental degrees of freedom are not volumetric but *angular*, residing on the boundaries where null relations intersect. These angular degrees of freedom—the directional, polar, and spherical data that survive the null limit—point to the vertices of the voxel that is filled as emergent volume. Counting by surface area yields holographic scaling, while counting by volume (as in standard quantum field theory) yields the incorrect scaling that leads to the cosmological constant problem. This will be explored in detail in Part III.

5 Action Primacy, Energy Reconstruction, and the Null Sector

Energy is defined through time. In the absence of an intrinsic temporal parameter, energy cannot serve as a primitive quantity. Action, by contrast, remains well defined. It appears in phase factors, boundary terms, and variational principles independent of any chosen clock.

Principle 1 (Action primacy)**.** *In null kinematics, where $d\tau = 0$, total action S is the primitive invariant. Energy emerges only after an intrinsic temporal scale T becomes available through timelike reconstruction, via*

$$E = \frac{S}{T}.$$

This represents a change of description rather than a dynamical assumption.

This distinction is critical. Every form of energy in physics, including kinetic energy, potential energy, rest mass, and dark energy, represents the cost of projecting spacetime structure. Energy is not stored in space. It is the cost of creating spacetime from null substrate. The cosmological constant, in particular, corresponds to the baseline projection cost per unit volume that becomes visible only at cosmic scales.

5.1 Temporal reconstruction

For a bounded causal region, the intrinsic duration is defined by the maximum proper time attainable along timelike curves it contains. In an asymptotically de Sitter spacetime, two natural timescales appear.

The first is the geometric timescale H_Λ^{-1} associated with the horizon radius. The second is the thermal timescale

$$\beta_\Lambda = \frac{2\pi}{H_\Lambda},$$

required for Euclidean regularity of the static patch [18]. When converting horizon action into a stationary energy scale, the thermal period provides the correct normalization.

5.2 Energy as projection cost

Reconstruction into a face frame introduces a time parameter t. Combining $S = N\hbar$ with the defining relation $E = dS/dt$ yields

$$E = \hbar \, \frac{dN}{dt}. \tag{8}$$

Definition 1 (Resolution rate)**.** *Define the resolution rate*

$$\Gamma := \frac{dN}{dt}. \tag{9}$$

Then

$$\Gamma = \frac{E}{\hbar}. \tag{10}$$

Energy is therefore the projection cost. It measures the rate at which null action packets are rendered into timelike physics. Energy transfer is a timelike operation arising from frequency differences between vertices after reconstruction. There is no intrinsic energy flow in the null sector.

5.3 Newton's constant and action per face

Newton's constant fixes the conversion between action and rendered area,

$$\ell_P^2 = \frac{\hbar G}{c^3}. \tag{11}$$

In this framework, ℓ_P^2 is interpreted as the surface area of a minimal voxel face.

Lemma 1 (Action partition)**.** *A closed voxel carries total action $\hbar$. Since the voxel has four faces and no distinguished orientation, the action is equally partitioned:*

$$S_{\mathit{face}} = \frac{\hbar}{4}. \tag{12}$$

Thus $\hbar$ is the packet size required to render null structure into spacetime geometry rather than a postulate of quantum indeterminacy.

5.4 The null sector and entropy

Energy is defined as the rate of action accumulation,

$$E := \frac{dS}{dt}. \tag{13}$$

In the vertex frame, no time parameter exists. Energy is therefore not a primitive null observable.

What exists in the null sector is event count.

Definition 2 (Event count)**.** *Let N denote the number of resolved null vertices. The null-sector action is*

$$S = N \, \hbar. \tag{14}$$

Only entropy and action bookkeeping exist in the null sector. All energetic and dynamical quantities arise only after timelike reconstruction.

6 The Vertex Frame: A Null Thought Experiment

6.1 Two Classes of Frames

There exist two distinct and non-interchangeable notions of reference frame.

Definition 3 (Vertex–vertex frame)**.** *A vertex frame is defined by comparisons between null vertices. It encodes angular relations on the celestial sphere and internal phase labels. It admits no metric, no proper time, no spatial translations, and no notion of velocity.*

Definition 4 (Face–face (inertial) frame)**.** *A face frame is defined by comparisons between timelike faces. It supports metric structure, proper time, spatial displacement, momentum, and dynamical evolution.*

Lorentz transformations act only between face–face frames. No Lorentz transformation maps a face frame to a vertex frame.

6.2 The Thought Experiment

The standard statement that one cannot boost to a photon rest frame is correct, but it addresses a different question than the one relevant at null structure. It assumes *a priori* that all frames must be related by dynamical boosts acting on timelike worldlines.

We instead define the null frame by a thought experiment that involves no boosting: *assume one simply is a photon*. We ask what relational data exists in that situation.

As the velocity of a timelike observer approaches c, the Lorentz factor diverges,

$$\gamma(v) = \frac{1}{\sqrt{1 - v^2/c^2}} \xrightarrow{v \to c} \infty, \tag{15}$$

and the proper time increment vanishes,

$$d\tau = \frac{dt}{\gamma(v)} \xrightarrow{v \to c} 0. \tag{16}$$

These divergences arise because the boost attempts to compare *faces* whose separation is measured by a timelike metric. The divergence signals that the metric structure used to define the comparison is becoming degenerate.

In the null limit, this degeneration is exact. The induced metric along a null generator satisfies

$$ds^2 = 0, \tag{17}$$

so neither temporal nor radial separation is defined. All events along the null trajectory are coincident in the only admissible sense: there exists no invariant quantity by which they can be separated.

However, angular relations between vertices remain well-defined, as do internal labels such as helicity and phase. These quantities live entirely on the null vertex structure and require no propagation, signaling, or temporal evolution to be compared.

Because vertex–vertex comparisons do not invoke a metric, they do not involve Lorentz factors, proper time, or spatial displacement. The quantities that diverge in face–face comparisons simply do not appear. The divergence is not resolved; it is rendered irrelevant by a change in the class of comparison.

This has a direct physical consequence: there is no space in which motion could be defined. Momentum proportional to c is not a property of particles; it is a property of wave propagation in reconstructed spacetime. Waves are the mechanism by which spacetime relations are established between vertices. Particles themselves correspond to the vertices—static resolution points that define the geometric structure from which spacetime is later rendered.

The standard statement that a photon has no rest frame refers to the impossibility of defining a rest frame for a null *wave* under Lorentz boosts. The photon, understood as a vertex rather than a propagating face, does not lack a rest frame; rather, it admits a *perfect* one: a frame in which no time elapses, no distance is traversed, and no dynamical comparison is required.

Boosting is a dynamical operation acting on time-evolving face frames. Since the vertex frame contains no time parameter, it cannot be reached by any boost. This does not invalidate the vertex frame; it confirms that static relational structure is ontologically prior to dynamical spacetime description, just as static variables exist prior to program execution.

7 Vertices, Faces, Measurement Closure, and the Perfect Rest Frame

Photons are identified with null vertices. Massive observers do not observe vertices directly. They observe *faces* formed by closing vertices into rendered spacetime geometry. Vertices exist prior to geometry. Geometry is the record of vertex closure.

Definition 5 (Voxel)**.** *A voxel is the minimal closed spacetime unit formed by closing four null vertices into a tetrahedral configuration with four faces.*

A single vertex carries no metric information. Only through closure into faces does spacetime structure become available to timelike observers, consistent with the degeneracy of null hypersurfaces in Lorentzian geometry [8, 19].

7.1 Two observable algebras and the projection map

We distinguish the intrinsic vertex algebra from the reconstructed face algebra.

Definition 6 (Vertex observable algebra)**.** *Let $\mathcal{A}_V$ denote the algebra generated by functions of the celestial direction and internal labels,*

$$\mathcal{A}_V := \mathrm{Alg}\{f(\Omega),\, N,\, \omega\}, \qquad \Omega \in S^2, \tag{18}$$

with N the event count and ω internal quantum labels. No metric, no time parameter, and no spatial translations are assumed in $\mathcal{A}_V$.

Definition 7 (Face observable algebra)**.** *Let $\mathcal{A}_F$ denote the reconstructed algebra generated by spacetime and dynamical observables,*

$$\mathcal{A}_F := \mathrm{Alg}\{x^i,\, p_i,\, t,\, H,\, \cdots\}, \tag{19}$$

defined only after voxel closure and rendering into a face frame.

The reconstruction from vertex data to face observables is modeled as a projection map with finite capacity,

$$\Pi_N : \mathcal{A}_V \longrightarrow \mathcal{A}_F. \tag{20}$$

Postulate 1 (Nonhomomorphic projection)**.** *The projection is not an algebra homomorphism. In general,*

$$\Pi_N(fg) \neq \Pi_N(f)\,\Pi_N(g), \tag{21}$$

reflecting coarse graining and finite information capacity.

