Homework #6: Machine Learning

Kyler Little

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Problem #1

Let $A = U\Sigma V^T$ be the SVD of A, where $A \in R^{m\times n}$, $U \in R^{m\times m}$ and $V \in R^{n\times n}$ are orthogonal matrices, $\Sigma = \text{diag}(\sigma_1, \cdots, \sigma_r, 0, \cdots, 0)$, and $r = \text{rank}(A)$. Show that:
(a) The first r columns of U are eigenvectors of AA^T corresponding to nonzero eigenvalues.
(b) The first r columns of V are eigenvectors of A^TA corresponding to nonzero eigenvalues.

Problem #2

Given a symmetric matrix $A \in R^{3\times 3}$, suppose its eigen-decomposition can be written as:

$$A = \begin{pmatrix} u_{11} & u_{12} & u_{13} \\ u_{21} & u_{22} & u_{23} \\ u_{31} & u_{32} & u_{33} \end{pmatrix} \begin{pmatrix} 3 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u_{11} & u_{12} & u_{13} \\ u_{21} & u_{22} & u_{23} \\ u_{31} & u_{32} & u_{33} \end{pmatrix}$$

What is the singular value decomposition of this matrix?

Problem #3

Given a data matrix $X = \begin{bmatrix} x_1, & x_2, & \cdots & x_n \end{bmatrix} \in R^{p\times n}$ consisting of n data points, and each data point is p-dimensional,
- Outline the procedure for computing the PCA of X.
- State what is the minimum reconstruction error property of PCA.
- Prove the minimum reconstruction error property of PCA by using the best low-rank approximation property of SVD.

Problem #4

Use the similarity matrix in Table 1 to perform single (MIN) and complete (MAX) link hierarchical clustering. Show your results by drawing a dendrogram. The dendrogram should clearly show the order in which the points are merged.

Table 1: Similarity Matrix

	p1	**p2**	**p3**	**p4**	**p5**
p1	1.00	0.10	0.41	0.55	0.35
p2	0.10	1.00	0.64	0.47	0.98
p3	0.41	0.64	1.00	0.44	0.85
p4	0.55	0.47	0.44	1.00	0.76
p5	0.35	0.98	0.85	0.76	1.00

Problem #5

Summarize results of PCA implementation and attach images.