Chapter 2 Exercises

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1. **Exercise 2.1: In ϵ-greedy action selection, for the case of two actions and $\epsilon = 0.5$, what is the probability that the greedy action is selected?**

   0.75

2. **Exercise 2.2 (Bandit example): Consider a k-armed bandit problem with $k = 4$ actions, denoted 1, 2, 3, and 4. Consider applying to this problem a bandit algorithm using ϵ-greedy action selection, sample-average action-value estimates, and initial estimates of $Q1(a) = 0$, for all a. Suppose the initial sequence of actions and rewards is $A_1 = 1$, $R_1 = -1$, $A_2 = 2$, $R_2 = 1$, $A_3 = 2$, $R_3 = -2$, $A_4 = 2$, $R_4 = 2$, $A_5 = 3$, $R_5 = 0$. On some of these time steps the e case may have occurred, causing an action to be selected at random. On which time steps did this definitely occur? On which time steps could this possibly have occurred?**

   The epsilon case definitely occurred for $A_4 = 2$ because $A_2 = 2$ and $A_3 = 2$ received an average reward of -1 which would be less than the other initally zero valued states. It should also be noted, however, that any or all of these five actions could be epsilon cases and they randomly selected the greedy option.

3. **Exercise 2.3: In the comparison shown in Figure 2.2, which method will perform best in the long run in terms of cumulative reward and probability of selecting the best action? How much better will it be? Express your answer quantitatively.**

   The method utilizing epsilon greedy search with an epsilon value of 0.01 will perform best in the long run in terms of cumulative reward and probability of selecting the best action because it will converge to a probability of $1 - e$ of selecting the optimal action. Given that, it is trivial that $1 - 0.01 > 1 - 0.1$.

4. **Exercise 2.4: If the step-size parameters, α_n, are not constant, then the estimate Q_n is a weighted average of previously received rewards with a weighting different from that given by (2.6). What is the weighting on each prior reward for the general case, analogous to (2.6), in terms of the sequence of step-size parameters?**

$$\prod_{i=1}^{N}(1-\alpha_i)Q_1 + \sum_{i=1}^{N}(\alpha_i * \prod_{j=i+1}^{N}(1-\alpha_j) * R_i)$$

5. **Exercise 2.5 (programming): Design and conduct an experiment to demonstrate the difficulties that sample-average methods have for nonstationary problems. Use a modified version of the 10-armed testbed in which all the $q(a)$ start out equal and then take independent random walks (say by adding a normally distributed increment with mean 0 and standard deviation 0.01 to all the $q(a)$ on each step). Prepare plots like Figure 2.2 for an action-value method using sample averages,**

incrementally computed, and another action-value method using a constant step-size parameter, $\alpha = 0.1$. Use $\epsilon = 0.1$ and longer runs, say of 10,000 steps.

Figure 1: Average Reward

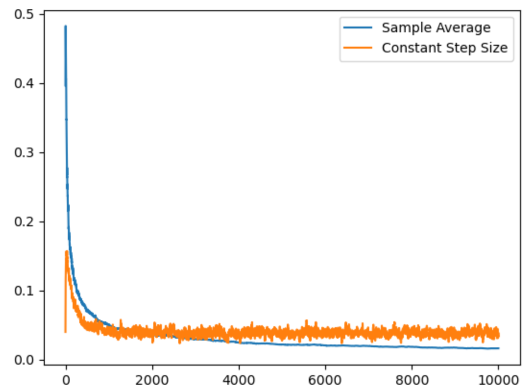
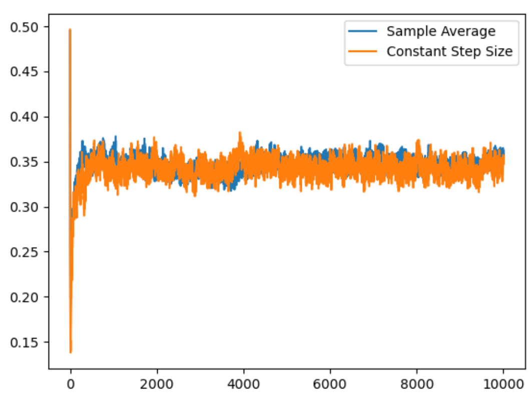


Figure 2: Optimal Action Frequency



Code in exercise_2_5.py

6. **Exercise 2.6 (Mysterious Spikes): The results shown in Figure 2.3 should be quite reliable because they are averages over 2000 individual, randomly chosen 10-armed bandit tasks. Why, then, are there oscillations and spikes in the early part of the curve for the optimistic method? In other words, what might make this method perform particularly better or worse, on average, on particular early steps?**

   The early oscillations and spikes are caused by the initial optimism and exploration of the agent. ϵ-greedy methods with initial values of 0 only take exploratory actions ϵ% of the time which leads to a more gradual convergence. In the case of the fully greedy optimistic initial values method this is different. Because it explores so much more freely early on, it will inevitably select the optimal action at a higher frequency.