Nonhomomorphic projections of this type are well known to induce effective noncommutativity even when the underlying algebra is classical [20, 21].

7.2 Measurement as face comparison and discrete action costs

Timelike measurement requires comparison of rendered geometry.

Postulate 2 (Two-face minimum)**.** *A single face does not define a spacetime observable. Any measurement requires comparison of at least two faces.*

Null structure cannot participate directly in dynamics. By definition, the null sector is static. Vertices do not evolve, propagate, or exchange energy. A timelike observer cannot act on a vertex. Interaction occurs only by reading rendered voxel faces, and is read only.

Definition 8 (Face read cost)**.** *Each rendered face read commits a fixed action cost*

$$S_{\mathit{face}} := \frac{\hbar}{4}. \tag{22}$$

Lemma 2 (Minimum measurement cost)**.** *Any measurement commits at least two face reads and therefore at least*

$$S_{\min} = 2S_{\mathit{face}} = \frac{\hbar}{2}. \tag{23}$$

Lemma 3 (Full cycle cost)**.** *Reading all four faces of a voxel accumulates total action*

$$S_{\mathit{cycle}} = 4S_{\mathit{face}} = \hbar. \tag{24}$$

7.3 Resolution rate and reconstructed energy

In a face frame with time parameter t, let $N(t)$ denote the cumulative number of face reads committed by the observer. The reconstructed action is

$$S(t) = \frac{\hbar}{4}\, N(t), \tag{25}$$

and the reconstructed energy is defined as

$$E := \frac{dS}{dt}. \tag{26}$$

This identification of energy as action per unit time follows the standard definition of energy in Hamiltonian mechanics [22].

Defining the face read rate $\Gamma := dN/dt$ yields

$$E = \frac{\hbar}{4}\, \Gamma. \tag{27}$$

If $C(t)$ denotes the number of completed voxel cycles, then $N(t) = 4C(t)$ and

$$S(t) = \hbar\, C(t), \qquad E = \hbar\, \frac{dC}{dt}. \tag{28}$$

Defining $\omega := dC/dt$ recovers the Planck relation

$$E = \hbar\omega, \tag{29}$$

originally introduced phenomenologically in quantum theory [23, 24].

Here ω is the reconstructed *voxel-cycle resolution rate* in the face frame (i.e. the observable frequency scale induced by rendering), not an additional primitive input.

7.4 Momentum as a face frame quantity

Momentum is defined only in face frames, where spatial translations exist [25].

Definition 9 (Vertex momentum)**.** *The vertex sector admits no translation generator. Therefore intrinsic vertex momentum is not defined as a dynamical observable. We denote the absence of a translation charge by*

$$p_{\mathit{vertex}} \equiv 0. \tag{30}$$

Photon momentum measured experimentally is therefore a reconstructed face frame quantity derived from face deltas after voxel rendering. It is not a vertex invariant.

7.5 The vertex frame as a perfect rest frame

A rest frame is usually defined operationally as a frame in which an object undergoes no spatial displacement as time evolves. This definition presupposes a time parameter and a notion of motion, as formalized in special relativity [26].

In the vertex frame,

$$d\tau = 0, \qquad ds^2 = 0, \tag{31}$$

and there is no translation generator. There is therefore no notion of velocity, acceleration, or momentum at the vertex level.

Proposition 1 (Perfect rest frame)**.** *The vertex frame is a perfect rest frame in the following precise sense. There exists no Hamiltonian H_V generating a one parameter automorphism on $\mathcal{A}_V$. Intrinsic time evolution is not defined.*

The standard divergence of the Lorentz factor as $v \to c$ [26, 27] reflects the breakdown of attempting to describe null structure within a face frame kinematics rather than a physical inconsistency.

7.6 Effective noncommutativity from coarse graining

Although $\mathcal{A}_V$ is classical, the projection Π_N can induce effective noncommutativity in $\mathcal{A}_F$.

Definition 10 (Projection defect)**.** *For $f, g \in \mathcal{A}_V$, define*

$$\mathcal{D}_N(f, g) := \Pi_N(fg) - \Pi_N(f)\Pi_N(g). \tag{32}$$

A concrete realization is provided by truncation on the sphere. Spherical harmonic truncation generates modes outside the truncated space under multiplication, yielding

$$\Pi_\ell(fg) \neq \Pi_\ell(f)\Pi_\ell(g), \tag{33}$$

a mechanism closely related to Toeplitz quantization and deformation quantization on compact phase spaces [28, 29].

7.7 Heisenberg scales as rendering bounds

The Robertson inequality for operators X, P in the face frame gives

$$\Delta X\, \Delta P \geq \frac{1}{2}\left|\langle [X, P] \rangle\right| [30]. \tag{34}$$

In this framework the right hand side arises from rendering and projection rather than intrinsic vertex fluctuations.

Finite angular capacity implies pixel area

$$a_{\mathrm{pix}} = \frac{4\pi r^2}{N}, \tag{35}$$

and transverse uncertainties

$$\Delta x_\perp \sim \sqrt{a_{\mathrm{pix}}}, \qquad \Delta p_\perp \sim \frac{E}{c}\, \Delta\theta. \tag{36}$$

Identifying the minimal rendered pixel with Planck area

$$\ell_P^2 = \frac{\hbar G}{c^3} \tag{37}$$

and restricting to modes resolved at scale r yields

$$\Delta x_\perp\, \Delta p_\perp \sim \hbar. \tag{38}$$

7.8 Summary

The vertex frame is a perfect rest frame because it admits no motion, no time, and no dynamics. Momentum, energy, and uncertainty do not exist at the vertex level as intrinsic observables. They arise only after voxel closure and face comparison through a nonhomomorphic projection into a finite capacity face frame. Quantum scales are the accounting units by which rendered spacetime geometry is paid for in action.

> **The photon is perfectly at rest. What fluctuates is not the vertex, but the cost of projecting it into spacetime.**

8 Superposition as Causal Ignorance and the Role of Virtual Quanta

In this framework, quantum superposition is not a statement about the ontology of the null substrate. It is a statement about epistemic incompleteness during the rendering of a causal graph when spacelike information is unavailable. At any given stage of rendering, the past causal structure is fixed, but the future causal vertex has not yet been determined. To preserve global consistency, the rendering algorithm must retain all causally admissible candidate vertices until sufficient information exists to commit one of them.

8.1 Candidate vertices and causal incompleteness

Let $\mathcal{G}_{\mathrm{past}}$ denote the realized causal graph consisting of committed vertices. At a rendering frontier Σ, the next vertex is not uniquely determined because spacelike-separated constraints have not yet been resolved. Instead, there exists a set of candidate vertices,

$$\mathcal{V}_{\mathrm{cand}}(\Sigma) = \{v_i \mid v_i \text{ is locally compatible with } \mathcal{G}_{\mathrm{past}}\}. \tag{39}$$

Each v_i represents a possible null-consistent continuation of the causal graph. These vertices are not yet part of the realized spacetime. They are elements of the unresolved future. Discarding any v_i prematurely risks constructing a globally inconsistent spacetime.

Quantum superposition is the representation of this set $\mathcal{V}_{\mathrm{cand}}$ together with weights encoding partial information. It reflects ignorance of which vertex will ultimately be committed, not indeterminacy of the null substrate itself.

8.2 Virtual quanta as unresolved causal links

In standard quantum field theory, interactions are represented by internal lines in Feynman diagrams. These internal lines correspond to virtual quanta and are integrated over all four-momenta. In the present framework, such objects admit a precise causal interpretation.

Virtual quanta correspond to candidate causal links connecting unresolved future vertices. They are not realized vertices in the causal past, but neither are they unphysical. They encode hypothetical causal connections whose consistency has not yet been decided. Their apparent off-shell character reflects the fact that they have not yet been constrained to lie on null generators of the realized graph.

Formally, off-shell propagation indicates that the vertex compatibility conditions have not yet been enforced globally.

8.3 The path integral as a causal search algorithm

The Feynman path integral provides a natural mathematical representation of this causal search. The transition amplitude between boundary configurations ϕ_i and ϕ_f is given by

$$\langle \phi_f | \phi_i \rangle = \int \mathcal{D}\phi \ \exp\left(\frac{i}{\hbar} S[\phi]\right), \tag{40}$$

where the integral ranges over all field configurations consistent with the boundary data.

In this framework, the integral is interpreted as a sum over all causally admissible completions of the unresolved future graph. Each field configuration ϕ corresponds to a particular assignment of candidate vertices and causal links. The action $S[\phi]$ acts as a consistency functional, assigning phase weights to different completions.

Virtual quanta appear as intermediate contributions within this sum. They are not physical excitations propagating through spacetime. They are components of the algorithm that explores which causal completions remain viable.

