Dynamic Mechanism Under Implied Volatility Via A Multifactor Model

10 June 2018

Presenter: Kai Xiang

What is implied volatility?

- Mathematically, the implied volatility of an option contract is the value which when input in an option pricing model (such as Black–Scholes) will return a theoretical value equal to the current market price of the option.

$$C(S_t, t) = N(d_1)S_t - N(d_2)Ke^{-r(T-t)}$$

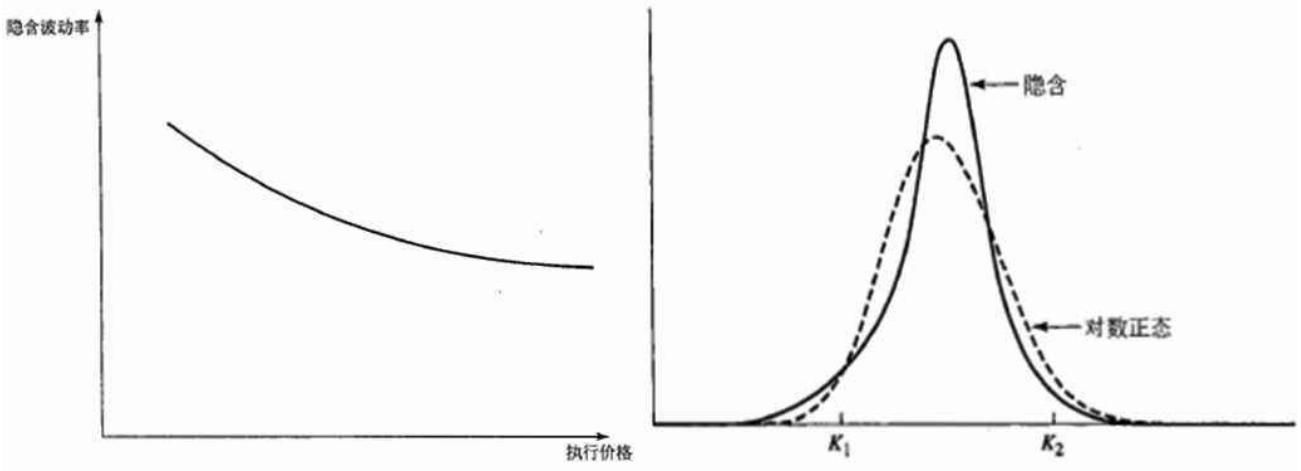
$$d_1 = \frac{1}{\sigma\sqrt{T-t}}\left[\ln\left(\frac{S_t}{K}\right) + \left(r + \frac{\boldsymbol{\sigma^2}}{2}\right)(T-t)\right]$$

$$d_2 = d_1 - \boldsymbol{\sigma}\sqrt{T-t}$$

- Economically, it shows how the market views where volatility should be in the future. It is a forward-looking indicator, that helps us gauge the sentiment about the volatility of a stock or the market. Note that it doesn't forecast the direction.

Implied volatility smile and fat tail phenomena

- Options based on the same underlying with different strike and expiration times will yield different implied volatilities (Non-constant).



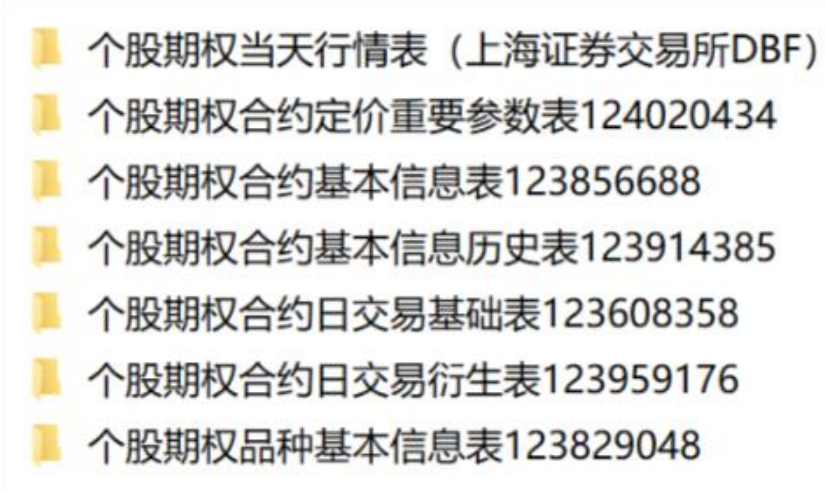
- In this project, I want to model the implied volatility from both cross-section and time series perspectives, then to capture some characteristics and use it to do further application.

