

RESEARCH STATEMENT

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1. INTRODUCTION

1.1. Short summary. Symplectic geometry is a rapidly evolving branch of mathematics with intimate connections to smooth topology, differential geometry, algebraic geometry, and mathematical physics. My research focuses on developing new tools to study symplectic manifolds, complemented by geometric constructions designed to test the boundaries of the field. My work has led me to study new symplectic invariants in terms of deformation theory and to apply these to new constructions of exotic symplectic manifolds. These results have also found applications to convexity questions in several complex variables and affine algebraic geometry. More recently, I have been systematically developing new symplectic capacities, based on various higher algebraic structures in Floer theory and symplectic field theory. These capacities contain strong information about high dimensional symplectic embedding problems, a mysterious area close to the heart of Hamiltonian dynamics. Current work in progress aims to further study, compute, and generalize these capacities and place them in the context of enumerative algebraic geometry and homological mirror symmetry.

1.2. Background. At face value, symplectic geometry studies symplectic manifolds, i.e. smooth manifolds M^{2n} equipped with a closed, nondegenerate two-form ω . Historically, Hamilton observed in the 1830's that Newton's laws can be reformulated by stating that a physical system follows the symplectic gradient flowlines of the total energy function $H : M \rightarrow \mathbb{R}$. For example, a particle moving along a circular wire is controlled by a Hamiltonian $H : T^*S^1 \rightarrow \mathbb{R}$. Classically (M, ω) is a cotangent bundle with its canonical symplectic form, but more general symplectic manifolds also arise in physics, for example as phase spaces in gauge theories. In addition to conserving energy, the Hamiltonian flow preserves the symplectic form ω and hence also the canonical volume form $\omega^{\wedge n}$, a classical observation known as Liouville's theorem in statistical physics. However, it was not until Gromov's seminal 1985 paper [Gro2] that symplectic geometry revealed itself to be significantly richer and more mysterious than volume geometry. For example, the famous "non-squeezing theorem" states roughly that symplectic transformations cannot squeeze a large ball into a narrow infinite cylinder, a feat easily accomplished by volume preserving maps. This result and its many followups have ramifications for classical physics and beyond, highlighting the quest to better understand the nature of symplectic transformations.

Gromov's proof of the non-squeezing theorem is based on the key geometric idea of pseudoholomorphic curves. Building on this, Floer soon introduced his famous homology theory to solve Arnold's conjecture on fixed points of Hamiltonian systems. Subsequently, pseudoholomorphic curves have been packaged into increasingly elaborate algebraic structures, giving birth to invariants such as Gromov–Witten theory, Fukaya categories, and symplectic field theory. These tools have revealed a remarkable amount of rigidity in symplectic geometry, with applications to smooth manifolds via their cotangent bundles and to complex varieties via their Kähler forms. Many of these pseudoholomorphic curve invariants also play a prominent role in mirror symmetry, an unexpected bridge between symplectic geometry and algebraic geometry first discovered by string theorists and later formulated in categorical terms by Kontsevich.

Simultaneously, the flexible side of symplectic geometry has seen major progress in the last few years. Flexibility refers to phenomena which, sometimes quite unexpectedly, satisfy an *h-principle*, reducing a geometric PDE to algebraic topology. Gromov [Gro3] pioneered the use of h-principles in symplectic geometry, and his work led to the consensus that symplectic objects of subcritical

dimension satisfy h-principles, whereas critical objects (e.g. Lagrangians and Legendrians) are rigid and must be studied using hard techniques such as pseudoholomorphic curves. More recently, a revision to this perspective has been needed to account for Murphy’s h-principle for *loose Legendrians* [Mur], which extends flexibility into the critical dimension. This result facilitated the discovery of a number of other novel flexibility phenomena: flexible Stein and Weinstein manifolds [CE1], Lagrangian caps [EM], Lagrangian immersions with few double points [EEMS], and various others. These interrelated results seem to herald a new chapter of symplectic geometry.

