Node stratification arises from simple walk-based preferential attachment rules
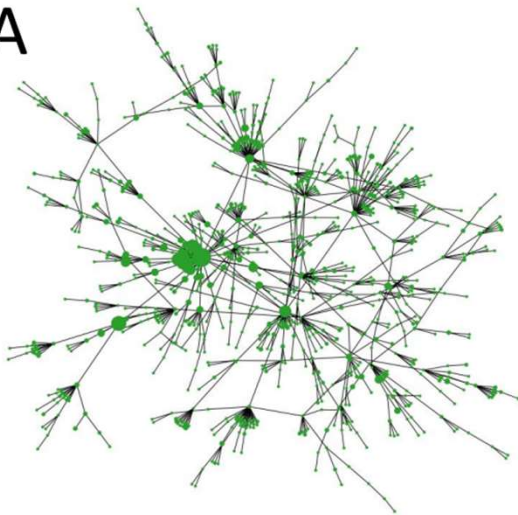
Kyle Soni[a], Anastasiya Salova[a], Gergely Odor[b], Balázs Maga[c], Pierfrancesco Dionigi[c], Miklos Abert[c], István A. Kovács[a]

[a]Northwestern University, Evanston, USA [b]Central European University, Vienna, Austria

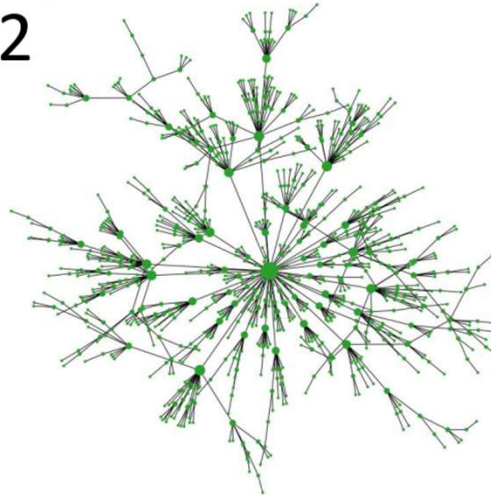[c]Alfréd Rényi Institute of Mathematics, Budapest, Hungary

k2 model

BA

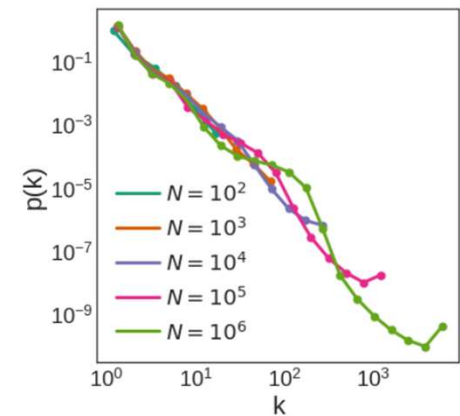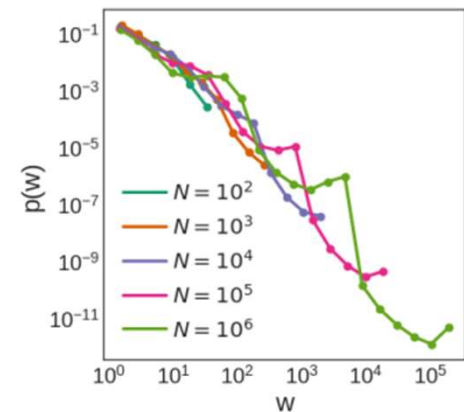k2

Degree Distribution

Weight Distribution

$$w_i \propto k_i$$

$$w_i \propto k_i^{(2)}$$

- Preferential attachment weight is proportional to walks of length 2 emanating from a node