  e.g. prediction, trading strategy, …

北京大学 PEKING UNIVERSITY

Data collection

- To study volatility, high quality dataset is necessary, especially when fit the surface, it highly rely on considerable data points.
- I use the option dataset from CSMAR(国泰安), for its through option information and long time duration.
- The original data files are divided into several modules. So I do some work to match these tables into a single excel file.



F	G	H	I	J	K	L
trikePric	ExerciseDate	TradingDate	lyingScrt	mainingTe	sklessRa	icalVolai
1.855	2014-12-24	2014-11-03	2.189	0.1397	3	0.1372
2.099	2014-12-24	2014-11-03	2.189	0.1397	3	0.1372
2.1	2014-12-24	2014-11-03	2.189	0.1397	3	0.1372
2.148	2014-12-24	2014-11-03	2.189	0.1397	3	0.1372
1.953	2014-12-24	2014-11-03	2.189	0.1397	3	0.1372
2.15	2014-12-24	2014-11-03	2.189	0.1397	3	0.1372
2	2014-12-24	2014-11-03	2.189	0.1397	3	0.1372
2.197	2014-12-24	2014-11-03	2.189	0.1397	3	0.1372
2.2	2014-12-24	2014-11-03	2.189	0.1397	3	0.1372
2.001	2014-12-24	2014-11-03	2.189	0.1397	3	0.1372
2.2	2014-11-26	2014-11-03	2.189	0.063	3	0.1372
2.05	2014-12-24	2014-11-03	2.189	0.1397	3	0.1372
2.245	2014-12-24	2014-11-03	2.189	0.1397	3	0.1372
2.25	2014-12-24	2014-11-03	2.189	0.1397	3	0.1372
2.3	2014-11-26	2014-11-03	2.189	0.063	3	0.1372
2.099	2014-12-24	2014-11-03	2.189	0.1397	3	0.1372
2.1	2014-12-24	2014-11-03	2.189	0.1397	3	0.1372
2.148	2014-12-24	2014-11-03	2.189	0.1397	3	0.1372

Data collection

- Important and essential information is contained such as
  key: unique identification for each item.
  - Sample: 20900000181211000103 2015-01-09
  ShortName: A brief Chinese name for every contract.
  - Sample: 180ETF沽3月3400, 沽-put, 购-call.
  CallOrPut: Option type.
  - 1: call, 0: put
  RemainingTerm: the time left before maturity
  - Sample: 0.127, unit: year
  ExerciseDate: The maturity date.
  TradingDate: All the price, volumn, volatility are collected on this day.
  HistoricalVolatility/ImpliedVolatility
  OpenPrice/ClosePrice/HighPrice/LowPrice
  Volume: Trading volume on that day.