8.4 Interference as constraint propagation

Interference arises when different candidate completions impose incompatible constraints. In the path integral, such incompatibilities manifest as phase cancellations. From the causal perspective, destructive interference eliminates globally inconsistent vertex assignments, while constructive interference reinforces assignments compatible with all known constraints.

The wave character of quantum phenomena thus reflects the structure of the constraint space. It is not evidence of oscillation at the vertex level, but of the geometry of ignorance over unresolved causal possibilities.

8.5 Collapse as commitment under information completion

Collapse occurs when sufficient information becomes available to uniquely select a consistent vertex from $\mathcal{V}_{\rm cand}$. At that moment,

$$\mathcal{V}_{\rm cand} \ \longrightarrow \ \{v_*\}, \tag{41}$$

and the selected vertex v_* is committed to the causal past. Faces are closed, action is paid, and the remaining candidates are discarded.

No physical discontinuity occurs. The collapse reflects an update of the causal graph once ignorance has been resolved. Virtual quanta disappear because their role as hypothetical causal links is no longer required.

8.6 Relation to standard quantum field theory

All standard predictions of quantum field theory are recovered. External legs of Feynman diagrams correspond to committed vertices and therefore satisfy on-shell conditions. Internal lines correspond to candidate causal links and are therefore integrated over off-shell configurations.

Energy, momentum, and locality are enforced only at committed vertices, not along unresolved causal links. The apparent nonlocality and indeterminism of quantum theory arise from attempting to describe this causal search using fully rendered spacetime variables.

8.7 Summary

Quantum superposition is the bookkeeping of causal ignorance required to render a globally consistent spacetime under incomplete information. Virtual quanta are elements of the unresolved future of the causal graph. The path integral is the algorithm that explores all admissible completions. Collapse is the act of committing a vertex once consistency is assured.

> **Quantum mechanics does not describe fluctuating reality. It describes the search for a causally consistent one.**

9 Summary of the Foundations

The starting point of this framework is a literal reading of special relativity. Not as an approximation, not as a model with a limited regime of validity, but as an exact statement about the null limit. The mathematics is unambiguous. As $v \to c$, the Lorentz factor diverges, proper time vanishes,

$$d\tau = 0, \tag{42}$$

and length contraction eliminates spatial extension along the propagation direction. These are not asymptotic tendencies. They are exact properties of massless motion. This is not an interpretive choice but a direct consequence of the theory, confirmed by more than a century of experiment.

If this is accepted, then the rest follows necessarily. A worldline with $d\tau = 0$ admits no clock. Without a clock there is no ticking, no evolution, and no intrinsic notion of before or after. Dynamics requires a time parameter. When time does not exist, dynamics is undefined. A null vertex is therefore static, not because it is frozen by some mechanism, but because the concept of change does not apply.

When metric structure collapses, what survives is not arbitrary. Directional information remains well defined. Angular relations on the celestial sphere, polarization, helicity, and phase survive the null limit. Special relativity already tells us that massless particles carry precisely this information and no more. This framework does not impose angular structure by assumption. It recognizes what null kinematics preserves once distances and durations are no longer meaningful.

Action is the only invariant quantity that remains meaningful when time disappears. Energy is defined as

$$E = \frac{dS}{dt}, \tag{43}$$

and therefore cannot be primitive in a context where dt does not exist. Action, by contrast, appears in phase factors, variational principles, and quantization conditions independent of any clock. The quantum $\hbar$ must therefore be an action quantum. It cannot be an energy quantum at the null level because energy has not yet been defined.

The factor $\hbar/4$ is not mysterious. It is geometric. The minimal closed three dimensional region requires four faces. This is a topological fact, not a parameter choice. A voxel carrying total action $\hbar$ with no preferred orientation must distribute that action equally across its four faces. The same counting appears in horizon entropy, Newtonian gravity, and cosmological constant calculations because all three are bookkeeping the same quantity: action on null boundaries.

Vertex frames and face frames are categorically distinct. One cannot boost into the null regime because Lorentz boosts presuppose clocks, rulers, and a time parameter. The divergence of the Lorentz factor as $v \to c$ does not indicate a physical breakdown. It indicates that face to face comparisons are being applied outside their domain of definition. Face frames compare metrics and proper time. Vertex frames compare angles and action counts. The singularity of γ marks a boundary between these types of description.

Energy emerges as a projection cost. If spacetime geometry arises by closing null vertices into volumetric elements, then rendering spacetime requires action. We observe this cost as energy only after a temporal scale exists to convert total action into a rate,

$$E = \frac{S}{T}. \tag{44}$$

The cosmological constant is not energy stored in space. It is the baseline action cost per unit volume of maintaining the projection. Quantum field theory computes energy densities within already rendered spacetime. That calculation addresses a different question.

Entropy bookkeeping is primitive. Energy bookkeeping is reconstructed. What exists is counting: events, degrees of freedom, action quanta. Energy appears only when this accounting is projected onto a timelike worldline equipped with a clock. This is why horizon thermodynamics works. It operates at the level where entropy is still the native currency.

None of this requires new physics. It requires taking seriously what special relativity already says about the null limit. To avoid this picture one must assert that something changes at $v = c$, that new physics intervenes to soften the exact consequences of time dilation and length contraction. There is no experimental evidence for such an intervention. Every observation of massless particles confirms that they behave exactly as null kinematics predicts.

The alternative is to treat photons as almost timelike but not quite, to regard the null regime as a limit that is approached but never realized, or to declare questions about null experience meaningless. That position is not supported by the mathematics. The mathematics states that $d\tau = 0$ exactly. Either the implications of that statement are accepted, or new physics must be proposed to modify it.

Until such physics is demonstrated, this framework is not optional. It is what consistency demands.

Part II

The Geometry of Null

10 Null Geometry and the Celestial Sphere

10.1 Degenerate Null Geometry

Let $\mathcal{H}$ be a null hypersurface generated by a null vector field k^a. The induced (pullback) metric q_{ab} on $\mathcal{H}$ satisfies

$$q_{ab}k^b = 0, \tag{45}$$

so q_{ab} is degenerate with intrinsic signature $(0, +, +)$ [8, 9]. The null direction is the zero mode.

The consequence is simple and important. Along the generators of $\mathcal{H}$, there is no intrinsic length. Only the transverse two-dimensional geometry on spacelike cross-sections is nondegenerate. A null surface carries transverse shape, but it does not carry intrinsic distance along its flow.

10.2 The Celestial Sphere and Angular Primacy

Null vectors are physically equivalent up to positive rescaling. If k^a is null, then λk^a is also null for $\lambda > 0$, and they represent the same physical direction. The space of distinct future null directions at p is therefore the projective quotient

$$\mathcal{S}_p = \mathcal{N}_p^+/\mathbb{R}^+ \cong S^2, \tag{46}$$

where $\mathcal{N}_p^+$ is the set of future-directed null vectors at p. This is the celestial sphere [31, 32].

Principle 2 (Angular Primacy)**.** *The intrinsic geometric content of null kinematics is angular and projective. Euclidean distance and volume require timelike reconstruction and are not intrinsic to null structure.*

This is the geometric reason area shows up everywhere in gravitational physics. Null boundaries naturally support an angular measure, not a volumetric one.

10.3 Angular Propagation and the 4π Measure

At a null emission event, the intrinsic data is a distribution over directions

$$\Omega = (\theta, \phi) \in S^2. \tag{47}$$

A simple way to express flux conservation in this setting is

$$\int_{S^2} \rho(\Omega)\, d\Omega = \text{constant}. \tag{48}$$

The factor 4π is not a convention here. It is the total solid angle of the celestial sphere. It is the same 4π that appears in Gauss laws, horizon thermodynamics, and holographic bounds [2, 33].

11 The Vertex-Face Equality Constraint

A null vertex represents a complete angular state on the celestial sphere S^2, specifying direction, polarization, and phase. Since null geometry supports only angular structure (Principle 2), both vertices and faces represent angular divisions of the same sphere.

The constraint $N_v = N_f$ follows from three non-negotiable requirements:

(1) Fixed total area: Newton's constant $G = \ell_P^2 c^3/\hbar$ states that $\hbar$ of action corresponds to exactly ℓ_P^2 of boundary area. This is not a choice—it is the geometric meaning of the empirical constant G.

(2) Complete spherical coverage: Outward null projection from a vertex must cover the full celestial sphere, with total solid angle 4π steradians.

(3) Angular primacy: Null geometry does not support intrinsic radial or temporal structure. The only well-defined degrees of freedom are angular. Therefore, vertices (angular positions) and faces (angular sectors) both count angular divisions of S^2.

Partitioning a fixed total—both area ℓ_P^2 and action $\hbar$—into angular sectors requires the number of vertices to equal the number of faces. Any mismatch $N_v \neq N_f$ would force either:

- Unequal action per angular degree of freedom (violating isotropy), or
- Incomplete spherical coverage (violating the 4π requirement), or
- Variable total action or area (violating $G = \ell_P^2 c^3/\hbar$).

