Node stratification from simple walk-based preferential attachment rules

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Introduction

The Barabási-Albert (BA) model is a well-known growth model that generates scale-free networks via preferential attachment [1]. At each time step, a new node is connected to an existing node with probability proportional to its degree. In this model, the age of the node is the sole determinant of its expected degree. We consider walk-based modifications of the BA attachment rule [2], where instead of attaching to the endpoint of a randomly selected edge of length 1 (BA), the new node connects to a randomly selected walk of length 2 or higher. In our models, connecting to the leading node leads to faster degree growth. This emergent node hierarchy may be relevant to real-life networks, where new agents joining the network, e.g., scientists looking for collaborations, attempt to maximize their degree or impact.

Basic Network Properties

We consider the version of the model where the new node connects to a random endpoint of a randomly chosen walk of length 2 ("**k2**" model, equivalent to Ref.[3]) or 3 ("**k3**" model). 100 networks are generated for various sizes *N*. The probability of attachment is proportional to the node's weight: number of walks of length 2 or 3 emanating from the node. These weights are distinct from degree, unlike the BA model.

Visualization:

Degree Distribution: p_k – probability of degree, k – degree

Weight Distribution: p_w – probability of weight, w – weight

k2

k3

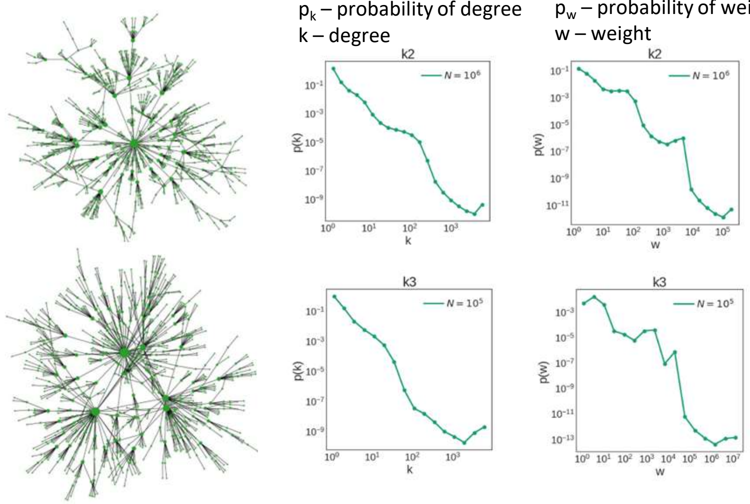

Figure 1: Summary of a networks generated according to the rules of the k2 (first row) and k3 (second row) models. Networks have size $N = 10^6$ and 10^5 respectively and are averaged over 100 trials.

Node Stratification with Scaling

We observe the emergence of three distinct groups: the highest degree node (the boss), its neighbors (followers), and the remaining low degree nodes. These groups have distinct scaling behavior.

k2 **k3**

Degree Dynamics:
k – degree
t – time

Mean Degree Evolution:
k_{mean} – degree
N – # of nodes

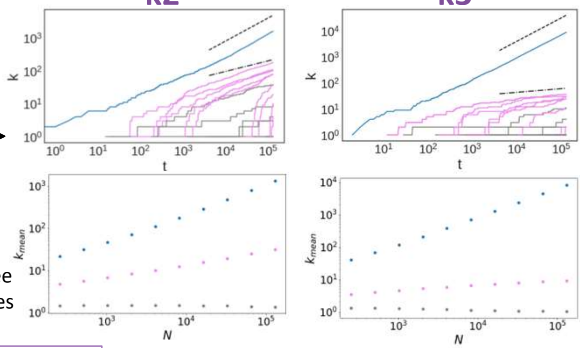

Figure 2: (*Top*) Degree dynamics. The theoretical predictions for k2 exponents are demonstrated on the bottom left. Obtaining the theoretical predictions for k3 requires further work. (*Bottom*) Mean degree of the three groups as a function of network size.

Boss	$k_i(t) \propto t^{2/3}$
Followers	$k_i(t) \propto t^{1/3} - t_i^{1/3}$
Other nodes	$k_i(t) \propto O(\log t)$

The scaling exponents for the boss and its followers can be obtained analytically from the evolution of their degree distributions. For k2, these estimates are close to the theoretical predictions.

Boss **Followers**

k2

k3

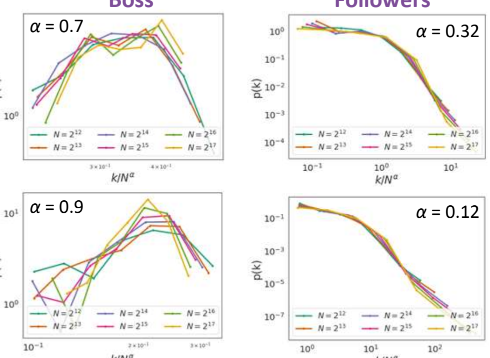

Figure 3: Collapsed degree distributions for boss and neighbors at different *N* (powers of 2). Exponents α were obtained from finite size scaling.

In both models, we observe that a node's expected degree is determined both by its age *and* its connection to the boss. This differs from the BA model's sole focus on age, and it shows that simple modifications can produce new characteristics that may be relevant to studying real-life networks (e.g. scientific collaborations).

References & Acknowledgements

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