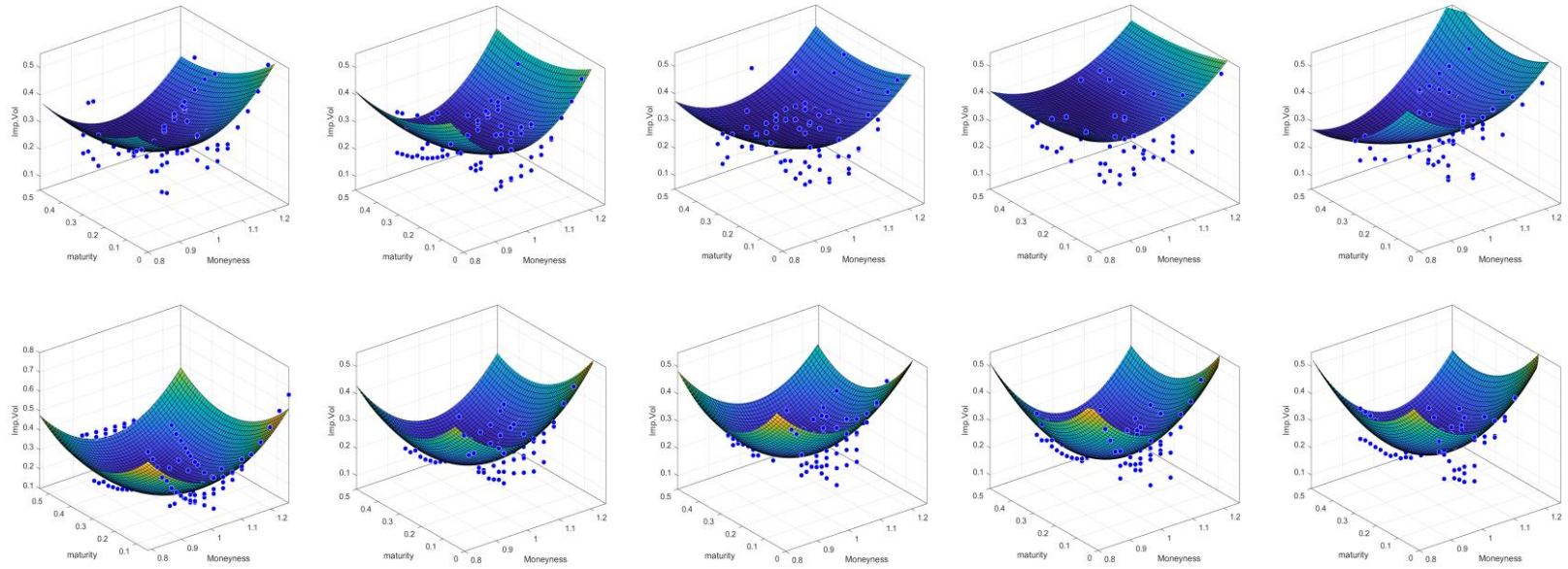
Data collection

```
% ====================== Part Two: Clean Data ==========================
% The project intend to analyze the mechanism of implied volatility, this
% task highly rely on high quality data, so a filtration and clean is
% necessary. Here are some assumptions while we choose data:
%   1. Any contract that have too few trading volumn is invalid. Say, if
%   trading  volumn is less than 10, we don't use it.
%   2. After (1), any date that has no more that three type maturity is
%   invalid. Because we want to fit the implied volatility surface, this
%   condition is compulsory, otherwise, it will be definitely overfitted.
%   3. Any contract with abnormal implied volatility is invalid. We ignore
%   some very strange data, most of them are at near maturity.
%
%   PS. Some procedures are done in python script clean.py for convenience.
```

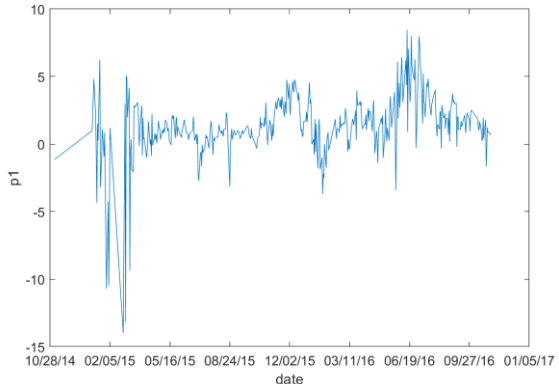
Fit the surface

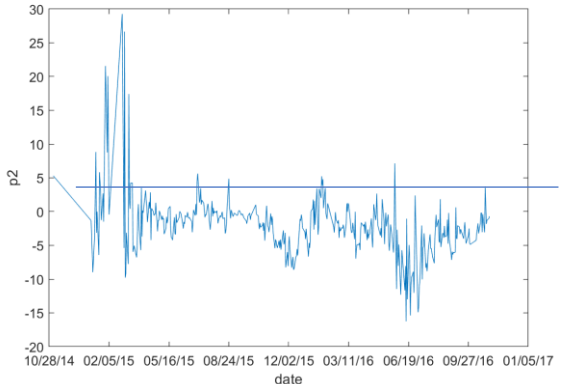
- If plot volatilities of different strike and maturity in one graph, we obtain an implied volatility surface, however, we want to find a approximation, a model that can use several parameters to determinate the surface.
- Here I adopt a five-factor model, the details about the model choosing is my report and code.