1.3. My research. My research program can be broadly categorized into three interrelated lines of inquiry:

- (1) **New tools to approach the flexibility versus rigidity dichotomy.** In joint work [MS] with Emmy Murphy we discovered the surprising phenomenon that flexible Stein manifolds such as \mathbb{C}^n admit plurisubharmonic functions with symplectically rigid sublevel sets. These sublevel sets, which we call *subflexible*, are exotic symplectic manifolds with trivial symplectic cohomology. Unlike previously known constructions of exotic Stein manifolds, their analysis requires more delicate deformed versions of symplectic cohomology as developed in [Sie1]. We give a general procedure for constructing subflexible symplectic manifolds and use them to resolve some questions about flexibility posed by Cieliebak–Eliashberg in [CE1]. We also apply subflexibility to convexity questions in several complex variables, giving new examples of polynomially convex domains and disproving a conjecture from [CE2].

Although (sub)flexibility exists only in real dimension greater than four, in joint work [NS] with Stefan Nemirovski we investigate analogous questions for four-manifolds. We give new constructions of singular Lagrangian surfaces in \mathbb{C}^2 and show how to obstruct others using gauge theory and three-dimensional contact geometry, generalizing the well-known nonexistence theorem for Lagrangian Klein bottles in \mathbb{C}^2 ([She, Nem]). The result is a classification theorem for rationally convex domains in \mathbb{C}^2 , and as a by-product we obtain a complete classification of Lagrangian surfaces in \mathbb{C}^2 with open Whitney umbrella singularities, resolving a question first posed by Givental [Giv] in 1986.

- (2) **Symplectic capacities using higher algebraic structures in Floer theory and symplectic field theory.** In [Sie2] I construct a new family of capacities $g_{k,l}$, $k, l \in \mathbb{N}$, which are defined in every dimension and often outperform the Ekeland–Hofer capacities in symplectic embedding problems. One key property is that they are invariant under taking products with \mathbb{C} , and I prove that they give sharp obstructions in at least some cases of the stabilized ellipsoid embedding problem. Their construction is based on the filtered \mathcal{L}_∞ structure S^1 -equivariant symplectic cochains, or alternatively using the filtered \mathcal{L}_∞ structure on linearized contact homology.

In [Sie3] I show that these capacities can also be defined by counting curves with certain tangency conditions, thereby relating them to Gromov–Witten theory with gravitational descendants and recent work on superpotentials [Ton]. I also describe how to embed these capacities into a more general symplectic field theory framework involving point constraints, tangencies conditions, nodal singularities, and also potentially higher genus curves, thereby defining a vast class of capacities whose investigation is still in its infancy.

- (3) **Deformation theory in symplectic geometry.** In [Sie1] I prove that a certain local system version of symplectic cohomology is nontrivial for the subflexible examples from [MS]. In fact, I prove a structural theorem stating that any Stein manifold can be made subflexible in such a way that its symplectic cohomology becomes trivial but is nevertheless remembered in the presence of an appropriate local system. More surprisingly, there is also a derived local system version using bulk deformed symplectic cohomology which involves higher dimensional moduli spaces of curves and has no obvious geometric counterpart. The

proof is based on a new noncommutative analogue of the fragility of squared Dehn twists about Lagrangian spheres, a phenomenon which is apparently only visible through the lens of bulk deformed Fukaya categories.

In the forthcoming [Sie4], I take this a step further by embedding twisted and bulk deformed symplectic cohomology into a more general family of “quantitative” deformations of symplectic cohomology. These more general deformations contain important information about symplectic embeddings and can be used to extend the new capacities to non-exact settings.

In the following, I give further background and details on these developments and their applications and outline plans for future research.

2. NEW TOOLS TO APPROACH THE FLEXIBILITY VERSUS RIGIDITY DICHOTOMY

2.1. Context. A natural question related to the classification of symplectic manifolds is whether there exist symplectic manifolds which are diffeomorphic but not symplectomorphic to Euclidean space. Gromov produced such an exotic \mathbb{C}^2 in [Gro2], while more recent results [SS, McL1, May, MS, AS, Har] have produced a number of examples with convex boundary, in fact affine varieties. The principle tool for detecting nonflexibility has been *symplectic cohomology* (SH), which is the natural version of Hamiltonian Floer theory for open symplectic manifolds, analogous to quantum cohomology in the closed case. The expectation is that all pseudoholomorphic curve invariants are trivial for flexible Stein manifolds. We prove several versions of this in [MS], raising the question of what happens for examples with trivial symplectic cohomology.

