We introduce a mathematical model based on both a classic population dynamics set of differential equations, while inserting stochastic elements to better represent natural phenomena in order to determine the effect of localized biodiversity on drought resistance in plant communities. Beginning with a competitive Lotka-Volterra model of population dynamics, we enrich the model with both stochastic and deterministic processes to account for the effects of the drought cycle, pollution, and habitat loss. We hypothesize that our model will reflect a positive relationship between biodiversity of a plant community and its ability to weather droughts.

While we believe that our model provides an accurate approximation to the natural process that we attempt to model, we recognize that there are shortcomings to any model and always room to improve finite models representing real-world processes. We thus detail the strengths and weaknesses of our system, as well as the logic behind our decisions, encouraging future researchers to determine paths to expand upon our approach.

We then detail our approach to coefficient estimation and the parameters we use in testing our model, justifying our decisions and estimates. We detail how we choose our coefficients not just based on our research into the topic, but also considering potential inaccuracies. We thus attempt to implement our model in a way that insulates our approach to large errors in estimates from research.

We present our results, both by answering the question at hand (the relationship between the biodiversity of a plant community and drought resistance) and by testing the various aspects of our model. We demonstrate support for our hypothesis that increased biodiversity in a plant community is associated with increased ability to resist and recover from droughts. Additionally, we isolate sections of our model to see how these sub-processes may affect and transform our model as a whole. Finally, we examine different types of relationships between plants species to try to generalize our results beyond our initial competitive model.

We propose further improvements to our model, as well as steps forward for any future researchers following down this or a similar path. We hope to provide solutions to potential shortcomings we see in our model or areas in which we believe further research would be fruitful.

Finally, we conclude by examining our process in its entirety and what our results imply about survivability of plant communities in the face of droughts and natural phenomena. We believe that this report provides benefits to not only ecologists studying drought resilience in plants, but also to future researchers hoping to examine related issues or implement similar models by detailing our thought process and where our approach both succeeds and can be improved.

# Contents

# 1 Introduction

In this paper, we examine the problem of determining the relationship between localized biodiversity of plant communities and drought resistance. Our goal was to create a model that predicts plant species population dynamics, considering competition between members of the same species (intraspecific competition) and competition between members of different species (interspecific competition), external factors, and habitat limits. We develop a model based on a classic competitive Lotka-Volterra ecological population dynamic model to model plant species population growth while incorporating stochastic processes to represent droughts, pollution, and habitat loss.

Our model is based on a series of differential equations modeling the population trajectories of multiple heterogeneous plant species. The population of each species is determined by growth rate, a damping factor that prevents unlimited growth, and competition between species for limited resources.

We enrich this base model with a discrete-time Markov chain which determines drought conditions for all of the plants. This allows for environmental conditions to change randomly over the course of any given simulation, allowing us to test different drought frequencies and thus, different climates. We also incorporate a random walk process determining pollution levels in the plant community. This allows us to include pollution processes that are not determined by the base system and a linear function of habitat loss. Finally, we also add a maximum total carrying capacity of the system, ensuring the total population of the system does not exceed some parameter.

Because of the generality and power of our model, we are able to test different relationships and environments for our plant community. We test different types of interspecific plant relationships, such as increased competitive, parasitic, and symbiotic relationships. This allows us to examine the population dynamics of a variety of plant communities.

We hypothesize that our results will demonstrate that a more bio-diverse local plant community will lead to improved resistance to droughts. We will examine this in three different climates with varying levels of droughts to measure our hypothesis. We will also determine how each aspect of our model impacts our results by isolating them individually and seeing how our results vary.

# 2 The Model

## 2.1 Model choice

### 2.1.1 Base competitive Lotka-Volterra model

For our model, we took inspiration from a traditional competitive Lotka-Volterra model of ecological population dynamics between multiple competing species. The Lotka-Volterra model is a system of differential equations modeling the growth and interaction between multiple species, classically examining "predator-prey" population relationships and dynamics. The version of the model that we chose as a base (the competitive Lotka-Volterra) is specifically for modeling population dynamics between multiple species competing with each other for shared resources (i.e. a "predator-predator" relationship, which would be analogous to multiple plants all competing for similar resources).

As mentioned above, the competitive Lotka-Volterra model typically uses ordinary differential equations to model population dynamics in a multi-species environment, focusing on how species compete with each other for shared resources. For the two species case, it is specified as follows:

$$\begin{cases} \frac{dx}{dt} = a_1 x + b_1 x^2 + c_1 xy \\ \frac{dy}{dt} = a_2 y + b_2 y^2 + c_2 xy \end{cases}$$

where $x$, $y$ represent the populations of the two species, $a_i > 0$ represent their growth rates, $b_i < 0$ represent their inability to grow beyond their carrying capacity (with an implicit $K_i$ such that $b_i = -\frac{a_i}{K_i}$), and $c_i < 0$ represents the competition between the two species. This value is negative due to competition's detrimental impact on the growth rate under non-drought conditions.Compactly:

$$a_i > 0$$
$$b_i = \frac{a_i}{K_i}$$
$$c_i < 0$$

We see that intraspecific competition (competition between members of the same species) is incorporated implicitly through a lower growth rate. Additionally, the maximum carrying capacity reflects and the inability for any species to grow continually, while also providing a damping factor also representing intraspecific competition. This model can clearly be extended to a case with a larger number of species, creating a series of differential equations modeling the population changes for each individual species and including competition terms pairwise between each species. For example, a generalized equation with $n$ total species would take the form of:

$$\frac{dx_i}{dt} = a_i x_i + \sum_{j=1}^{n} c_{ij} x_i x_j \tag{1}$$

with $c_{ii}$ representing the $b_i$ from the two species model, enforcing the marginal carrying capacity.

For the purpose of this problem and to better model reality, we enriched this model in the following ways. We added a stochastic process to model the drought cycle, another stochastic process to model pollution, a maximum joint carrying capacity for the plant community, and a function of habitat loss. We will discuss each of these in turn.

### 2.1.2 Stochastic drought cycle

As the problem is directly concerned with the drought resistance of plant communities, our formulation of the drought cycle is particularly important. Because of the often unpredictable of droughts, we decided to model it as a stochastic process, specifically a discrete-time Markov chain. As a discrete-time Markov chain, our drought cycle can be thought of as a random walk on a graph, allowing for transitions from any of the drought states to another. Our model accounts for three discrete states for the drought cycle: normal (non-drought) conditions, moderate drought, and severe drought. At each time interval, the environment may "randomly walk" to a new state (i.e. transition from one environmental state to another). We weight these transition probabilities using data on frequency of droughts. Because this model is so general, it is also easily modified to experiment with high- and low-frequency drought regions.