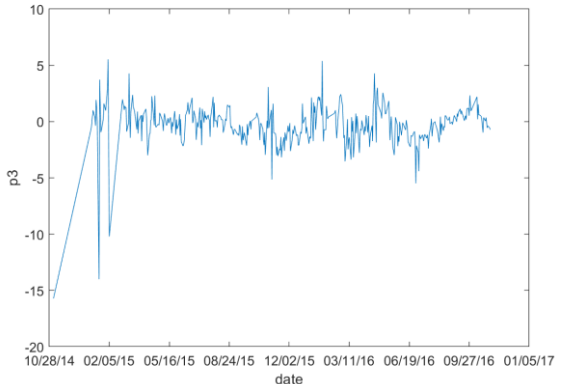
$$\sigma(m,\tau) = \beta_1 + \beta_2 * m + \beta_3 * t + \beta_4 * m^2 + \beta_5 * t * m + \beta_6 * t^2$$

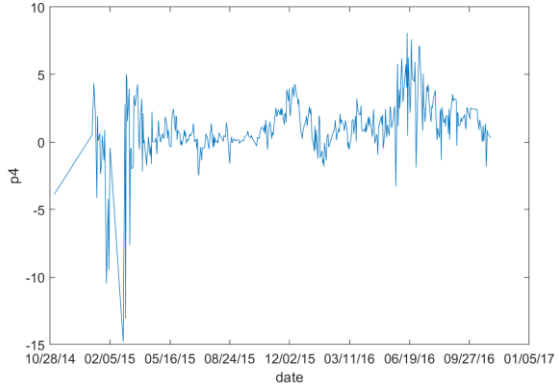


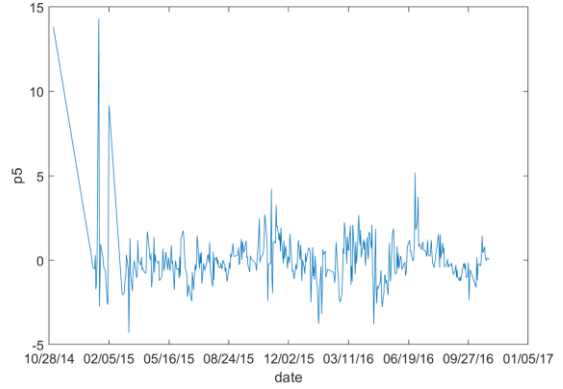