Combined with the topological requirement that closure in three dimensions requires $N_v \geq 4$ and $N_f \geq 4$ (Euler characteristic), this uniquely determines $N_v = N_f = 4$: tetrahedral structure with equal-area faces.

The assignment $\hbar/4$ per face follows immediately from equal partition of total voxel action $\hbar$ over four boundary elements. This is not a calibration—it is forced by the empirical value of G, the geometry of null surfaces, and the topology of three-dimensional closure.

12 Tetrahedral Voxel Construction: Forced by Geometric Constraints

Null geometry supplies angular structure but not volume. To obtain emergent volume, angular data must close into volumetric elements. We now show that the tetrahedral voxel is not a choice but the unique solution to a set of non-negotiable geometric constraints.

12.1 Geometric Picture: The Voxel as Spherical Projection

Consider a null vertex as the fundamental object. To discover what volumetric element it participates in, we project spherically outward in all directions at the speed of light. This spherical wavefront connects the vertex to its neighbors—other null vertices that bound the same reconstructed volume element.

- The null vertex sits at the center
- Outward projection sweeps out the celestial sphere S^2
- The sphere partitions into angular sectors, each pointing toward a neighboring vertex
- These sectors define the faces of the voxel
- The voxel "fills in" between the vertex and its neighbors as bulk geometry

Projection forward and backward in time connects the vertex to both past and future neighbors, building the full 4D causal structure. But at any given "time slice," the spatial voxel structure is determined by the angular partition of S^2.

12.2 The Constraint System

The voxel structure must satisfy seven independent geometric requirements:

1. **Action Quantization:** Total action per voxel equals $\hbar$ (quantum postulate).
2. **Area-Action Identity:** Total surface area must equal ℓ_P^2, where $\ell_P^2 = \hbar G/c^3$ (from Newton's constant). This is not a choice—it is the geometric meaning of G.

3. **Equal Face Areas:** All faces must have identical area. When neighboring voxels share a face, both must assign the same pixel content to that shared boundary. Without this, action bookkeeping becomes inconsistent across voxel interfaces.
4. **Angular Isotropy:** Each angular sector must subtend the same solid angle when projected from the central vertex. For a partition of the full celestial sphere S^2 (total solid angle 4π), isotropy requires:
$$\Delta\Omega_{\text{sector}} = \frac{4\pi}{N_{\text{sectors}}} \tag{49}$$
5. **Linear Vertex-Face Scaling:** The number of vertices must equal the number of faces: $N_v = N_f$. Since null geometry carries only angular structure (no intrinsic radial scale), both vertices (angular positions) and faces (angular sectors) count divisions of the same sphere S^2. This constraint ensures action per angular degree of freedom remains uniform and total area remains fixed as prescribed by G.
6. **Shape Flexibility (Curvature Encoding):** The voxel must be deformable while preserving constraints (1-5). This allows curvature to be encoded through deficit angles and shape degrees of freedom, exactly as in Regge calculus [34]. The voxel must be able to assume irregular shapes while maintaining the equal-area constraint.
7. **Volumetric Closure:** The angular sectors must close into a finite 3D polyhedron with no open edges or vertices at infinity.

12.3 Unique Solution: Four Faces, Tetrahedral Closure

These seven constraints uniquely determine the voxel structure through forced logical steps.

Step 1: Minimal closure forces 4 vertices and 4 faces.

From constraint (7), we need a closed 3D polyhedron. The minimal such object is the 3-simplex (tetrahedron) with 4 vertices. From constraint (5), $N_v = N_f$, which immediately gives:
$$N_{\text{vertices}} = N_{\text{faces}} = 4 \tag{50}$$
This is topologically forced. Euler's formula for a closed polyhedron states $V - E + F = 2$. For $V = 4$ and minimal connectivity:

- Each vertex connects to all others (complete graph)
- This gives $E = \binom{4}{2} = 6$ edges
- Check: $V - E + F = 4 - 6 + F = 2 \implies F = 4$ ✓

Step 2: Solid angle per sector is forced.

From constraints (4) and (5):
$$\Delta\Omega_{\text{sector}} = \frac{4\pi}{4} = \pi \text{ steradians} \tag{51}$$
Each of the four angular sectors subtends exactly π steradians—one quarter of the full celestial sphere.

Step 3: Face areas are forced.

From constraints (2), (3), and (5):
$$\text{Total voxel surface area} = \ell_P^2 \tag{52}$$
$$\text{Number of faces} = 4 \tag{53}$$
$$\implies \text{Area per face} = \frac{\ell_P^2}{4} \tag{54}$$

Combined with $G = \ell_P^2 c^3/\hbar$, each face carries action:
$$S_{\text{face}} = \frac{\hbar}{4} \tag{55}$$

This is not a fit parameter. It is forced by vertex-face equality (constraint 5) and the empirical value of Newton's constant.

Step 4: Faces must be triangular (Regge calculus requirement).

Constraint (6) requires that we can deform the voxel to encode curvature while maintaining equal face areas. This is where the unique solution emerges.

Consider a tetrahedron with four faces, each constrained to have area $\ell_P^2/4$:

- **In flat space:** The tetrahedron is regular. All edge lengths are equal, all vertex angles equal. The four vertices lie on a sphere, and the deficit angle at each vertex is zero.
- **In curved space:** The tetrahedron becomes irregular. Edge lengths vary, vertex angles vary, but the four face areas remain equal to $\ell_P^2/4$. The deficit angle at each vertex—the amount by which the sum of face angles falls short of 2π—encodes the local curvature, exactly as in Regge calculus [34].

A tetrahedron has 6 edge lengths as degrees of freedom. After accounting for 3 constraints from overall scale and orientation, this leaves 3 independent shape parameters—sufficient to encode the 3 independent components of spatial curvature at a point while respecting the equal-area constraint.

Crucially, this works only for triangular faces. If we attempted to partition the sphere into 4 non-triangular faces (e.g., quadrilaterals), the equal-area constraint combined with closure would over-constrain the geometry, eliminating the deformation freedom needed to encode curvature. A quadrilateral has 4 edge lengths; with 4 such faces sharing edges, the equal-area constraints would leave no freedom for deficit angle variations.

Triangular faces are forced by the requirement that the voxel remain deformable for curvature encoding. This is not an aesthetic choice—it is the only topology that satisfies all seven constraints simultaneously.

12.4 Relationship Between Solid Angle and Face Area

It is important to distinguish two related but different quantities:

- **Solid angle per sector:** $\Delta\Omega = \pi$ steradians (scale-free, pure angular measure)
- **Area per face:** $A_{\text{face}} = \ell_P^2/4$ (physical area on the voxel boundary)

The solid angle tells you angular coverage on the celestial sphere. The face area tells you action cost and is fixed by the identity $G = \ell_P^2 c^3/\hbar$.

When we project the celestial sphere outward from the central vertex to the voxel boundary, each angular sector of solid angle π steradians intercepts a triangular face. For a spherical cap at radius r, the naive relationship would be $A = r^2\Delta\Omega = \pi r^2$. But the tetrahedron's faces are *flat* triangles, not spherical caps.

The projection from spherical sectors to flat triangular faces introduces geometric factors that depend on the tetrahedron's shape. However, the constraint $A_{\text{face}} = \ell_P^2/4$ fixes this geometry uniquely through the normalization provided by Newton's constant. The area is not derived from the solid angle—both are independently constrained, and consistency between them determines the voxel geometry.

12.5 Volume from Three Null Directions

Each voxel is bounded by four faces, meaning four null directions emanate from the central vertex. But only three independent null directions are needed to span 3D space—the fourth ensures closure but is redundant for spanning.

Taking any three non-coplanar null directions $\{\Omega_1, \Omega_2, \Omega_3\} \subset S_v^2$ from the vertex, during timelike reconstruction each advances with $\Delta r_i = c\,\Delta t$:

$$\Delta V \sim \Delta r_1 \Delta r_2 \Delta r_3 \sim (c\,\Delta t)^3 \quad \Longrightarrow \quad \frac{dV}{dt} \sim c^3 \tag{56}$$

This gives the direct geometric meaning of the c^3 factor in $G = \ell_P^2 c^3/\hbar$:

- Volume is constructed at rate c^3 from three null directions
- Each null direction advances at speed c
- The product gives the volumetric growth rate
- The fourth face ensures closure but is redundant for spanning the space

12.6 The Fundamental Identity

We can now state the complete identity encoded in Newton's constant.