When a drought occurs (i.e., the random walk falls on a drought event), our original competitive Lotka-Volterra model is modified. We create indicator variables that are equal to 1 in the presence of the specific weather event and 0 otherwise (one each for moderate and severe droughts). These indicator variables "turn on" new coefficients that affect our original coefficients. Incorporating these coefficients, our original model shown in (1) is expanded to:

$$\frac{dx_i}{dt} = a_i x_i + \sum_{j=1}^{n} c_{ij} x_i x_j + \gamma(t) \left[ d_i x_i + \sum_{j \neq i} e_{ij} x_i x_j \right] + \Gamma(t) \left[ f_i x_i + \sum_{j \neq i} g_{ij} x_i x_j \right] \tag{2}$$

where $\gamma$ is an indicator variable indicating when the environment is experiencing a moderate drought and $\Gamma$ is an indicator variable indicating when the environment is experiencing a severe drought. Here, our terms $d_i$ and $f_i$ represent the slowdown (and negative) growth experienced by plant species in moderate and severe drought respectively. As droughts are costly for plants in terms of resources, we expect each of these coefficients to be negative. Additionally, we expect severe droughts to be more difficult to survive in, so:

$$|d_i| \leq |f_i| \ \forall i$$

According to our research, moderate droughts can create an environment in which species face increased intraspecies competition from competition for similar (and now more sparse) resources relative to interspecies competition. These intraspecies effects can lead to less dominant species gaining a relative advantage, as dominant species with higher populations dies off at higher rate. This conclusion is drawn from the stress gradient hypothesis, which states that facilitation and mutual support is more common than competition during environmentally stressful conditions. This has been observed during moderate drought conditions, where diverse species with different rooting patterns partition soil and utilize water resources more efficiently underground. Thus, during a moderate drought, interactions between plant species will have a positive effect on the growth rate of the plants due to efficient and mutually beneficial interactions. This effect, however, is not found in severe drought environment because the non-dominant species face intense intraspecfic competition due to the sheer lack of resources. Because of this, we also make the following distinctions:

$$e_{ij} > 0$$
$$g_{ij} < 0$$

Through equation (2), we then have modeled our stochastic drought process. Through this process, our differential model has elements of stochasticity, leading to non-deterministic outcomes even in simulations with identical coefficients and initial conditions. This process is modified through the pollution process, which is discussed below.

### 2.1.3 Stochastic pollution

Just as with droughts, pollution, whether it be from air or chemicals or other sources/avenues, cannot be perfectly predicted and its effects are numerous. Because its occurrence can be difficult to ascertain, we incorporate pollution as a bounded random walk.

Because pollution is often detrimental to plant growth due to contaminated resources and depleted soil nutrients, we enforce a negative effect on the linear plant growth in the face of pollution, on a continuous scale. This combines with our model in equation (2) to create:

$$\frac{dx_i}{dt} = a_i x_i + \sum_{j=1}^{n} c_{ij} x_i x_j + \gamma(t) \left[ d_i x_i + \sum_{j \neq i} e_{ij} x_i x_j \right] + \Gamma(t) \left[ f_i x_i + \sum_{j \neq i} g_{ij} x_i x_j \right] + \beta(t) [h_i x_i] \quad (3)$$

where $\beta(t)$ is our random walk process with drift. $h_i$ is the coefficient determining how detrimental the pollution level is to plant life. Because we expect pollution to be a net negative effect for all plants, we then specify:

$$h_i < 0$$

However, pollution does not solely affect plant growth; it can also affect drought frequency. Rain withheld in polluted deep clouds freezes at twice the altitude as non-polluted clouds and form ice crystals, reducing precipitation in dry regions and causing more frequent droughts. Because of this, we modify our drought process in the presence of pollution. As pollution grows, the characteristic transition matrix of our drought condition Markov chain changes. We modify the transition probabilities by factors of pollution. Specifically, we make our probabilities of both entering and staying in non-drought weather lower by the factor of pollution, while we increase the probabilities of entering and staying in moderate- and severe-drought weather by factors of pollution. We ensure that these factors all sum to zero, so that our transition matrix still holds the necessary properties to model a random walk on a graph. This also provides us with clear parameters for our random walk process $\beta(t)$. We scale our step term such that if we take solely positive walks for the entirety of the simulation, the transition matrix will have no negative values, protecting our matrix from becoming a non-probability matrix. This choice means that our drought process will not fail, and we can scale our coefficient $h_i$ to still reflect any level of direct effects of pollution that we wish to incorporate into our model.

### 2.1.4 Maximum joint carrying capacity

While we accounted for the total population that can be supported by the resources in the community for each species in the carrying capacity $K_i$, our model does not enforce a single joint carrying capacity for the overall system. While the model has an implicit joint carrying capacity equal to $\sum_{i=1}^{n} K_i$, this value will never be reached so long as the model has negative competition coefficients (the effects from $a_i$ and $b_i$ will cancel each other out, leading to the negative effect of the competition coefficients). In addition, this implicit joint carrying capacity scales with the number of species incorporated in

the model, making comparisons of population levels and changes between simulations with varying numbers of species difficult to interpret.

Because of this, we enforce a maximum joint carrying capacity, modeled by bounding a linear combination (in this case, a weighted sum) of the populations. The weights may be adjusted to account for the different levels of resources/land each member of a given species may take up (e.g., a given species may be a tree which takes up more land for its roots and has canopies that absorbs lots of sunlight, while another species may be a small weed with small root systems and no leaves). This approach allows for flexibility based on the desired species of study.

Maximum joint carrying capacity is implemented by subtracting the mean amount of excess growth per species from each species' growth.

### 2.1.5 Habitat loss

To examine habitat loss, we decrease the maximum joint carrying capacity by a linear factor. We scale this factor based on the original maximum joint carrying capacity, ensuring that we end up with a specific proportion of the original habitat space by the end of the simulation. Thus, our habitat loss is a linear effect that depresses the total land/resources available to the plants in our system.

## 3 Assumptions

**Drought Resistance** Plants living in drought-prone environments can undergo genetic changes after droughts to adapt to limit resources in the soil. However, our current model does not account for adaptations due to previous droughts.