Regard parameters as time series

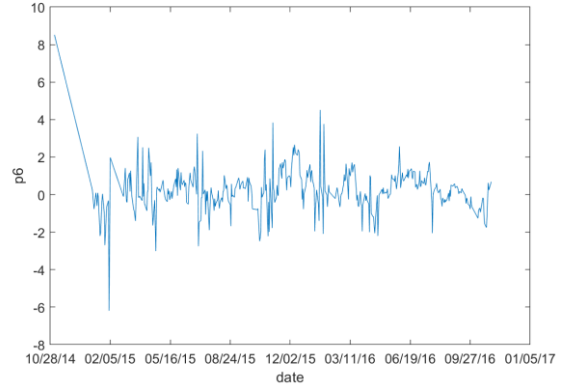






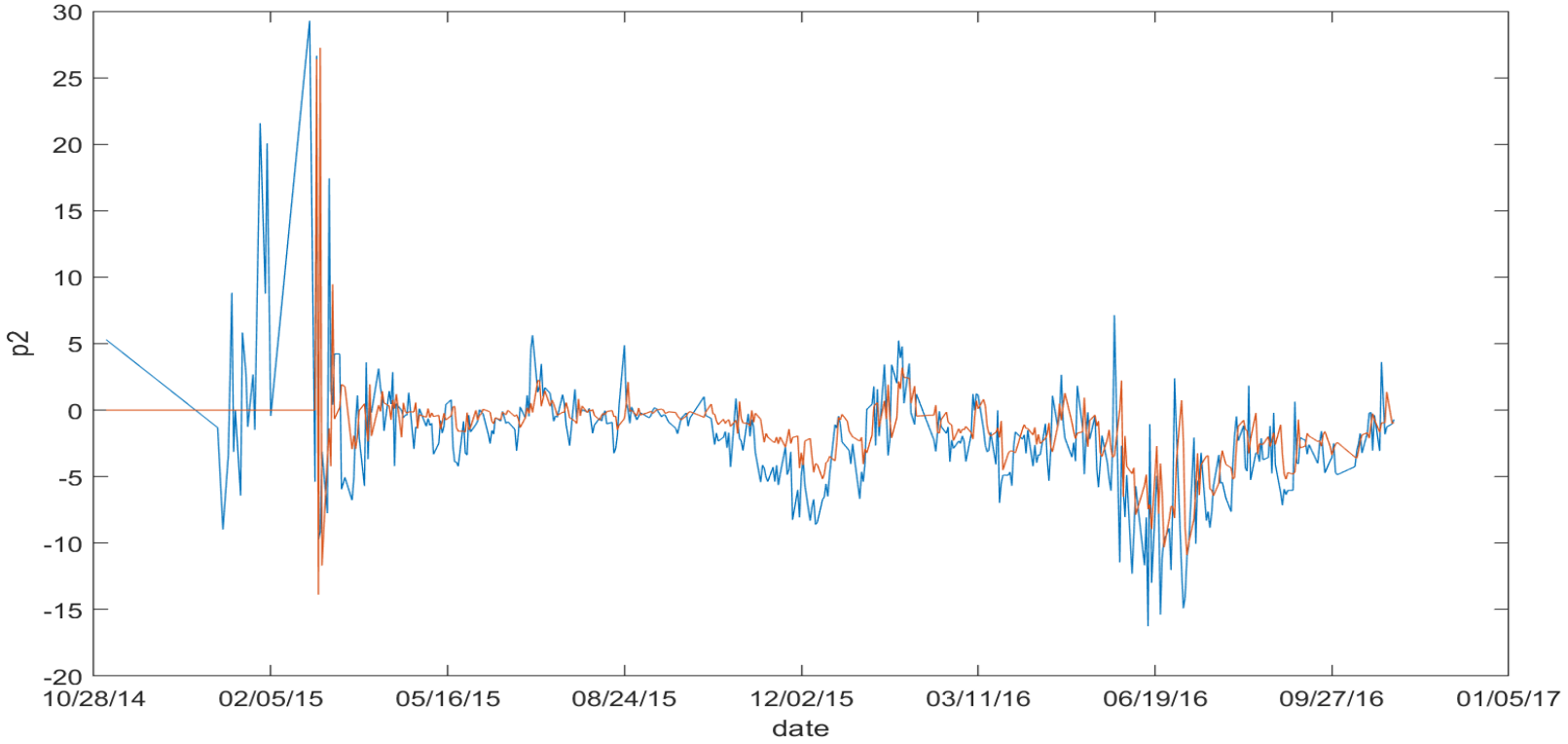






- Very unstable, mean reverting.
- Try to build a time series model to predict, say, ARMA(p,q)