For one minimal voxel centered on a null vertex:

$$\text{Number of vertices} = 4 \quad \text{(minimal 3-simplex)} \tag{57}$$

$$\text{Number of faces} = 4 \quad (\text{from } N_v = N_f) \tag{58}$$

$$\text{Total boundary area} = \ell_P^2 \quad (\text{from } G = \ell_P^2 c^3/\hbar) \tag{59}$$

$$\text{Area per face} = \frac{\ell_P^2}{4} \tag{60}$$

$$\text{Action per face} = \frac{\hbar}{4} \tag{61}$$

$$\text{Total action content} = \hbar \tag{62}$$

$$\text{Volumetric generation rate} = c^3 \quad (\text{from 3 null directions at speed } c) \tag{63}$$

Newton's constant equates boundary and bulk:

$$G = \frac{\ell_P^2}{c^3 \hbar} \quad \Longleftrightarrow \quad \ell_P^2 = \frac{\hbar G}{c^3} \tag{64}$$

Meaning: Creating ℓ_P^2 of boundary costs the same $\hbar$ of action as creating volume at rate c^3. The boundary and the bulk have identical action budgets. **This is holography at the single-voxel level**, encoded in the fundamental constants since Newton.

12.7 Summary: Zero Free Parameters

The tetrahedral voxel structure is completely determined by three empirical inputs:

1. The measured value of Newton's constant G
2. The quantum of action $\hbar$
3. Null geometry carrying only angular degrees of freedom (geometric fact)

From these, everything follows by geometric necessity:

Quantity	Value
Number of vertices	4 (minimal 3-simplex)
Number of faces	4 (from $N_v = N_f$)
Face topology	Triangular (forced by Regge calculus)
Total surface area	ℓ_P^2 (forced by $G = \ell_P^2 c^3/\hbar$)
Area per face	$\ell_P^2/4$ (forced by equal division)
Action per face	$\hbar/4$ (forced by G)
Solid angle per face	π steradians (forced by isotropy)
Shape flexibility	Irregular tetrahedra (forced by curvature encoding)

There are zero adjustable parameters. The factor of $1/4$ in black hole entropy, cosmological constant calculations, and throughout gravitational physics is not mysterious. It is the forced consequence of:

- Minimal 3D closure requiring 4 vertices and 4 faces (topology)
- Linear vertex-face scaling from angular primacy (null geometry)
- Triangular face topology from curvature encoding (Regge calculus)
- $G = \ell_P^2 c^3/\hbar$ (empirical constant)

The tetrahedron is not chosen—it is the unique geometric object satisfying all constraints simultaneously.

13 Why 4π and Why the Quarter Shows Up Everywhere

The constants that show up in gravitational physics are not arbitrary. Many of them track back to the geometry and topology of S^2.

13.1 Gauss-Bonnet and 4π

The Euler characteristic of the sphere is $\chi(S^2) = 2$. Gauss-Bonnet gives

$$\int_{S^2} K\, dA = 2\pi\chi(S^2) = 4\pi, \tag{65}$$

and since $K = 1/r^2$ and $A = 4\pi r^2$, the total solid angle is

$$\Omega_{S^2} = \frac{A}{r^2} = 4\pi. \tag{66}$$

This is radius independent and topological.

13.2 How it Propagates

In Newtonian gravity, flux conservation over S^2 forces the inverse square law and fixes the 4π in Gauss's law [2]. In general relativity, the 8π in Einstein's equations is fixed by matching to the Newtonian limit [8]. In black hole thermodynamics, the same π factors appear through horizon area and Euclidean regularity and reduce to the clean $A/(4\ell_P^2)$ result [18, 4].

The main point for this paper is simple. The 4π is a geometric invariant of the celestial sphere, and the quarter is the minimal closure factor for a tetrahedral bookkeeping of angular data.

14 Black Hole Entropy and Terminal Vertices

The tetrahedral bookkeeping gives a concrete geometric reading of why horizon entropy is an area law and why the coefficient is $1/4$.

14.1 Terminal Structure

A black hole singularity can be treated as a terminal feature of the causal network, in the sense that null propagation does not extend beyond it. In the tetrahedral picture, the horizon is the part of the boundary that remains accessible. The bookkeeping consequence is that a single face per voxel contributes to what an exterior observer can count.

This matches the standard semiclassical result that horizon entropy is proportional to area, not volume [3, 4, 33].

14.2 Pixel Counting

If one face corresponds to solid angle π, the number of face-pixels on a horizon of area A is

$$N_{\text{pix}} = \frac{A}{\ell_P^2/4} = \frac{4A}{\ell_P^2}. \tag{67}$$

Each pixel (face) contributes action $\hbar/4$. In the null-first accounting, entropy is the *dimensionless* count of independent action quanta accessible to the exterior observer. We therefore identify the (dimensionless) horizon entropy (setting $k_B = 1$) with total action measured in units of $\hbar$:

$$\frac{S_{\text{BH}}}{k_B} \;\equiv\; \frac{S_{\text{tot}}}{\hbar} \;=\; \frac{1}{\hbar}\left(N_{\text{pix}} \cdot \frac{\hbar}{4}\right) \;=\; \frac{N_{\text{pix}}}{4} \;=\; \frac{A}{4\ell_P^2}. \tag{68}$$

Equivalently, restoring k_B gives the Bekenstein–Hawking formula $S_{\text{BH}} = k_B\, A/(4\ell_P^2)$.

Part III

Dissolution of the Cosmological Constant via Action Pixelation

15 The Cosmological Constant Problem

15.1 Statement of the Discrepancy

A Planck-scale cutoff estimate for vacuum energy density in quantum field theory gives

$$\rho_{\mathrm{QFT}} \sim \frac{\hbar c}{\ell_P^4}, \tag{69}$$

while observations are consistent with a late-time de Sitter scale

$$\rho_\Lambda = \frac{3H_\Lambda^2 c^2}{8\pi G}. \tag{70}$$

Convention note. In this paper ρ denotes an *energy density*. Some cosmology conventions use ρ for mass density, in which case the corresponding relation is $\rho_{\Lambda,\text{mass}} = 3H_\Lambda^2/(8\pi G)$ and $\rho = \varepsilon/c^2$.

The ratio is of order 10^{122} [5, 35]. The standard presentation treats this as a fine-tuning crisis.

15.2 The Category Error

In the null-first picture, the mismatch is better understood as a category error with two parts.

First, the QFT estimate counts bulk volumetric modes as if they were fundamental. Gravitating systems obey holographic scaling, where independent degrees of freedom scale with boundary area rather than bulk volume [13, 14, 33].

Second, the QFT estimate treats energy as primitive in a regime controlled by null structure. In null kinematics, energy is not intrinsic because it requires a time scale. What is intrinsic is action, which is defined without clocks and appears directly in quantum phases and boundary terms.

The resolution in this paper is therefore not a modification of gravity. It is a change in what is being counted. We count horizon pixels and their action, then render to energy using the physically correct de Sitter time scale.

16 From Horizon Pixels to Total Action

16.1 De Sitter Horizon Geometry

For an asymptotically de Sitter universe, the cosmological horizon radius is

$$R_\Lambda = \frac{c}{H_\Lambda}, \tag{71}$$

with horizon area

$$A_\Lambda = 4\pi R_\Lambda^2. \tag{72}$$

The number of Planck-area pixels on the horizon is

$$N_{\text{pix}} = \frac{A_\Lambda}{\ell_P^2}. \tag{73}$$

Numerically, this is of order 10^{122} for the observed late-time scale. In this framework, that number is not a pathology. It is simply the horizon pixel count.

16.2 Action Pixelation

From tetrahedral closure of null geometry, each fundamental face carries action

$$\Delta S_{\text{pix}} = \frac{\hbar}{4}, \tag{74}$$

consistent with the semiclassical horizon entropy normalization [3, 4, 33]. The total horizon action is therefore

$$S_{\text{tot}} = \Delta S_{\text{pix}}\, N_{\text{pix}} = \frac{\hbar}{4}\frac{A_\Lambda}{\ell_P^2} = \frac{\pi\hbar R_\Lambda^2}{\ell_P^2}. \tag{75}$$

Using $\ell_P^2 = G\hbar/c^3$ gives

$$S_{\text{tot}} = \frac{\pi R_\Lambda^2 c^3}{G} = \frac{\pi c^5}{G H_\Lambda^2}. \tag{76}$$

This is action on a null boundary, not energy stored in a bulk volume.

17 Rendering Action into Energy

17.1 Which Timescale

Timelike observers measure energies and energy densities, so we convert action to energy via

$$E = \frac{S}{T}. \tag{77}$$

For a de Sitter horizon there are two natural time scales: the Hubble time H_Λ^{-1} and the thermal (KMS) period

$$\beta_\Lambda = \frac{2\pi}{H_\Lambda}. \tag{78}$$

The thermal period is fixed by Euclidean regularity of the static patch and equals the inverse Gibbons–Hawking temperature [18]. Since we are rendering boundary action into a stationary energy scale associated with horizon equilibrium, β_Λ is the relevant time scale.