**Droughts as a Discrete Process** We simplified the U.S. Drought Monitor drought categories to three discrete classifications: no drought, moderate drought, and severe or extreme drought. These categories were based on the SPI index and were used to create our Markov transition matrix. However, in reality, transitions from one drought category to another are continuous processes, not discrete ones.

**Proportional Population Growth** Since our model uses a differential equation that represents rate of change, we assume that the population of plants grows proportionally to its current level.

**Stress Gradient Hypothesis** The coefficients we chose to model the moderate drought conditions were determined by the Stress Gradient Hypothesis, which states that facilitation is generally more common during stressful conditions. However, the Stress Gradient Hypothesis does not consider variation between species' responses to stress: while many plant species may develop mutually beneficial relationships during environmentally stressful conditions, other plant species could develop competitive relationships due to similar rooting patterns forcing them to compete for the same limited resources in the soil during a moderate drought; our model, however, assumes that different rooting patterns and canopy structures among bio-diverse plants do not cause competition during moderate stress but rather increase facilitation.

# 4 Strengths and Weaknesses

## 4.1 Strengths

1. **Simplicity**
   One of the main draws of this model is its simple and elegant setup, which allows for powerful conclusions from a relatively simplistic starting point. Both for the purposes of explanation/interpretation and implementation, the simplicity of the model is very attractive. Though we complicate and add nuance to the base model, each of these layers is easily peeled back and can be displayed to audiences both quantitative and not. Because of this, we believe that the simplicity aids in the communicability of the model. In addition, our model folds in many factors that do not have to each be implemented or estimated separately. By incorporating a simple population growth rate, we do not have to account for how much individual factors lead to growth (e.g., levels of sunlight, non-drought rainfalls, etc.). Because of this, it may be easier to find data to tune our parameters, rather than a model that attempts to estimate each factor individually.

2. **Adaptability**
   The base model itself consists of only a series of differential equations, yet it can map complicated systems and allow us to infer general population dynamics. For our sake, the simplicity allows us to add complexity and richness to the model in other ways and add properties to better reflect reality and the problem at hand. We have already modified the competitive Lotka-Volterra model with stochastic models of drought and pollution, as well as a maximum joint carrying capacity. Each of these pieces are relatively easy to implement as complementary pieces to our model. Because of this, if we were to adapt this problem to examine other areas more deeply, we would be able to quite easily make additions or subtractions from the current setup.

3. **Generality and customizability**
   In addition, the competitive Lotka-Volterra model is very general, allowing us to test various types of plants, generalized only to their base aspects involving population dynamics (i.e. their growth rate, their competitive interactions with other species, etc.). This generalization allows for better testing and richness. Our model allows for many layers of customizability. For example, the types of species that we model We can model species with relationships that are competitive, parasitic, or symbiotic, merely by changing competition coefficients (in the latter cases, some of these coefficients will be positive as the presence of the partner species will encourage growth). These two features make our model attractive not only for testing in this setting but also in future settings with either similar or different problems.

4. **Nuance**
   Because of our formulation, we allow for many nuanced relationships between species at different times, allowing for fine changes and manipulation. By splitting our coefficients between non-, moderate drought, and severe drought conditions, we allow for the nuance of the qualitative difference in interspecific competition. This allows us to accurately model the benefits of localized biodiversity in plant communities. This nuance may be lost in a more simplistic model that merely has droughts affect the growth rate of plants via simple addition or magnification.

5. **Stochastic drought and pollution**
   The processes determining how droughts and pollution occur are incredibly complicated in real

life. Because mapping weather patterns and perfectly predicting droughts is both beyond the scope of this project and an extremely important open problem that researchers much smarter than us have constantly attempted to solve, we decided that taking these processes as stochastic would be accurate and powerful. We feel comfortable with this assumption because these processes are basically random from the perspective of the plant community. There is little that our local community of plants can do that would affect general weather patterns, let alone droughts. Additionally, pollution is a macro process that is often either created by humans or external factors unaffected by the plants themselves.

Because of these facts, we believe that a stochastic process is the most beneficial for our model. The strength of this is both ease of computation/implementation and our ability to reflect real-life data. While pollution data is not widely available (especially when accounting for all of the different types of pollution), drought data is very common. We can thus use this frequency data to inform our parameters for the stochastic drought process, allowing us to easily and simply model our droughts off real-world data. Because we cannot replicate this exactly with pollution, we settle for another simple model that leverages the pseudo-random nature of the process.

## 4.2   Weaknesses

1. **Deterministic equations**
   The processes that we are modeling here are quite complicated and likely cannot be accurately modeled using entirely deterministic differential equations. While we allow and account for stochastic processes in our drought and pollution sub-models, we assume no noise in our differential equation model. While we account for this slightly by allowing for perturbations in the parameters at the beginning of each simulation, these parameters are not updated throughout the simulation. Thus, each simulation is entirely deterministic. This does not accurately reflect the randomness of daily and monthly processes like sunlight, rainfall (in a non-drought environment), and external factors (e.g., migration of herbivores, forest fires, etc.)

2. **Predetermined parameters**
   While we do fold in many measures that are difficult to individually estimate into single parameters, we still have to choose best guesses for coefficients describing quite complicated processes. Though we have conducted research to best ascertain what realistic coefficients may resemble, these areas are not often researched, likely because of their narrow utility. Because of this, we must make best guesses and tune our parameters based on what is often qualitative data. This makes it difficult to determine our success in modeling the exact impacts of individual factors, as we can only evaluate the total outcomes. When comparing to data, this makes it more difficult to test than a model that would potentially compact many of our parameters into many fewer ones.

3. **Implicit parameters** Conversely, we have many effects that are only included implicitly. For example, intraspecific competition is an especially difficult factor for us to explicitly determine. We implicitly include it in both the damping factor that enforces the carrying capacity and the lower than otherwise growth rate. While we may have estimates for each of these parameters, we cannot exactly determine the effect that intraspecific competition has. This helps us because we then do not have to make an additional assumption, but it does make it harder to evaluate this implicit value. Were there to be trustworthy and general data that we thought we would be

beneficial to include, we could add this term to the model (it would be equivalent to our current model, but with an explicit term for intraspecific competition), but we do not for this version because of the lack of this data.

