

Time Analysis of Recursion

$$T(n) = r \cdot T(n/c) + f(n) \quad \text{recursive definition}$$

$$= \sum_{i=0}^L r^i \cdot f(n/c^i) \quad \text{series definition}$$

where $L = \log_c n$ // num levels

Decreasing case ($T_{i+1} < T_i$)

$$O(f(n))$$

$$\begin{array}{l} \textcircled{n} \\ \textcircled{n/2} \\ \textcircled{n/4} \\ \vdots \end{array} \quad \begin{array}{l} n \\ + \\ n/2 \\ + \\ n/4 \\ \vdots \end{array} = n + n/2 + n/4 + \dots$$

$$\begin{array}{l} \textcircled{n} \\ \textcircled{n/2} \\ \textcircled{n/4} \\ \vdots \end{array} \quad \begin{array}{l} n \\ + \\ n/2 \\ + \\ n/4 \\ \vdots \end{array} \leq \lim_{L \rightarrow \infty} n \left(\frac{1 - \frac{1}{2}^{L+1}}{1 - \frac{1}{2}} \right) = n \cdot 2$$

Constant case

$$O(f(n) \log n)$$

$$\begin{array}{c} \textcircled{n} \\ / \quad \backslash \\ \textcircled{n/2} \quad \textcircled{n/2} \\ / \quad \backslash \quad / \quad \backslash \\ \textcircled{n/4} \quad \textcircled{n/4} \quad \textcircled{n/4} \quad \textcircled{n/4} \\ \vdots \end{array} = \begin{array}{c} n \\ + \\ n \\ + \\ n \\ \vdots \end{array} = \begin{array}{c} \log n \\ \hline n + n + n \dots \\ n = n \log n \end{array}$$

Increasing case ($T_{i+1} > T_i$)

$$O(n \log_c r)$$

$$\begin{array}{c} \textcircled{n} \\ / \quad \backslash \\ \textcircled{n} \quad \textcircled{n} \\ / \quad \backslash \quad / \quad \backslash \\ \textcircled{n} \quad \textcircled{n} \quad \textcircled{n} \quad \textcircled{n} \\ \vdots \end{array} = \begin{array}{c} n \\ + \\ 2n \\ + \\ 4n \\ \vdots \end{array} = \begin{array}{c} \log n \\ \hline n + 2n + 4n \dots \\ 4n \leq 2^{\log n} + 2^{\log n} + 2^{\log n} \dots \end{array}$$