17.2 Why the Thermal Time β_Λ Is the Natural Scale

A potential point of confusion is the appearance of a *thermal* time scale, β_Λ, in what is otherwise a geometric and kinematical construction. We emphasize that β_Λ is not introduced as a thermodynamic assumption, but arises unavoidably from the structure of null horizons and the absence of any intrinsic time scale on the null substrate.

In the null-first framework, null relations carry no proper time ($d\tau = 0$) and therefore cannot support a primitive notion of energy. The only invariant quantity that can be accumulated along null structure is *action*. Time, energy, and temperature emerge only after rendering null relations into a timelike description appropriate to massive observers.

For spacetimes with a cosmological horizon, this rendering is unique. A causal horizon enforces a finite information-access boundary, and the corresponding observer necessarily experiences a Kubo–Martin–Schwinger (KMS) state with inverse temperature

$$\beta_\Lambda = \frac{2\pi}{H_\Lambda}, \tag{79}$$

where H_Λ is the de Sitter expansion rate. This is not a choice: it is the only time scale that can be constructed from the null geometry of the horizon itself.

Crucially, β_Λ is the *thermal time* associated with horizon closure, not a microscopic cutoff. Planck time does not play this role, because the Planck scale governs *resolution* of action quanta, not the rate at which null structure must be rendered into a consistent timelike evolution. The horizon instead fixes the global pacing of action accumulation visible to the observer.

From the null perspective, the appearance of temperature is therefore a bookkeeping artifact of partial access to null relations. The inverse temperature β_Λ measures the minimal temporal interval required for the observer to resolve independent horizon-crossing events. It is the macroscopic shadow of null closure, in exactly the same sense that horizon area is the geometric shadow of action counting.

This explains why β_Λ, and not an arbitrary cosmological or Planckian timescale, governs the effective energy density associated with the cosmological constant. Once action quanta of size $\hbar$ are rendered at a rate fixed by β_Λ^{-1}, the observed vacuum energy density follows as a consistency condition, not as a dynamical input.

17.3 Spacelike Constraint and Indirect Gravitational Influence

A subtle but important point concerns the role of spacelike-separated structure in gravitational and thermodynamic phenomena. Degrees of freedom beyond a causal horizon do not locally interact with the observer and therefore do not directly gravitate with them in the usual sense: no stress–energy can be exchanged, and no local force is exerted.

However, this absence of direct coupling does not imply irrelevance. Gravity is not solely a local interaction but a global constraint on geometry. Spacelike-inaccessible degrees of freedom remain coupled to degrees of freedom that *do* interact with the observer, and through this coupling they participate indirectly in determining the consistent spacetime geometry.

In particular, horizons impose closure conditions on null structure. Although the observer cannot access the full null graph, the requirement that null relations close consistently across the horizon constrains the admissible timelike rendering of spacetime. This constraint manifests not as a force or energy flux, but as fixed geometric and thermodynamic parameters, such as surface gravity, horizon temperature, and the associated thermal time scale β_Λ.

From this perspective, spacelike-separated structure does not "gravitate on" the observer dynamically, but it does gravitate *geometrically* by fixing the global consistency conditions under which the observer's local physics must operate. The resulting thermal activity is therefore not a consequence of microscopic emission or radiation, but of enforced equilibrium arising from partial access to null structure.

This clarifies why horizon-induced thermal behavior is unavoidable in the null-first framework. Once time is imposed on a system with spacelike-excluded null relations, the observer necessarily experiences thermal activity as the timelike shadow of global geometric constraint.

17.4 Energy and Energy Density

The rendered horizon energy is

$$E_\Lambda = \frac{S_{\rm tot}}{\beta_\Lambda} = \frac{\pi c^5}{G H_\Lambda^2} \cdot \frac{H_\Lambda}{2\pi} = \frac{c^5}{2 G H_\Lambda}. \tag{80}$$

The emergent static-patch volume enclosed by the horizon is

$$V_\Lambda = \frac{4\pi}{3} R_\Lambda^3 = \frac{4\pi c^3}{3 H_\Lambda^3}. \tag{81}$$

The corresponding energy density is

$$\rho_\Lambda = \frac{E_\Lambda}{V_\Lambda} = \frac{c^5/(2GH_\Lambda)}{4\pi c^3/(3H_\Lambda^3)} = \frac{3H_\Lambda^2 c^2}{8\pi G}. \tag{82}$$

This matches the standard Friedmann form for a Λ-dominated universe, but here it appears as an accounting identity: horizon action (area scaling) rendered into energy (thermal time) and distributed over reconstructed volume.

Theorem 1 (Natural de Sitter Scale from Pixelated Horizon Action)**.** *For an asymptotically de Sitter horizon, assigning $\hbar/4$ of action per Planck-area pixel and rendering over the thermal period $\beta_\Lambda = 2\pi/H_\Lambda$ yields*

$$\rho_\Lambda = \frac{3H_\Lambda^2 c^2}{8\pi G}. \tag{83}$$

No vacuum energy assumption is required.

17.5 Why the 2π Matters

If one uses $T = H_\Lambda^{-1}$ instead of β_Λ, the result is off by a factor of 2π. The 2π is not a fit. It is the same Euclidean regularity and KMS periodicity factor that fixes Hawking and Gibbons–Hawking temperatures in horizon thermodynamics [4, 18].

18 Why the 10^{122} Appears

The QFT estimate is internally consistent given its assumptions. The problem is that those assumptions treat Planck-scale degrees of freedom as independent and local in the bulk. Holography says otherwise [13, 14, 33].

A compact way to see the mismatch is to compare the two scales directly. The pixelated horizon result can be written as

$$\rho_\Lambda = \frac{3H_\Lambda^2 c^2}{8\pi G} \sim \frac{\hbar c}{\ell_P^2 R_\Lambda^2}, \tag{84}$$

using $\ell_P^2 = G\hbar/c^3$ and $R_\Lambda = c/H_\Lambda$. The QFT cutoff estimate scales as

$$\rho_{\text{QFT}} \sim \frac{\hbar c}{\ell_P^4}. \tag{85}$$

The ratio is therefore

$$\frac{\rho_{\text{QFT}}}{\rho_\Lambda} \sim \frac{\hbar c/\ell_P^4}{\hbar c/(\ell_P^2 R_\Lambda^2)} = \frac{R_\Lambda^2}{\ell_P^2} = \left(\frac{R_\Lambda}{\ell_P}\right)^2 \sim 10^{122}. \tag{86}$$

This is the square of the ratio between the horizon scale and the Planck scale. It is also the order of magnitude of the horizon pixel count $N_{\text{pix}} = A_\Lambda/\ell_P^2$.

In this framework, the mismatch is the numerical signature of counting bulk modes when only boundary pixels are fundamental.

19 Friedmann as Global Accounting

Equation (82) can be rewritten as

$$H_\Lambda^2 = \frac{8\pi G}{3}\rho_\Lambda. \tag{87}$$

In the present interpretation, this is a consistency condition tying together three ingredients:

- the area scaling of null boundary degrees of freedom,
- the conversion from action to energy using the de Sitter thermal period,
- the geometric relation between horizon area and enclosed volume.

The standard dynamical derivation from Einstein's equations remains valid. The point here is that the same relation also follows from boundary action bookkeeping once the null structure is treated as fundamental.

20 Expansion as the Geometric Trace of Action Accumulation

Standard cosmology interprets cosmic expansion as the dynamical response of spacetime geometry to matter and energy distributions. The null-first framework reverses this causal relationship. Rather than metric evolution driven by stress-energy, the fundamental process is the accumulation of action through sequential resolution of null relations.

We begin with the geometric interpretation of Newton's constant,

$$G = \frac{\ell_P^2}{\hbar}, \tag{88}$$

which reveals that spatial area serves as a natural bookkeeping quantity for accumulated action. In this view, action—not energy density—is the fundamental invariant.

Event Count and Action Ledger. Consider N_v as the total number of resolved null vertices in an observer's causal past. Each vertex contributes approximately one quantum of action, $s \approx \hbar$, giving a total accumulated action of

$$S = N_v \hbar. \tag{89}$$

The geometric identity encoded in G then yields

$$\Delta A = G\,\Delta S = \ell_P^2\,\Delta N_v. \tag{90}$$

This relationship suggests that cosmological expansion is not a stretching of pre-existing space, but rather the geometric consequence of adding new null relations to the causal network. The universe doesn't expand into a pre-existing vacuum; instead, the vacuum itself emerges as a holographic projection of the growing causal structure.

From this perspective, the Hubble parameter H loses its character as a dynamical driver of expansion. Instead, it quantifies the rate at which new events are resolved relative to the existing causal set, determining how rapidly new geometric area must be generated to preserve consistency.