4. **Discrete approximations to continuous phenomena**
   While any model that involves computer simulation will necessarily quantize any continuous process, our model explicitly turns continuous processes into discrete processes. We see three key discrete approximations in our model that may lead to inaccuracies in our analysis: our drought process and its effects, our pollution process, and the carrying capacity/habitat loss process.
   Our drought conditions are modeled as discrete environmental conditions without continuous transitions to and from drought states. This is clearly inaccurate because while some weather processes have log or nearly discrete characteristics, processes like humidity or temperature are continuous. This is also a flaw in our pollution process, which is also modeled with a discrete random walk.
   Finally, while marginal carrying capacity is not a discrete process, as the damping factor ($b_i$ in the two-species model and $c_{ii}$ in the general multi-species model), the maximum joint carrying capacity is modeled as a discrete process. In reality, we expect the borders of any local plant environment to be malleable, as the local habitat may expand or contract depending on the growth rate and capabilities of the plant species being modeled. Modeling the joint carrying capacity as a stochastic or malleable constraint may further enrich this non-discrete phenomenon.

5. **Potential chaos**
   While we do not find qualitatively large amounts of chaos in our problem, we recognize that there may be subspaces of our input space that may display chaotic behavior in simulations. It is difficult to determine exactly where these chaotic subspaces may lie because our stochastic processes do lend themselves well to direct analysis. While stochastic differential equations may be analyzed for chaotic behavior (and stochastic Lotka-Volterra models have been), this model incorporates random behavior on different levels. This makes our model difficult to analyze from a chaotic perspective. While we can still analyze how our model will act under any given set of competition coefficients (e.g., during non-drought conditions), it is much more difficult to comprehensively analyze it, as it does not behave as a typical stochastic differential equation. A simpler or more direct model that incorporates these random processes directly in the differential equations may thus be simpler to analyze given initial inputs. We do not believe this factor to be impossible to determine, but it is unfeasible for our purposes. Thus, we examine how small perturbations act on our system without systematic analysis.

# 5   Testing

## 5.1   Coefficients

For each simulation, we chose our coefficients from a normal distribution based on the above criteria to account for randomness in environmental phenomena and minor errors in our coefficient choices The coefficient $c_{i,j}$ presented below is the interaction coefficient for competitive interactions between

species. In the **Different interspecific relations** section, we detail the changes we made to this coefficient based on the specific relation.

- $a_i \sim \mathcal{N}(0.3, 0.05)$

- $c_{ij} \sim \mathcal{N}(-0.0007, 0.00012)$

- $d_i = -0.9 * a_i$

- $e_{ij} = -0.7 * c_{ij}$

- $f_i = -1.5 * a_i$

- $g_{ij} = 3.0 * c_{ij}$

## 5.2   Transition matrix

The U.S. Drought Monitor is a map that shows the areas in the United States that are in drought based on four drought categories: moderate (D0), severe (D2), extreme (D3), and exceptional (D4). These categories are measured by the Standard Precipitation Index (SPI) characterizes drought on various timescales. The SPI allows meteorologists to compare precipitation in different regions using a probability distribution that models precipitation data.

For our drought conditions probability transition matrix, we simplified the above drought classifications to three categories: no drought, moderate drought, and severe or extreme drought. These categories are constructed from the following SPI values:

| Drought Category | SPI Value |
|---|---|
| No Drought | $\geq 0$ |
| Moderate drought | $0$ to $-1.49$ |
| Severe/extreme drought | $\leq -1.5$ |

We then created a probability transition matrix for three regions with varying drought intensities and frequencies. We ran our simulation for these three regions to observe how the frequency and intensity of droughts affect plant growth.

### 5.2.1   Mild-drought climate

We based our transition matrix for less drought-prone areas based on weekly precipitation measurements in Massachusetts in 2022:

|  | No Drought | Moderate Drought | Severe/Extreme Drought |
|---|---|---|---|
| No Drought | 0.9440 | 0.0560 | 0.0 |
| Moderate Drought | 0.0952 | 0.8570 | 0.0478 |
| Severe/Extreme Drought | 0.0 | 0.60 | 0.40 |

### 5.2.2 Drought-prone climate

We also modeled transition matrix for the drought-prone region off of drought conditions in Algeria from 2005 to 2006, where 75% of the region is a desert zone. The transition matrix is as follows:

|  | No Drought | Moderate Drought | Severe/Extreme Drought |
|---|---|---|---|
| No Drought | 0.774 | 0.225 | 0.001 |
| Moderate Drought | 0.065 | 0.7365 | 0.1985 |
| Severe/Extreme Drought | 0.050 | 0.128 | 0.822 |

### 5.2.3 Abnormally wet climate

Finally, we created a transition matrix for abnormally wet regions based on the above two matrices as follows:

|  | No Drought | Moderate Drought | Severe/Extreme Drought |
|---|---|---|---|
| No Drought | 0.980 | 0.02 | 0.0 |
| Moderate Drought | 0.997 | 0.002 | 0.001 |
| Severe/Extreme Drought | 0.60 | 0.40 | 0.0 |

# 6 Results

For all of our results, unless otherwise specified, we include the model for pollution, a 10% habitat loss over the course of the simulation, a mild-drought climate, competitive interactions between all plant species, and 20 species of plants.

## 6.1 Recovery rate versus species count

To evaluate our system and the drought resistance of the plant community against the number of plant species in the community, we used "recovery rate". We calculate recovery rate as the the disparity in post-drought population and its mean after a severe drought normalize by the time between the two points. That is,

$$\text{Recovery Rate} = \frac{\text{population}_{\text{post-drought}} - \text{population}_{\text{mean}}}{\text{time}_{\text{post-drought}} - \text{time}_{\text{mean}}} \tag{4}$$

where $\text{time}_{\text{mean}}$ is the time where the population first returns to the mean after the severe drought. We take the mean of this value across 20 trials against the number of plant species in the community and plot the results below.