Averaged over 1000 networks for various sizes
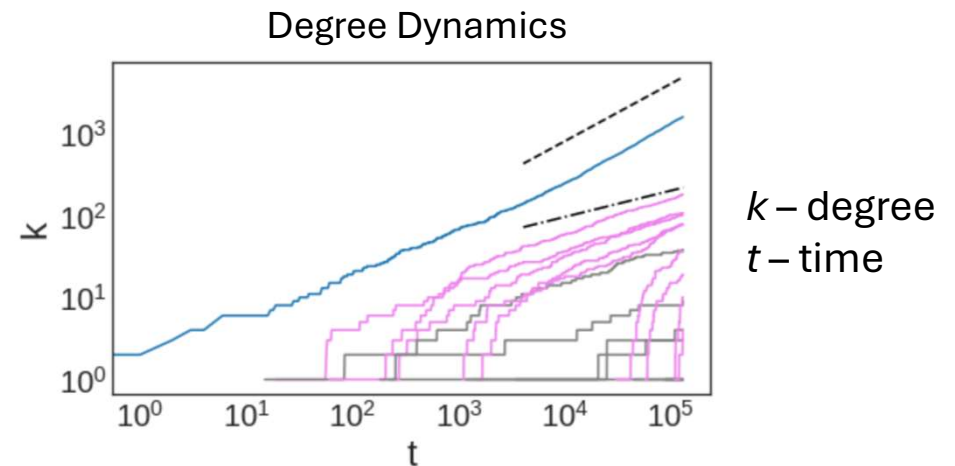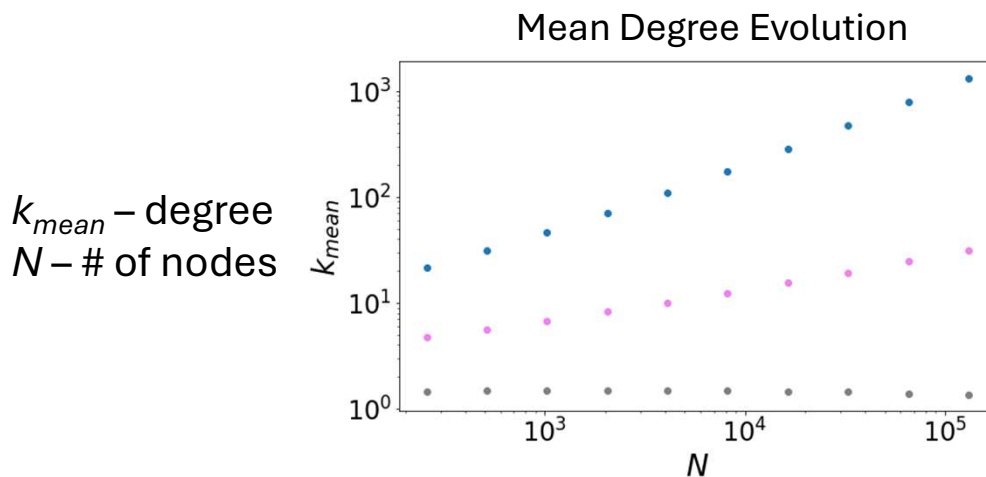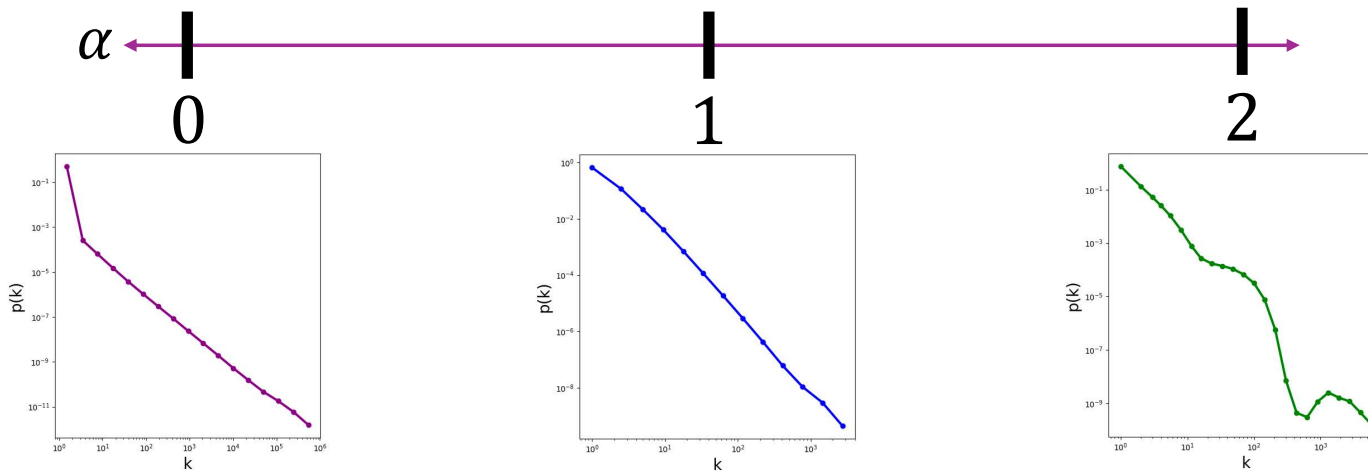
Node Stratification

- Boss, followers, low degree nodes
  - Approximated as highest degree node, its neighbors, and remaining nodes
- Distinct scaling for each group

Boss	$k_i(t) \propto t^{2/3}$
Followers	$k_i(t) \propto t^{1/3} - t_i^{1/3}$
Other nodes	$k_i(t) \propto O(\log t)$

Mean Degree Evolution

k_{mean} – degree
N – # of nodes

Degree Dynamics

k – degree
t – time

Averaged over 1000 networks for various sizes

Step Model

- Preferential attachment with two steps:
  - A node is chosen via probability distribution p_i
  - Incoming node attaches to a random neighbor instead

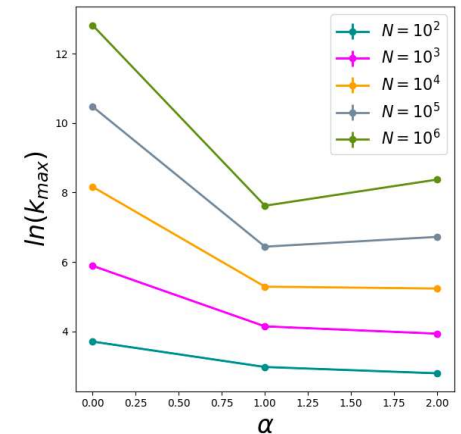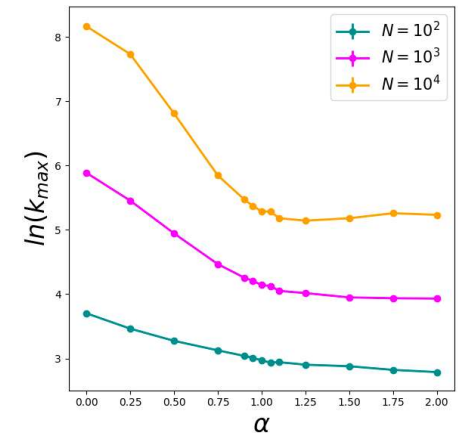- $p_i \propto k^\alpha$ where $0 \le \alpha \le 2$

α number line with marks at 0, 1, 2

| One-Step Friend | Barabási-Albert | k2 |

All degree distributions averaged over 100 networks with $N = 10^6$

Max Degree vs. α

- Less α values but larger sizes

4

References and Acknowledgements

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Saramaki, J., & Kaski, K. (2004). Scale-free networks generated by random walkers. Physics A: Statistical Mechanics and its Applications, 341, 80.

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