2.2. Past and current work. In [MS] we describe a general procedure which inputs a Stein manifold of dimension at least six and outputs a subflexible one of similar topology, denoted by $SF(X)$. A question left unresolved in the work of Cieliebak–Eliashberg (see [CE1, Remark 11.30]), is whether flexibility is preserved under homotopies. The following theorem shows that this is false.

Theorem 1. [MS] *Every flexible Stein manifold has, after a Stein homotopy, a nonflexible sublevel set.*

The proof of this theorem involves symplectic Lefschetz fibration techniques and understanding how they interact with flexibility, together with deformed versions of symplectic cohomology as described below.

As a beautiful application of advances in flexibility, Cieliebak–Eliashberg [CE2] gave a complete topological classification of rationally and polynomially convex domains in $\mathbb{C}^{n \geq 3}$, answering an old question in the theory of several complex variables. They also conjectured that all polynomially convex domains must be flexible, which we disprove by combining subflexibility with complex analytic techniques:

Theorem 2. [MS] *There exist nonflexible polynomially convex domains in \mathbb{C}^n for $n \geq 3$.*

In another direction, in joint work with Stefan Nemirovski we study rational convexity of disk bundles over surfaces, the simplest class of topologically allowed four-manifolds. Letting $D(\chi, e)$ denote the disk bundle with Euler number e over the orientable surface of Euler characteristic χ , and defining $\tilde{D}(\chi, e)$ similarly for nonorientable base, we found the complete classification:

Theorem 3. [NS] *There exist strictly pseudoconvex domains in \mathbb{C}^2 with rationally convex closures diffeomorphic to the following disk bundles:*

- $D(\chi, 0)$ for $\chi \neq 2$.
- $\tilde{D}(\chi, e)$ for $(\chi, e) \neq (1, -2)$ or $(0, 0)$ and $e \in \{2\chi - 4, 2\chi, 2\chi + 4, \dots, -2\chi - 4 + 4\lfloor \chi/4 + 1 \rfloor\}$.

Moreover, these are the only possibilities.

The answer turns out to be much more complicated than the higher dimensional analogue. In addition to some deep but now well-known obstructions coming from Seiberg–Witten theory and the Whitney–Massey theorem on embeddings of nonorientable surfaces, we find some additional obstructions related to the classification of tight contact structures on certain three-manifolds and their symplectic fillings. We balance these obstructions with various constructions of Lagrangian caps, partly inspired by [Lin].

2.3. Future directions. An important follow-up question in symplectic flexigidity is:

Question 1. *Is there any Stein manifold (or more generally symplectic manifold with convex boundary) which has trivial symplectic cohomology and is diffeomorphic but not symplectomorphic to Euclidean space?*

We can also formulate analogous questions, such as whether any contact manifold with trivial contact homology algebra is necessarily overtwisted, or whether any Legendrian with trivial Legendrian contact homology algebra is necessarily loose. To date, there is no result in symplectic geometry which deduces flexibility from an algebraic criterion, and these questions are presently out of reach with known symplectic invariants. Since geometric criteria are typically quantitative in nature, I expect new quantitative invariants such as those described below will play an important role in bridging the gap.

3. SYMPLECTIC CAPACITIES USING HIGHER ALGEBRAIC STRUCTURES IN FLOER THEORY AND SYMPLECTIC FIELD THEORY

3.1. Context. Roughly, a symplectic capacity is a symplectic invariant c such that $c(M, \omega) \leq c(M', \omega')$ whenever there is a symplectic embedding $(M, \omega) \hookrightarrow (M', \omega')$. Symplectic volume is an obvious example, but there are others which more faithfully encode nonsqueezing phenomena. Prominent examples are the embedded contact homology (ECH) capacities [Hut1] $c_i^{\text{ECH}}, i \in \mathbb{N}$, which give quite strong symplectic embedding obstructions in dimension four but are not defined in higher dimensions, and the Ekeland–Hofer (EH) capacities [EH1, EH2] $c_i^{\text{EH}}, i \in \mathbb{N}$, which are defined in any dimension but do not typically give sharp obstructions. The former are gauge-theoretic in nature and the latter are defined by a variational problem, although Gutt–Hutchings [GH] recently gave a conjecturally equivalent definition using filtered S^1 -equivariant symplectic cohomology.