### 6.1.1 Mild-drought climate

**Recovery rate against number of plant species
in a mild-drought climate**
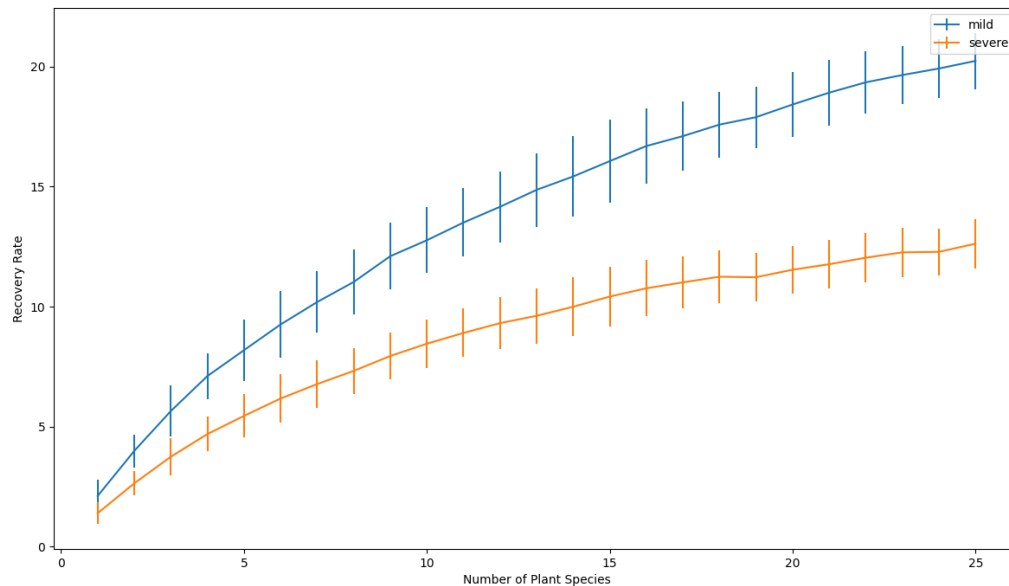


We see that any level of biodiversity helps the recovery rate of the system as a whole with diminishing returns. We immediately see a benefit to localized biodiversity, as the plant community as a whole proves more resilient to moderate droughts because of the lack of strong dominance any individual species. In environments with fewer species, there is a higher likelihood of dominance by individual or few species. Thus, their plant communities are more devastated by droughts. Meanwhile, in communities with a high count of different plant species, the plant community is quicker to recover because their is less competition with a dominant species.

Additionally, we see that severe droughts are more detrimental to the plant community, which we expect, but this effect is larger the greater number of plants species in the community. This is likely because of the Stress Gradient Hypothesis, as in the mild drought conditions encourage non-dominant species to take up resources previously utilized by the dominant species.

Overall, the mild-drought climate supports the hypothesis that localized biodiversity leads to increased drought resistance through our metric of recovery rate.

We note that the marginal increase in the recovery rate after reaching 10 plant species is small compared to the marginal increases in the previous recovery rates. Our model also suggests that as the number of species increases, the recovery rate also increases.

### 6.1.2 Drought-prone climate

We repeat the above process, but in the drought-prone climate, dictated by the drought-prone climate Markov chain.
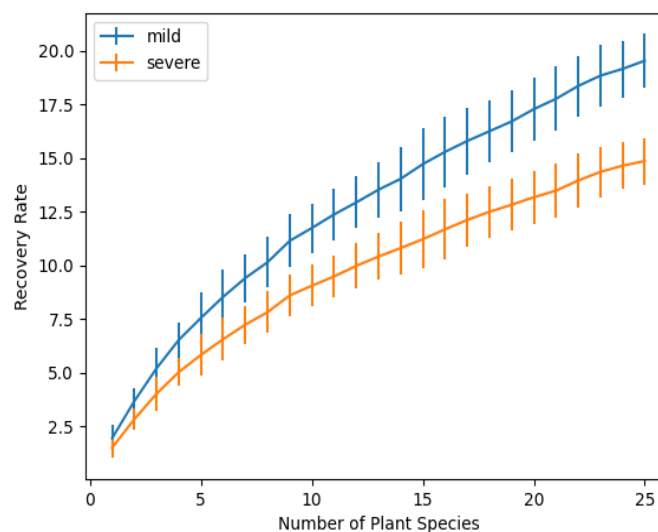
**Recovery rate against number of plant species
in a drought-prone climate**
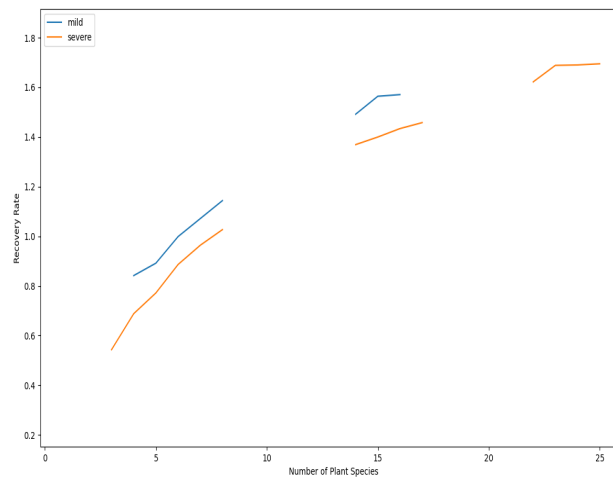


We see that our data has many missing values, representing simulations in which the community failed to reach its mean value again after a severe drought. This is due to the prolonged severe droughts in this climate that often cause species to go extinct or to at least never recover to their previous mean value. Overall, however, from the limited data remaining, we believe that this follows a similar general path to the previous mild-drought climate. This again supports our hypothesis, but the data contains much more noise because of the severity of the drought climate.

### 6.1.3 Abnormally wet climate

**Recovery rate against number of plant species in an abnormally wet climate**



Here, we see very similar results to the mild-drought climate results, but with a lower rate of increase

of recovery rate across the number of plant species. We believe that this result is because droughts are much less common in this environment, and, when they occur, they are shorter on average. This also supports our hypothesis.

## 6.2 Habitat loss effects

We examine the effects of habitat loss on our model.