7. **Exercise 2.7 (Unbiased Constant-Step-Size Trick): In most of this chapter we**

have used sample averages to estimate action values because sample averages do not produce the initial bias that constant step sizes do (see the analysis leading to (2.6)). However, sample averages are not a completely satisfactory solution because they may perform poorly on nonstationary problems. Is it possible to avoid the bias of constant step sizes while retaining their advantages on nonstationary problems? One way is to use a step size of $\beta_n = \alpha_n/\bar{o}_n$, to process the nth reward for a particular action, where $\alpha > 0$ is a conventional constant step size, and $\bar{o}_n$ is a trace of one that starts at 0: $\bar{o}_n = \bar{o}_{n-1} + \alpha(1 - \bar{o}_{n-1})$ for $n \geq 0$, with $\bar{o}_0 = 0$. Carry out an analysis like that in (2.6) to show that Q_n is an exponential recency-weighted average *without initial bias.*

Q_n is an exponential recency-weighted average without initial bias if the step size α obeys the following two rules: (1) $\sum_{n=1}^{\infty} \alpha_n(a) = \infty$ and (2) $\sum_{n=1}^{\infty} \alpha_n^2(a) < \infty$.

Q_n is an exponential recency-weighted average using the step-size of β_n,

$$
\begin{aligned}
Q_{n+1} &= Q_n + \beta_n[R_n - Q_n] \\
&= \beta_n R_n + (1 - \beta_n)Q_n \\
&= \beta_n R_n + (1 - \beta_n)[\beta_{n-1} + (1 - \beta_{n-1}Q_{n-1}] \\
&= \prod_{i=1}^{n}(1 - \alpha/\bar{o}_i)Q_1 + \sum_{j=1}^{n} \alpha/\bar{o}_j R_j \prod_{k=j+1}^{n}(1 - \alpha/\bar{o}_k)
\end{aligned}
$$

Q_n does not have initial bias,

$$
\begin{aligned}
\sum_{n=1}^{\infty} \beta_n(a) &= \sum_{n=1}^{\infty} \alpha/\bar{o}_n = \alpha \sum_{n=1}^{\infty} 1/\bar{o} = \infty \\
\sum_{n=1}^{\infty} \beta_n^2(a) &= \sum_{n=1}^{\infty} \alpha^2/\bar{o}_n^2 = \alpha^2 \sum_{n=1}^{\infty} 1/\bar{o}^2 < \infty
\end{aligned}
$$

8. **Exercise 2.8 (UCB Spikes): In Figure 2.4 the UCB algorithm shows a distinct spike in performance on the 11th step. Why is this? Note that for your answer to be fully satisfactory it must explain both why the reward increases on the 11th step and why it decreases on the subsequent steps. Hint: if c = 1, then the spike is less prominent.**

   The UCB algorithm treats unvisited actions as maximizing actions, so after each action action has been visited once, the agent begins to select actions greedily which increases the average returns.

9. **Exercise 2.9: Show that in the case of two actions, the soft-max distribution is the same as that given by the logistic, or sigmoid, function often used in statistics and artificial neural networks.**

$$P(A_t = 1) = 1/(1 + \exp^{-x})$$

10. **Exercise 2.10: Suppose you face a 2-armed bandit task whose true action values change randomly from time step to time step. Specifically, suppose that, for any time step, the true values of actions 1 and 2 are respectively 10 and 20 with probability 0.5 (case A), and 90 and 80 with probability 0.5 (case B). If you are not able to tell which case you face at any step, what is the best expected reward you can achieve and how should you behave to achieve it? Now suppose that on each step you are told whether you are facing case A or case B (although you still don't know the true action values). This is an associative search task. What is the best expected reward you can achieve in this task, and how should you behave to achieve it?**