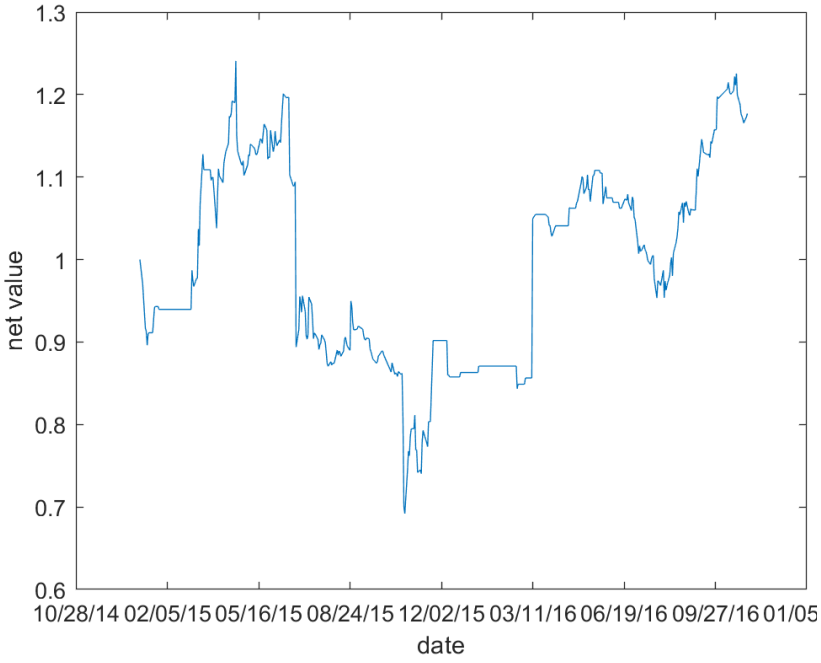
ARMA(2,1) model



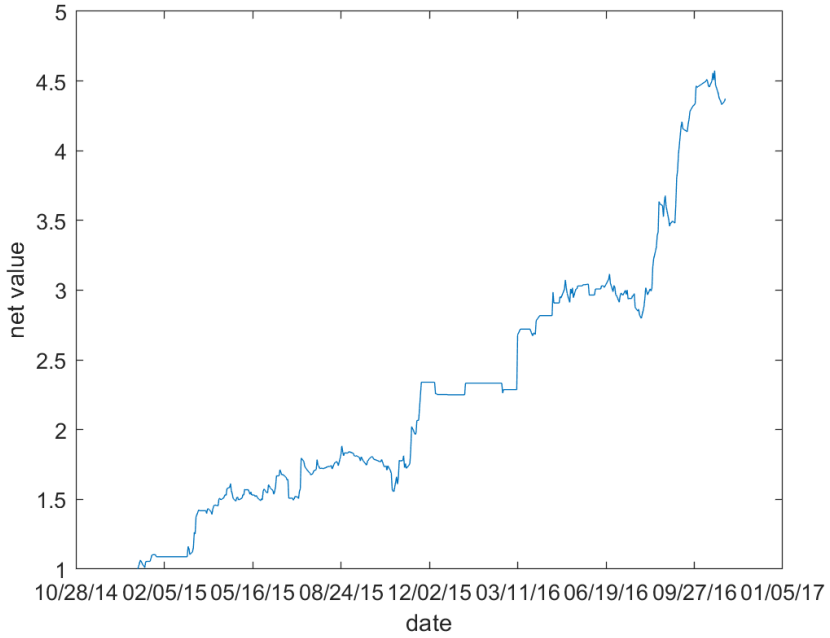
- Here I adopt a ARMA(2,1) model to predict next day's parameters, the details about the model choosing is my report and code, mainly using AIC rule.
- We will use our prediction in following part.

Some trading strategy demos

- Long underpriced contracts, and short overpriced contracts.
- Buy those contract whose implied volatility is greater than theoretical value, vice versa.
- The trading signal is fixed, so the most important part here is position management.