21 Null Closure, Energy, and the Emergence of the Friedmann Scaling

This section presents a closed kinematical derivation of the cosmological density–scale relation without assuming an expansion rate as a primitive input. The only geometric datum is a finite null screen of areal radius R, carrying a finite angular resolution capacity at Planck area. All dynamical quantities emerge from consistency conditions on null resolution.

21.1 Angular Capacity of a Null Screen

Let the null screen be a compact spacelike cross section with areal radius R and area

$$A = 4\pi R^2. \tag{91}$$

Under Planck area resolution, the number of independent angular pixels is

$$N_{\text{pix}} = \frac{A}{\ell_P^2} = \frac{4\pi R^2}{\ell_P^2}. \tag{92}$$

21.2 Resolution Rate and the Energy Identity

Let Γ denote the null resolution rate, defined as the number of resolved action quanta per unit reconstructed time. Resolution occurs in discrete units $\Delta S \simeq \hbar$. If $S(t)$ denotes the total resolved action in a causal region, then

$$\frac{dS}{dt} \simeq \Gamma\hbar. \tag{93}$$

Independently, by definition of action,

$$S = \int E(t)\,dt \qquad \Longrightarrow \qquad \frac{dS}{dt} = E. \tag{94}$$

Equating (93) and (94) yields

$$\Gamma = \frac{E}{\hbar}. \tag{95}$$

Thus the null resolution rate is not an independent parameter but is fixed directly by the energy content of the region.

21.3 Stationarity and Terminal Face Counting

Stationarity of null reconstruction requires that, over one characteristic reconstruction timescale β, the number of resolved action quanta saturates the available angular capacity of the null screen.

However, not all angular pixels contribute equally to the exterior causal boundary. In the tetrahedral voxel construction, each spacetime voxel possesses four faces, but only one face contributes to the terminal null screen. The remaining faces close internally through past, future, or interior adjacency. Therefore, the effective exterior capacity is reduced by a factor of four.

The stationarity condition is thus

$$\Gamma\,\beta = \frac{N_{\text{pix}}}{4}. \tag{96}$$

Substituting (95) and (92) into (96) gives

$$\frac{E}{\hbar}\,\beta = \frac{1}{4}\,\frac{4\pi R^2}{\ell_P^2} = \frac{\pi R^2}{\ell_P^2}. \tag{97}$$

21.4 Elimination of $\hbar$

Using the relation

$$\ell_P^2 = \frac{G\hbar}{c^3}, \tag{98}$$

equation (97) becomes

$$E\,\beta = \frac{\pi R^2 c^3}{G}. \tag{99}$$

This relation is purely kinematical and contains no reference to any expansion rate.

21.5 Energy Density at Fixed Closure Radius

Define the energy density ρ_E in the causal region as

$$\rho_E \equiv \frac{E}{V}, \tag{100}$$

where the associated causal volume is taken as

$$V = \frac{4\pi}{3} R^3. \tag{101}$$

Dividing (99) by (101) yields

$$\rho_E \, \beta = \frac{\pi R^2 c^3}{G} \cdot \frac{3}{4\pi R^3} = \frac{3c^3}{4GR}. \tag{102}$$

21.6 Thermal Time of the Null Screen

A null screen carries a natural thermal timescale associated with its surface gravity. For a spherical null screen of radius R, the intrinsic reconstruction timescale is

$$\beta = \frac{2\pi R}{c}. \tag{103}$$

Substituting (103) into (102) gives

$$\rho_E = \frac{3c^3}{4GR} \cdot \frac{c}{2\pi R} = \frac{3c^4}{8\pi G R^2}. \tag{104}$$

Expressed as a mass density $\rho_M = \rho_E / c^2$, this becomes

$$\rho_M = \frac{3c^2}{8\pi G R^2}. \tag{105}$$

21.7 Derived Definition of the Expansion Rate

For comparison with standard cosmological notation, one may define

$$H \equiv \frac{c}{R}. \tag{106}$$

This is not an input but a label for the null closure scale. Equation (104) then takes the familiar Friedmann form

$$\rho_E = \frac{3H^2 c^2}{8\pi G}. \tag{107}$$

21.8 Spectral Closure and the 8π Identity

The null closure scale can also be encoded spectrally. Modeling the null screen as the sphere S^2_R, the Laplace–Beltrami eigenvalues are

$$\lambda_\ell = \frac{\ell(\ell+1)}{R^2}, \tag{108}$$

so the spectral gap is

$$\lambda_1 = \frac{2}{R^2}. \tag{109}$$

Combining (109) with (92) yields the exact identity

$$\lambda_1 \, \ell_P^2 \, N_{\rm pix} = 8\pi. \tag{110}$$

This relation expresses a fixed conversion between spectral rigidity of the null screen and its finite angular capacity. The appearance of the factor 8π matches the normalization appearing in the Einstein field equations, but arises here purely from kinematical closure rather than dynamical coupling.

21.9 Interpretation and Scope

The derivation above determines the functional relation between energy density, closure radius, and reconstruction timescale without assuming an expansion rate or field equations. The numerical value of the closure radius R is a boundary condition reflecting the total energy content of the causal patch. This is analogous to the role of initial data in general relativity or state selection in quantum mechanics.

The result shows that once a stationary null screen exists, finite angular capacity and action quantization force the scaling $\rho \propto 1/(GR^2)$ and fix the normalization exactly. Dynamics and evolution of the energy content are addressed separately.

22 A Complete Derivation of the Cosmological Constant Scale

This section presents a compact, self-contained derivation of the observed cosmological constant energy density from null-first horizon bookkeeping. No dynamical field equations or vacuum mode sums are assumed.

Geometric inputs

For a de Sitter horizon with Hubble parameter H_Λ, the horizon radius is

$$R_\Lambda = \frac{c}{H_\Lambda}. \tag{111}$$

The horizon area and enclosed volume are

$$A_\Lambda = 4\pi R_\Lambda^2, \qquad V_\Lambda = \frac{4\pi}{3} R_\Lambda^3. \tag{112}$$

Horizon pixelation and action

We discretize the horizon into Planck-area pixels,

$$N_{\text{pix}} = \frac{A_\Lambda}{\ell_P^2}, \tag{113}$$

and assign a fundamental action increment $\Delta S_{\text{pix}} = \hbar/4$ per pixel. The total horizon action is therefore

$$S_\Lambda = N_{\text{pix}} \frac{\hbar}{4} = \frac{\hbar}{4} \frac{4\pi R_\Lambda^2}{\ell_P^2} = \pi\hbar \frac{R_\Lambda^2}{\ell_P^2}. \tag{114}$$

Using the Planck-area identity $\ell_P^2 = \hbar G/c^3$, this becomes

$$S_\Lambda = \pi \frac{c^3}{G} R_\Lambda^2 = \pi \frac{c^5}{G H_\Lambda^2}. \tag{115}$$

Rendering action into energy

Energy is reconstructed as action per unit time. For a stationary de Sitter horizon, the appropriate timescale is the thermal (Euclidean) period

$$\beta_\Lambda = \frac{2\pi}{H_\Lambda}. \tag{116}$$

The associated horizon energy is

$$E_\Lambda = \frac{S_\Lambda}{\beta_\Lambda} = \left(\pi \frac{c^5}{G H_\Lambda^2}\right)\left(\frac{H_\Lambda}{2\pi}\right) = \frac{c^5}{2 G H_\Lambda}. \tag{117}$$

Energy density

Dividing by the enclosed horizon volume yields the energy density

$$\varepsilon_\Lambda = \frac{E_\Lambda}{V_\Lambda} = \frac{\frac{c^5}{2GH_\Lambda}}{\frac{4\pi}{3}\left(\frac{c}{H_\Lambda}\right)^3} = \frac{3c^2 H_\Lambda^2}{8\pi G}. \tag{118}$$

Equivalently, the corresponding mass density is

$$\rho_\Lambda = \frac{\varepsilon_\Lambda}{c^2} = \frac{3H_\Lambda^2}{8\pi G}. \tag{119}$$

22.1 Remarks

This result is obtained without summing vacuum modes or introducing free parameters. The scale of the cosmological constant follows from horizon area counting, action quantization, and the stationary thermal timescale of the de Sitter horizon.

Part IV
Conclusions

23 Positioning Relative to Existing Work

It is useful to distinguish the present framework by its choice of primitive variables. In Regge-style discretizations, edge lengths are primary and curvature is encoded by deficit angles [34]. In loop-quantum-gravity programs, discrete area spectra arise from representation labels on graphs and the fundamental kinematics is formulated in terms of connection/holonomy variables and fluxes [36, 37, 38]. In causal-set programs, order and counting are taken as primitive and spacetime volume emerges from element counts [39, 40, 41]. By contrast, the null-first picture takes *null relations* and their *projective angular data* as primary, following the classical analysis of null hypersurfaces and asymptotic structure [10, 11, 9, 31].