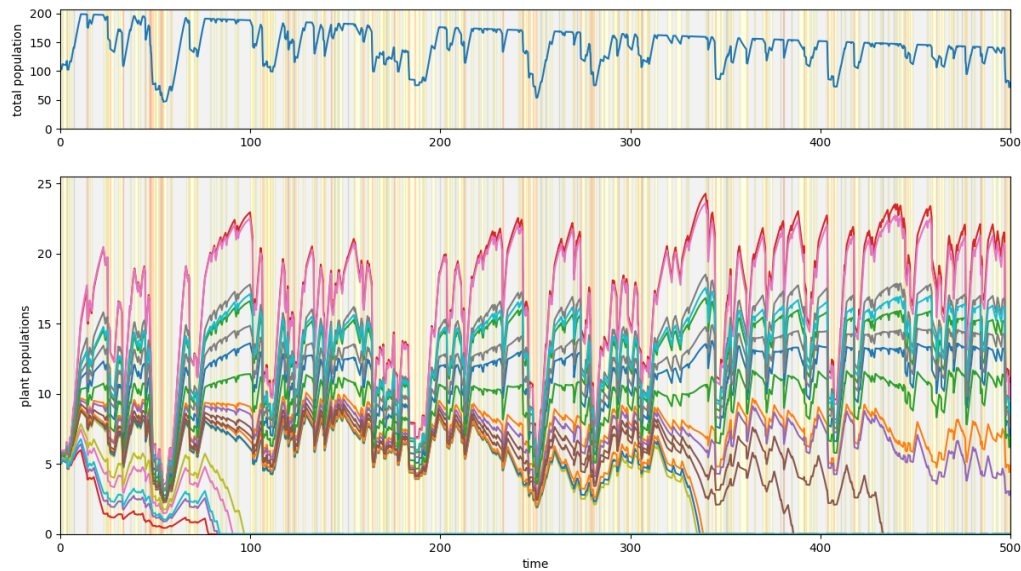**Population dynamics with 30% habitat loss over time**
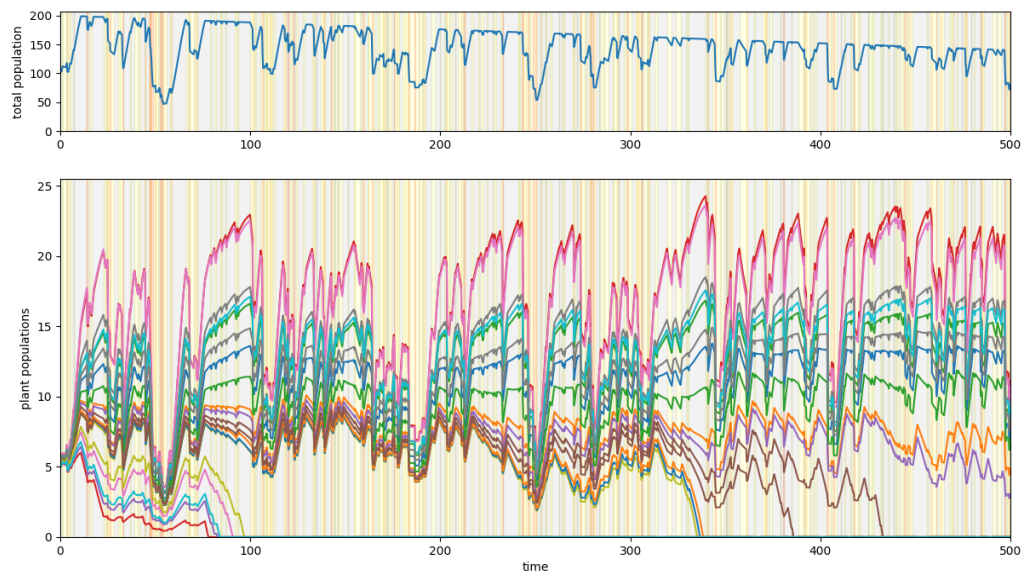


The first subchart shows total population of the community, while the second shows the population of each individual species, both over time. The yellow areas are mild droughts, while the orange areas are severe droughts.

We see that the habitat loss drastically impacts our results, leading to many species going extinct of the simulation period. The more dominant species all maintain their general population levels and seem to stabilize, while the non-dominant species lose their share of the resources and go extinct due to the habitat loss. This implies that habitat loss most drastically affects the plant species with the least amount of resources.
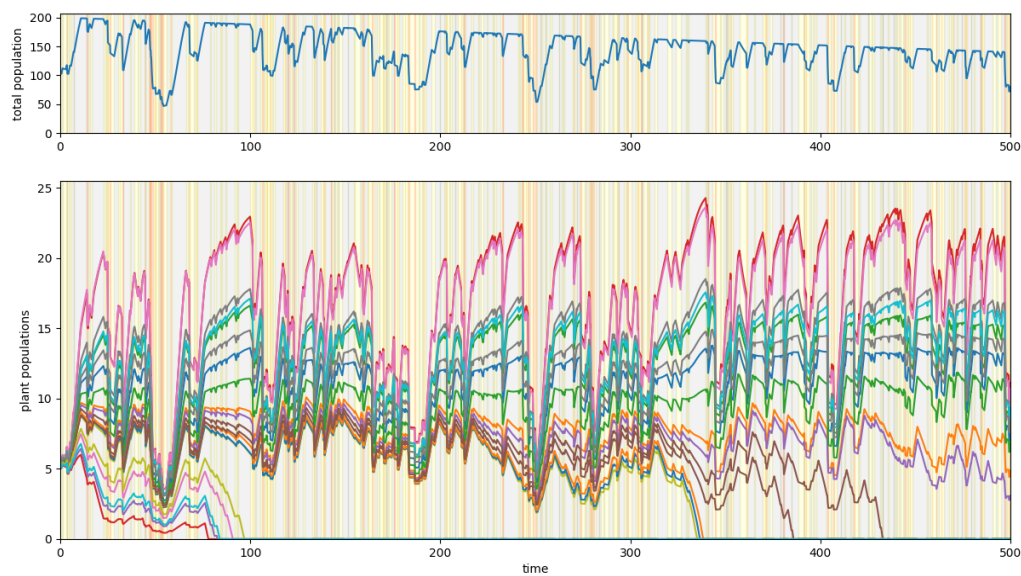
## 6.3 Pollution effects

We examine the effects of our pollution process on our model as a whole.

**Plant population dynamics with pollution**



**Plant population dynamics with no pollution**



These charts are of the same format as the above charts demonstrating the effects of habitat loss. We see that pollution has very little effect on the overall dynamics of the plant community population.

However, the drought frequency is indeed increased, even if marginally. This, however, does not appear to have a strong effect on the long-term behavior of this particular simulation. Similar numbers of species go extinct, and the population follows a similar pattern. This implies that our pollution model does not have a strong impact on the system as a whole.

## 6.4 Different interspecific relations

Here, we model different interspecific relations.