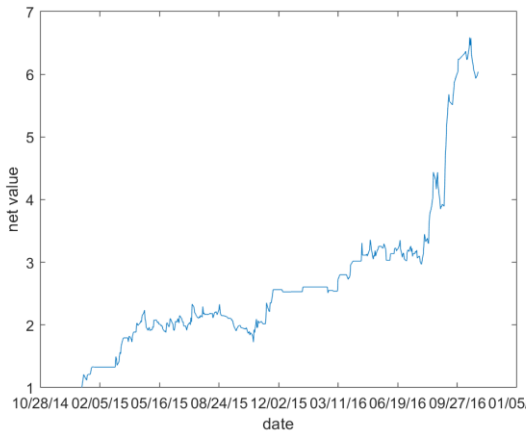


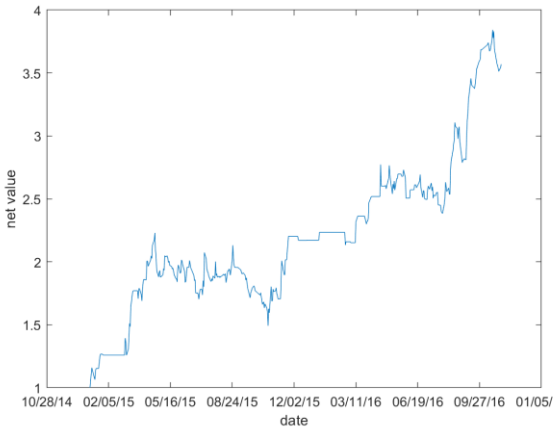
50% short, 50% long

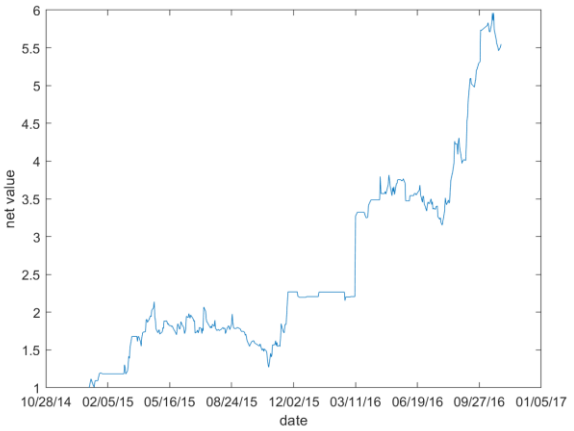


10% short, 20% long

Some trading strategy demos





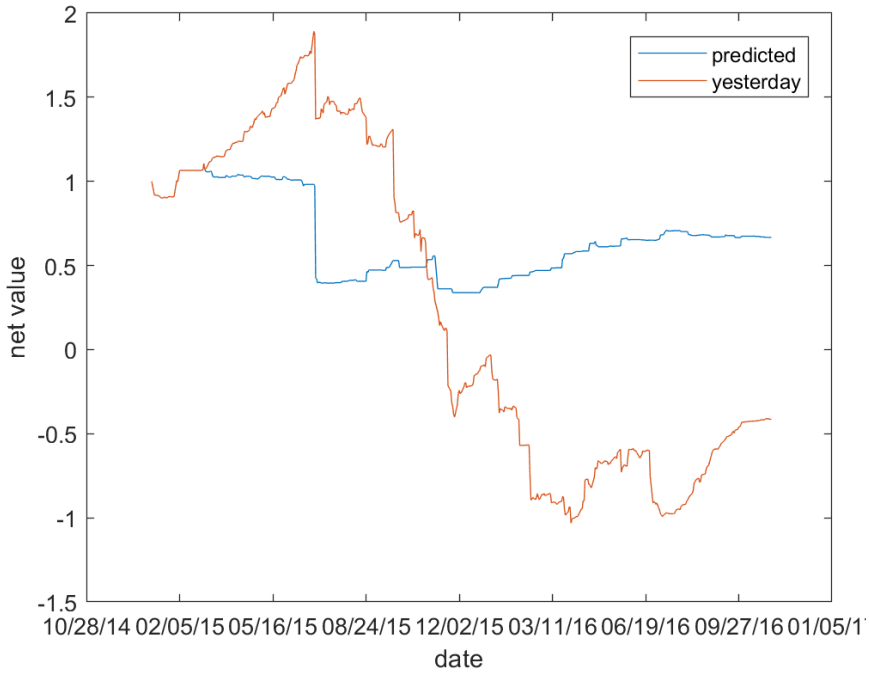


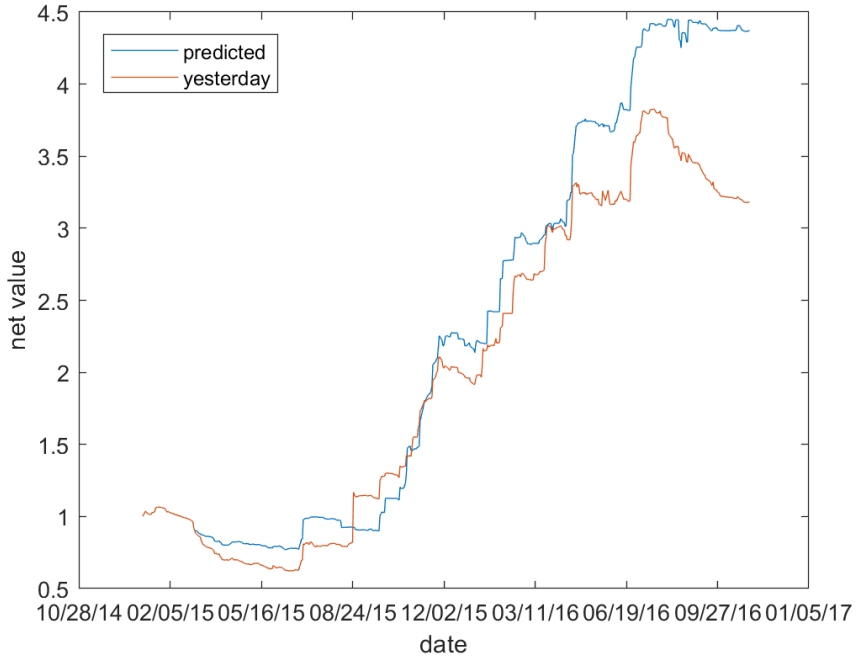
- Some concerns.
- How we diversify our money in different contract?
  Same volume; same amount; more complicated forms…
- Be careful about some details, the loss of short is unlimited, and price change of deep out of money option is very sensitive.

- Above are some different money distribution strategies' back-test curves.
- Adjust weight according to volatility, special treatment for extremely cheap options.

- Now let's use that time series model that we have derived.

Some trading strategy demos





- Still, left is the radical type, the right is more unadventurous.

  Position = f(price, imp.Vol, Number of contract)

One single day illustration

```
>> sum(position.*price)
sum(position.*openprice)

ans =

    0.1903


ans =

    0.0092
```

Short? 0.002 -> 0.2328?

```
>> [openprice price longShortType position]

ans =

    0.0090    0.0002    1.0000    0.2139
    0.0092    0.0030   -1.0000   -0.0004
    0.2336    0.0010   -1.0000   -0.0002
    0.0002    0.0117   -1.0000   -0.0011
    0.0002    0.0030   -1.0000   -0.0011