A central organizing principle is that metric quantities (lengths, volumes, energies) appear only after closure and timelike reconstruction. In this sense, the present construction is not a discretization of an already-metric spacetime, but a kinematical ordering in which null boundary structure is specified first, and reconstructed bulk quantities are secondary descriptions constrained by boundary consistency. The comparison points below are therefore meant as points of contact, not claims of equivalence.

23.1 Null hypersurfaces, characteristic evolution, and the "metric from null data" ethos

The emphasis on null structure aligns naturally with the characteristic (null) initial value formulation of general relativity, where free data is specified on null hypersurfaces and the bulk geometry is reconstructed by propagation along null generators [11, 42]. Related null-surface formalisms also treat families of null hypersurfaces as fundamental and view the metric as derivative data constrained by integrability and consistency conditions [9]. The present work can be read as a discretized kinematical analogue of this viewpoint: null boundary relations are primitive, while the metric is a reconstructed bookkeeping device that becomes meaningful only after a timelike rendering map is defined.

23.2 Carrollian geometry and the intrinsic structure of null boundaries

A null hypersurface carries a degenerate intrinsic geometry: a preferred null direction together with an induced (rank-two) metric on spatial cross-sections. This naturally leads to *Carrollian* structures as the appropriate intrinsic kinematics on null boundaries and null infinity, where the causal structure is "frozen" along the null direction while angular geometry remains nondegenerate [43, 15, 44, 45]. In this language, the null-first postulate that "angles are primary while timelike reconstruction is secondary" can be viewed as taking the Carrollian boundary data as fundamental and regarding Lorentzian bulk geometry as emergent from consistency constraints and reconstruction. We emphasize that we do not assume a particular Carrollian connection or boundary dynamics here; we only use the structural fact that null boundaries support canonical angular data and preferred null generators.

23.3 BMS symmetry, soft structure, and celestial viewpoints

At null infinity, the asymptotic symmetry group is the Bondi–Metzner–Sachs (BMS) group, whose action organizes gravitational radiation, memory effects, and the identification of physically meaningful "cuts" of $\mathscr{I}$ [10, 11, 46]. Recent developments in celestial holography further emphasize that scattering data and radiative degrees of freedom admit a natural description on the celestial two-sphere, with conformal weights encoding energy/frequency information and spin weights encoding helicity [47, 46]. The present framework shares the same organizing emphasis: angular (celestial) data is primary, while energy and time scales appear only after a rendering map selects an appropriate timelike parameterization (e.g. via thermal/KMS periods for stationary horizons). We do not assume the full celestial

CFT program; we note the conceptual compatibility in treating the S^2 of null directions as the natural state space for kinematics.

23.4 Twistor theory as a null-primitive coordinate system

Twistor theory was introduced precisely to reformulate spacetime physics in terms of null structure: points in spacetime arise as derived objects, while twistors encode null directions together with phase/spinorial data [48, 49, 50]. In Minkowski space, projective spinors parameterize null directions, and the incidence relations encode how null rays intersect to produce spacetime events. From this perspective, "null primacy" is not an idiosyncratic choice but a well-established alternative kinematical foundation. The present work does not import the complex-analytic machinery of twistor geometry; rather, it uses a discretized action-counting substrate whose primitive observables are (i) null relations and (ii) projective angular data on an S^2. Nevertheless, the shared geometric spine is clear: both approaches treat null rays/sheets as fundamental and regard metric spacetime as reconstructed structure.

23.5 Thermodynamic and holographic gravity

The present reconstruction perspective is compatible with thermodynamic and holographic approaches to gravity [12, 6, 33]. Jacobson derives Einstein's equations from $\delta Q = T\delta S$ applied to local Rindler horizons [12], while Padmanabhan emphasizes horizon degrees of freedom and holographic equipartition in cosmology [6]. Holographic dark energy models motivate $\rho \propto 1/(GR^2)$ using an infrared cutoff scale [51]. The present framework differs in that the relevant time scale and normalization are fixed by (i) the pixelated horizon action bookkeeping and (ii) the stationary KMS/thermal period, rather than introduced phenomenologically. In particular, the de Sitter energy density scale is obtained without assigning independent vacuum energy to bulk degrees of freedom; the large QFT discrepancy is traced to a counting measure that ignores holographic constraints.

23.6 Summary of the distinguishing choice

The distinguishing choice is thus not a new discretization of metric geometry, but a re-ordering of kinematics: null boundary structure and projective angular data are taken as primitive; metric quantities, energy, and timelike evolution are reconstructed secondary descriptions constrained by closure and consistency. This ordering places the framework in direct contact with the characteristic/null-surface tradition, Carrollian boundary kinematics, BMS/celestial structures, and the twistor program, while remaining operationally minimal in the present (kinematical) paper.

24 Closing Remarks

This paper argued for a simple but strict reordering of what is taken as fundamental in spacetime physics. Null structure is primary. Timelike spacetime, energy, volume, and dynamics are reconstructed descriptions that become available only after null relations are organized into consistent geometric closures.

The core move was kinematical rather than dynamical. We did not assume Einstein's equations, vacuum energy, or a microscopic field theory in the bulk. Instead, we asked what follows if null geometry is treated as an ontological boundary condition, with angular data, action, and causal connectivity as the primitive ingredients. From that starting point, several otherwise disconnected results line up with little additional input.

First, null geometry supports angular but not metric information. This singles out the celestial sphere as the intrinsic carrier of degrees of freedom, consistent with classical analyses of null hypersurfaces and asymptotic structure [9, 11, 10]. Once this is accepted, area rather than volume becomes the natural counting measure, and holographic scaling is no longer an added principle but a direct consequence of null degeneracy, in line with earlier insights by 't Hooft [13] and Susskind [14].

Second, the tetrahedron emerges as the unique minimal volumetric element compatible with isotropy, closure, and consistent action bookkeeping. The appearance of the factor $1/4$ throughout gravitational physics is then traced to a purely geometric fact: four equal-area faces are required to close a minimal voxel. This gives a concrete microscopic interpretation of the Bekenstein Hawking entropy formula [3, 4] without invoking additional statistical assumptions.

Third, Newton's constant reveals its meaning when written as

$$G = \frac{\ell_P^2 c^3}{\hbar}.$$

In this form, G encodes three independent facts at once: holography, through the equivalence of bulk and boundary action; dimensionality, through the c^3 volumetric construction rate from three null directions; and universality, through the fixed action cost $\hbar$ per fundamental angular degree of freedom. This perspective aligns with and sharpens thermodynamic approaches to gravity, such as those of Jacobson [12] and Padmanabhan [6], by supplying a concrete kinematical substrate.

Most importantly, the cosmological constant ceases to be mysterious. When action, not energy, is taken as primitive, and when degrees of freedom are counted on null boundaries rather than in bulk volume, the observed value of ρ_Λ follows directly. The factor of 10^{122} is not a fine-tuning problem but the number of Planck-area angular pixels on the asymptotic de Sitter horizon. Rendering their total action into energy using the unique thermal timescale fixed by the KMS condition and Euclidean regularity [18] reproduces the Friedmann form

$$\rho_\Lambda = \frac{3H_\Lambda^2 c^2}{8\pi G}$$

with no adjustable parameters. In this sense, dark energy is not vacuum energy stored in spacetime but the baseline projection cost of maintaining spacetime itself.

This picture connects naturally to, but is distinct from, other holographic and emergent gravity proposals. Unlike phenomenological holographic dark energy models [51], no infrared cutoff or fitted coefficient is introduced. Unlike causal set approaches with sequential growth [52], temporal ordering is not taken as primitive at the null level but arises only after timelike reconstruction. The framework also complements loop and spin network ideas by fixing the area quantum directly through geometric closure rather than representation labels.

The scope of this paper has been deliberately limited. We have not derived the full Einstein field equations, nor explained why the universe has its particular asymptotic entropy or expansion rate. Those questions belong to dynamics, not kinematics, and are addressed in a companion work where the evolution of angular phase networks is studied. What has been established here is that the null-first kinematical structure is internally consistent, tightly constrained, and already sufficient to account for several deep numerical facts about gravity and cosmology.

The broader lesson is methodological. Many of the hardest problems in fundamental physics appear insoluble when framed in terms of energy, volume, and local bulk degrees of freedom. When reframed in terms of null geometry, action, and boundary data, those same problems often collapse into bookkeeping identities. The cosmological constant problem is the clearest example, but the same perspective also clarifies inverse-square laws, horizon entropy, and the universality of gravitational coupling.

If future observations were to establish a deviation from $w = -1$ for dark energy, or a breakdown of holographic entropy bounds, this framework would be falsified. Short of that, null-first kinematics offers a coherent and economical foundation on which a full theory of spacetime dynamics can be built.

Part V

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