### 6.4.1 Increased competition

**Population dynamics with increased competition**
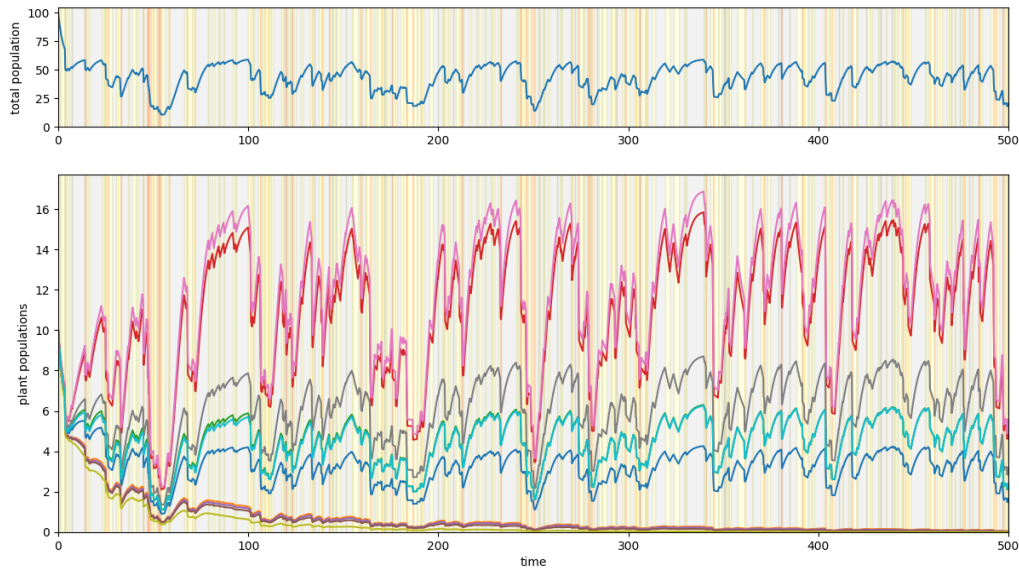


In this simulation, we model only 10 species. To represent increased competition, we decreased the negative mean of the normal distribution we chose our interaction coefficients from compared to the standard competitive case as follows: $c_{i,j} \sim \mathcal{N}(-0.01, 0.00012)$. The smaller negative value indicates how a species' growth rate is negatively affected by interactions to a greater degree in the increased competition case than in the standard competitive case. In an environment with increased competition (modeled as increases in the absolute value of the competition coefficients), we see that the system as a whole is more extreme, with individual species displaying much more dominance. There is less room for non-dominant species to grow once a few species have gained an upper hand. This implies that very competitive environments may lead to hegemonic population dynamics.

### 6.4.2 Parasitic

**Population dynamics with parasitic relationships**
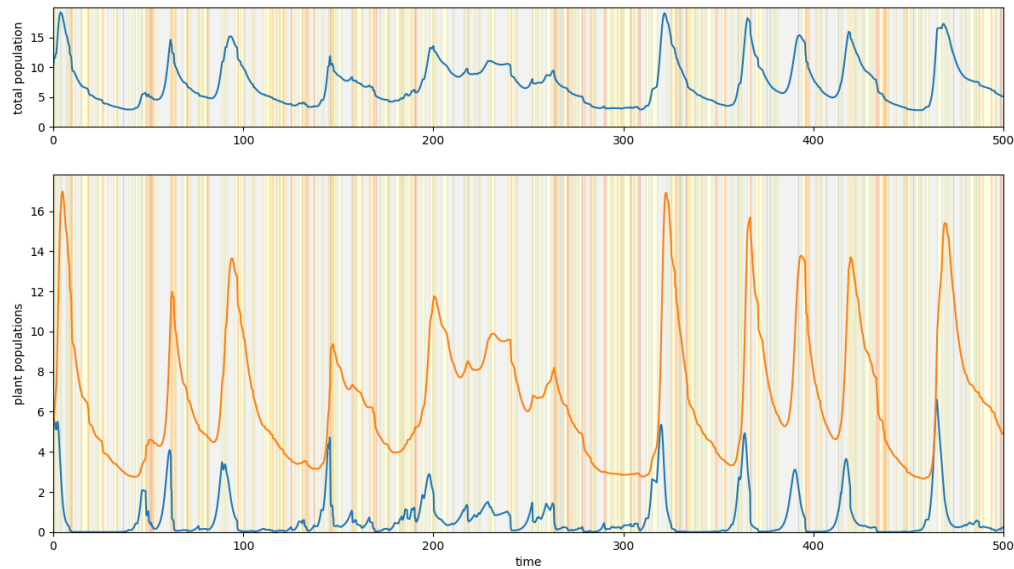


In this simulation, we model only 2 species, one a parasite (orange) and the other its host (blue). To represent this, we increased the mean of the normal distribution we chose our coefficient from as follows: $c_{ij} \sim \mathcal{N}(0.0, 0.002)$. In this case, we ensured that when the parasitic plant had a positive interaction coefficient, then the non-parasitic plant had a negative interaction coefficient. We see that the parasite grows rapidly whenever its host tries to recover, sapping the resources from the host until it approaches zero. The host remains near zero until it is able to recover. This process repeats continually, without the host ever truly reaching extinction. This implies that parasitism is not a truly sustainable system in this model with only 2 plant species.

### 6.4.3 Symbiotic

**Population dynamics with symbiotic relationships**
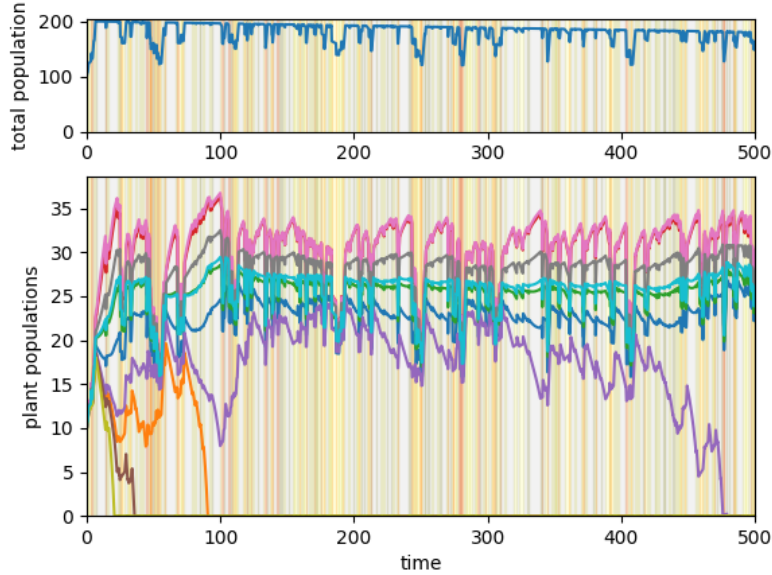


In this simulation, we model 10 species, each symbiotic (i.e. their competition coefficients are positive). We chose our competition coefficients from the following distribution: $c_{ij} \sim \mathcal{N}(0.0007, 0.00012)$. We see that they exhibit similar behavior to the standard competitive model, but with a higher overall population, as any growth by a species encourages further growth for the others. This leads to the overall population continually hitting the joint maximum carrying capacity.