An excellent laboratory for studying symplectic embeddings is the problem, largely open beyond dimension four, of when one ellipsoid

$$E(a_1, \dots, a_n) := \left\{ (z_1, \dots, z_n) \in \mathbb{C}^n : \sum_{i=1}^n \frac{\pi |z_i|^2}{a_i} \leq 1 \right\}$$

symplectically embeds into another ellipsoid $E(b_1, \dots, b_n)$. It is known [MS] that the ECH capacities give sharp obstructions to this problem in dimension four, and moreover these obstructions can sometimes (but certainly not always) be “stabilized” to give sharp obstructions to the problem $E(a, b) \times \mathbb{C}^k \hookrightarrow B^4(c) \times \mathbb{C}^k$ [CGH, HK, CGHM, McD].

3.2. Past and current work. By studying a filtered \mathcal{L}_∞ structure on S^1 -equivariant symplectic cochains, we prove the following:

Theorem 4. [Sie2] *There is a family of symplectic capacities $g_{k,l}$, $k, l \in \mathbb{N}$, such that $g_{k,1}$ is the k th capacity from [GH], and such that $g_{k,l}(M \times \mathbb{C}) = g_{k,l}(M)$.*

Since these capacities depend on rather elaborate curve counts, computing them in examples is quite nontrivial. We give an alternative formulation of $g_{k,l}$ in terms of a gravitational descendant \mathcal{L}_∞ augmentation defined in terms of curves with one point constraint and an order $k-1$ tangency condition ([CM, Ton]). We also prove that if an open symplectic manifold (M, ω) compactifies to

a closed symplectic manifold (X, η) , we have upper bounds for the capacities of (M, ω) in terms of descendant Gromov–Witten invariants of (X, η) . Moreover, at least for ellipsoids we also have lower bounds formulated in terms of ball packings. Combining these bounds, we prove the following result:

Theorem 5. [Sie2] *The capacities $g_{k,l}$ give sharp obstructions to the problem $E(1, 3d-1) \times \mathbb{C}^k \hookrightarrow B^4(c) \times \mathbb{C}^k$ for $d \in \mathbb{N}$.*

Note that the optimal embeddings were found by Hind [Hin] using a symplectic folding technique, and the sharp obstructions for these example were found earlier by McDuff [McD] using quite different methods.

In [Hut2], Hutchings sketched another family of capacities r_k , $k \in \mathbb{N}$, based on rational symplectic field theory. Unlike $g_{k,l}$, these are not robust with respect to dimensional stabilization. However, in [Sie2] we introduce the notion of a “blow-up capacity” and show that this gives a common generalization of both r_k and g_k , and moreover that the two families are related by recursive formulas which also involve certain curves with nodes and higher singularities.

3.3. Future directions. I expect that the capacities $g_{k,l}$ will give strong embedding obstructions in many other contexts. A current project in progress involves computing the capacities $g_{k,l}$ for two-dimensional ellipsoids $E(a, b)$. These capacities can be reformulated in terms of descendant relative Gromov–Witten invariants of weighted projective planes, which are computable by localization or recursion methods. It will be interesting to compare the resulting obstructions to the known obstructions for the stabilized ellipsoid embedding problem. I am also studying the problem of more general high dimensional ellipsoid embeddings $E(a_1, \dots, a_n) \hookrightarrow E(b_1, \dots, b_n)$, for example the “round regime” where $a_1 \approx \dots \approx a_n$. Here I expect the capacities r_k and more general blow-up capacities to play a more prominent role.

A more speculative question is to what extent do the higher genus analogues of $g_{k,l}$ stabilize with respect to dimension and what information they contain. Answering this will likely require new ideas in homological algebra such as those developed in [CFL].

I am also interested in algorithmic techniques to compute these various capacities. In the computational project [BT] we implemented a numerical algorithm to compute the first Ekeland–Hofer capacity of convex domains in Euclidean space. Subsequent work [HK] produced a combinatorial formula in the case of convex polytopes. I believe similar methods, using optimization techniques and/or exact combinatorial formulas, should extend to more general capacities and for more general domains. More speculatively, such computational methods could be applied to practical problems involving integrating Hamiltonian flows, such those arising in molecular dynamics and Hamiltonian Monte Carlo simulations.

