

INTRODUCTORY LECTURE

We begin with the abstract definition of a group;

Definition 0.0.1. *A group is an ordered pair (G, μ) , where G is a set and $\mu : G \times G \rightarrow G$ is a binary operation, satisfying the following axioms:*

- (1) *(associativity) $\mu(\mu(a, b), c) = \mu(a, \mu(b, c))$ for any $a, b, c \in G$*
- (2) *(identity) there exists $e \in G$ such that for any $a \in G$ we have $\mu(a, e) = \mu(e, a) = a$*
- (3) *(inverses) for any $a \in G$, there is an element $a^{-1} \in G$ such that $\mu(a, a^{-1}) = \mu(a^{-1}, a) = e$.*

Here is a simple first example of a group:

Example 0.0.2 (the integers). *Let \mathbb{Z} denote*