# 7 Future Improvements

1. **Parameter estimation**

   Because of our formulation, our model is very dependent on correct parameter estimation. While we obviously can communicate our assumptions and our estimates for parameters which helps from a communication and transparency standpoint, we sacrifice the ability to finely tune or provide direct justification for our decisions (aside from research into topics that are not typically explored). There are multiple ways to mitigate this problem, many of which we detail below because they overlap with other shortcomings of the model. Many of those add layers of complexity or reformulations to the model. However, a simpler improvement to our model would be more accurate parameters. This would lead to potentially more accurate results, or at the very least results that can be justified stronger. While we do not necessarily suspect our parameters of being inaccurate (otherwise, we would have made adjustments), we do only have a certain level of confidence in them. We hope to mitigate incorrect estimates of parameters through our draws from random variables, but we do recognize that the expected values of these random variables do carry an implication of confidence that may not reflect our outlook.

2. **Stochastic differential equations (SDEs)**

   We believe that this model could be significantly enriched through the use of stochastic differential equations (SDEs) to account for the randomness described above in the **Deterministic equations** subsection of **4.2 Weaknesses**. The randomness of everyday life for a plant includes variation in resources available and potential external threats that are impossible for us to directly model in a deterministic manner. A stochastic base model would likely help account for these and allow us to determine both how chaotic our system is (i.e. how sensitive it is to perturbations in input) and allow for richer trials with single estimates of parameters.

   Another benefit to this, as also discussed in the **Weaknesses** section would be our ability to directly analyze our system from an SDE standpoint. The field of SDEs is quite rich, with many tools and methods to analyze structures modeled by SDEs, including the chaos of systems, properties of equilibria, and asymptotic behavior. Thus, an SDE model would lead to simpler analysis as a base model (potentially even including drought effects as part of the SDE process as opposed to a separate stochastic process like we have done here).

   Finally, this will help our problem of directly estimating our parameters. With SDEs, we can allow for small error in our exact estimations in a richer way than randomly sampling from a selected distribution for each simulation. While we would still need assumptions to specify our SDE model, we believe that our model would be less reliant on the exact accuracy of parameters because the parameters would update with each step.

3. **Reverse formulation**

   As we discussed in each of the two previous possible improvements, we are heavily reliant on *a priori* estimations for our coefficients. This places a lot of faith in our research and our ability to accurately determine coefficients. A potential improvement to this would inverting our problem. Instead of specifying the coefficients to our problem based on prior research, we could reverse our process and specify the general form of our equations (without specifying our parameter estimates). Then, we could use a numerical differential equation method to estimate our coefficients based on a given set of data. There are potential issues with this formulation, some of which are why we did not pursue this model. For one, finding data that accurately maps this exact process is very difficult and may not exist in a feasible format. Additionally, training on any specific data set would cause us to lose generality and limit our ability to test other species. This overfitting would make any claims to general conclusions suspect. For these reasons, among others, we decided not to pursue this path. However, we believe that there may be potential in a model leveraging this.

4. **Continuous drought states**

   As discussed previously in **4.2 Weaknesses**, one weakness of our model is the discretization of droughts. A more accurate model would potentially model rainfall as a stochastic process, with a possibility of extended stretches of little rainfall. This would allow for a more continuous process of droughts. However, specifying the parameters of this potential model would be even more difficult than estimating the entries of our transition matrix corresponding to our drought state discrete-time Markov chain.

5. **Richer pollution effects**

   While we believe that our model does portray aspects of the effects that pollution can have on plant communities, there are additional aspects and nuances to pollution that our model does

not account for. Chief among these is the relationship between pollution and drought severity. We have found that pollution can exacerbate the effects of drought through both a factor of magnification and potentially through qualitative changes of effects. We forgo this to instead have pollution only affect drought frequency (which we have also found is a realistic assumption) and proxy for the pollution affect on drought severity by having pollution cause more frequent severe droughts. However, a more advanced model may choose to incorporate this directly in by some factor or some method that previous research may suggest. We also see that our pollution effects have had little impact on our results. This may or may not be realistic, so further research could determine the importance of pollution on these dynamics.

6. **More advanced habitat loss function**
Our current method of habitat loss may lack accuracy because it both assumes linearity of habitat loss and it does not accurately account for diminishing marginal carrying capacity for individual plant species. The linearity of habitat loss may be accurate, but a richer model would likely allow for nonlinear effects or even a stochastic process demonstrating how habitat loss is likely locally random, or at the very least pseudo-random. Additionally, our habitat loss only bounds the maximum joint carrying capacity of the system, not the marginal carrying capacities. An improvement could be to change the marginal carrying capacity $K_i$ for each individual species by some rate of the habitat loss function. This would allow for certain species to be more or less impacted by the habitat loss process. We do not think that this addition would affect our results drastically because we have not found the marginal carrying capacity to be binding often. However, this would certainly enrich the model.

# 8 Conclusion

We proposed a mathematical model for predicting the dynamics of a local plant population over time which accounts for random environmental factors that impact the growth and health of plant communities in both general and specific cases. We incorporated elements from classical ecological models alongside stochastic processes to test our hypothesis that increased biodiversity leads to increased drought resistance in local plant communities. We also dissected our model and explained both its strengths and shortcomings. Then we displayed our results under different simulations measuring our hypothesis and testing individual factors of our model. Finally, we suggested improvements for further research and other considerations for future models.

From our results, we found that as the number of plant species increases, the total plant population is able to better resist droughts as measured by recovery rate. These findings support our hypothesis that local plant communities benefit from any amount of biodiversity with marginally diminishing returns. To ensure the long-term viability of any plant community, our model suggests that protecting non-dominant species from extinction and encouraging interspecific competition (by preventing a singular dominant species from arising) promotes overall plant community health and provides resilience against droughts, specifically by encouraging rapid recovery post drought.

We believe that this effect scales, as our tests with even higher numbers of species follows this same trend, even though the marginal benefits diminish. We also believe that this can affect the larger environment by improving macro-level drought resilience through preservation of non-dominant plants, improving total plant community health. While we are hopeful that our results may help promote plant community health in the face of droughts, we believe that there is further modelling research to be done in this field, especially in the areas that we have outlined above.

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