    0.0935    0.0643   -1.0000   -0.0002
    0.2500    0.0016   -1.0000   -0.0001
    0.0114    0.0004    1.0000    0.1998
    0.0300    0.0568   -1.0000   -0.0003
    0.0006    0.0735   -1.0000   -0.0008
    0.0004    0.1008   -1.0000   -0.0009
    0.0090    0.0237   -1.0000   -0.0004
    0.0128    0.0060    1.0000    0.1932
    0.0002    0.2328   -1.0000   -0.0011
    0.3501    0.6461   -1.0000   -0.0001
    0.0001    0.0482   -1.0000   -0.0014
    0.0027    0.0500   -1.0000   -0.0006
    0.0008    0.0007   -1.0000   -0.0008
    0.0004    0.0321   -1.0000   -0.0009
    0.0004    0.0162   -1.0000   -0.0009
    0.0001    0.0038    1.0000    0.7414
    0.0002    0.0066   -1.0000   -0.0011
    0.0010    0.2166    1.0000    0.3988
    0.0008    0.1466    1.0000    0.4242
    0.0010    0.1278   -1.0000   -0.0007
    0.0353    0.0543   -1.0000   -0.0003
    0.0019    0.0580   -1.0000   -0.0006
    0.0034    0.1518   -1.0000   -0.0005
    0.0024    0.0094   -1.0000   -0.0006
    0.0014    0.0234   -1.0000   -0.0007
    0.0058    0.0805    1.0000    0.2427
    0.0002    0.0312    1.0000    0.6181
```

further extension

- Difference between put and call
- Practicable implement of arbitrage: straddle, calendar arbitrage…
- More flexible position management
- More accurate model

Thank you!