4. DEFORMATION THEORY IN SYMPLECTIC GEOMETRY

4.1. Context. Symplectic cohomology is a fundamental invariant of open symplectic manifolds (M, ω) , the homology of a chain complex generated by periodic orbits of a Hamiltonian $H : M \rightarrow \mathbb{R}$, with differential counting pseudoholomorphic cylinders. A local system version called *twisted symplectic cohomology* ($\mathrm{SH}^{\mathrm{tw}}$) involves picking an ambient closed two-form and weighing pseudoholomorphic curve counts by the integral of Ω over a given curve. In [Sie1] I also develop the theory of *bulk deformed symplectic cohomology* ($\mathrm{SH}^{\mathrm{bd}}$), building on the work on Fukaya–Oh–Ohta–Ono [FOOO] and Usher [Ush]. This can be viewed as a derived local system analogue of $\mathrm{SH}^{\mathrm{tw}}$ which is based on a class in $H^{2k>2}(M; \mathbb{R})$, defining roughly by picking an ambient smooth cycle Q and counting pseudoholomorphic curves with marked points passing through Q .

4.2. Past and current work. Since $\mathrm{SF}(X)$ becomes flexible after subsequent Weinstein handle attachments, a standard consequence of the Viterbo transfer map is that $\mathrm{SF}(X)$ has vanishing symplectic cohomology. In spite of this, we have:

Theorem 6. [Sie1] *Assume $\dim X = 6$. For a certain choice of closed two-form on $SF(X)$, there is an isomorphism $SH^{tw}(SF(X)) \cong SH(X)$.*

As a consequence, if $SH(X)$ is nontrivial to begin with, it can be hidden in a twisted version and used to detect symplectic rigidity. As with ordinary symplectic cohomology, computing SH^{tw} tends to be highly nontrivial. The proof of Theorem 6 uses some powerful ideas [Sei2, BEE] about pseudoholomorphic curves in Lefschetz fibrations to translate it into a statement about Fukaya categories.

For $\dim_R X \geq 8$, we have examples for which $H^2(SF(X); \mathbb{R})$ is trivial, and we extend the above result using bulk deformations:

Theorem 7. [Sie1] *Assume $\dim X = 4k + 2$ for $k \geq 2$. For a certain choice of smooth cycle in $SF(X)$, there is an isomorphism $SH^{bd}(SF(X)) \cong SH(X)$.*

The key ingredient is a new noncommutative analogue of Seidel’s result [Sei1] on the fragility of squared Dehn twists about Lagrangian spheres. In essence, Seidel observed that the squares of four-dimensional Dehn twists are smoothly trivial but symplectically nontrivial, and moreover they become symplectically nontrivial after a small perturbation of the symplectic form. In higher dimensions, by Moser’s theorem there are typically no nontrivial perturbations of the symplectic structure, but we can view bulk deformations of Fukaya categories as an algebraic proxy, and it turns out that fragility extends to this setting.

Theorem 8. [Sie1] *Let L and S be Lagrangian spheres in a Liouville domain (M^{4k}, θ) . Assume that L and S intersect once transversely, and let Q be a smooth half-dimensional cycle in $(M, \partial M)$ which is disjoint from L and intersects S once transversely. Then L and $\tau_S^2 L$ are not quasi-isomorphic in $\text{Fuk}(M, \theta)$ but are quasi-isomorphic in $\text{Fuk}_Q(M, \theta)$.*

In order to address Question 1 above, a natural approach is to consider more general deformations of symplectic cohomology which do not require classes in $H^*(M)$. Indeed, the differential on symplectic cochains extends to the structure of a filtered \mathcal{L}_∞ algebra, and hence we can define quantitative deformations via Maurer–Cartan theory, with SH^{tw} and SH^{bd} appearing naturally as special cases. These are quantitative deformations in contrast with formal deformations, which are necessarily trivial for any acyclic \mathcal{L}_∞ algebra by a corollary of the homological perturbation lemma. Moreover, in [Sie4] we construct candidate exotic Stein manifolds with vanishing symplectic cohomology and give natural Maurer–Cartan elements in their symplectic cochain complexes which could potentially be used to generalize Theorems 6 and 7 above.

4.3. Future directions. A current research project involves trying to compute the filtered \mathcal{L}_∞ structures for a wide class of Liouville domains and to characterize their Maurer–Cartan spaces. Some important examples come from Weinstein neighborhoods of model singular Lagrangians in special Lagrangian torus fibrations (c.f. [GS]), which have canonical quantitative deformations related to gluing maps in SYZ mirror symmetry.

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