Simulating Blackjack Card Counting Strategy

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Abstract
This study simulated card counting strategies of the popular casino game Blackjack on MATLAB software. Each game consisted of a single player versus the dealer. The player started out with $1,000.00 and played basic strategy until winning or losing $500. The variables that were studied included: the threshold of the count before raising amount bet, and the maximum bet that would be placed when the count exceeded the threshold. The goal was to try to find an optimal strategy for increasing the player's bet when the count was high to maximize the probability of the player ending the game with a profit. Data was collected in MATLAB and then copied into an Excel spreadsheet to analyze. An ANOVA test proved that both variables of interest had a significant effect on win percentage. Also, a hypothesis test proved that win percentage slightly increases when the count is higher.

Algorithm Description

How the code works

The script *game.m* first initializes all parameters that are necessary for trial, game, and hand. At the beginning of each hand the player places a wager based on the current count; and then two cards are dealt to the player and the dealer. The two hands are then sent to the function *blackjack.m* which determines if the hand has a pair, an ace, or neither and calls the appropriate function to determine the strategy and outcome for the hand. The three functions that could be called are *pairhand.m, softhand.m,* or *hardhand.m*. Each of these functions will first determine the player's strategy by referencing *pair.m, soft.m* or *hard.m*; then play out the hand and compare it to the dealers to determine the outcome. The function *pairhand.m* includes calls to *hardhand.m* or *playpair.m* depending on whether or not the player chooses to split the pair. The play pair function is used only in the event of a split to compare the player's two hands to the dealers one hand. After the outcome is determined it is sent back to *game.m* which will then adjust the player's money and collect data. Throughout the program the count is adjusted in *deal.m* anytime a card is taken from the deck.

Data Collection Methods

During data collection, we set decks equal to 6, and had the player leave the table only when they had made or lost $500. We then ran the simulation with various combinations of count threshold and maximum bet, with n = 1,000, recording the percentage of games "won" (ended with $1500), the variance of that percentage, and the percentage of times a hand was won at each value of the count.

In the code submitted with this paper, we have set the number of decks equal to 1, set the player to leave the table after winning or losing only $100, set the threshold equal to 5 and the max bet equal to $25. Changes were made to simplify grading and make the code easier to follow.

Validation and Verification

In order to verify the model, all three team members checked through the code. Outputs of the shuffle algorithm were checked visually for randomness through many trials. Outputs of various hands played were checked for logic, which pointed out errors with splitting aces. The outputs were tested for reasonableness after changing factors such as number of decks in the shoe and the initial amount of money, as well as the threshold and maximum bet, which were later analyzed.

Efforts were also made to comment the model well.

Validation was somewhat more difficult. There was only one random input variable, the shuffle order of the cards, which was assumed to be completely random. Shuffles are generally assumed to be random in statistics, and casinos have a vested interest in randomizing a deck as much as possible to minimize cheating. However, we could not find another way to test the assumption that casino blackjack shoes are entirely random. Most of our data analysis amounts to sensitivity analysis, but without knowing real world sensitivities, we cannot say if the model is accurately sensitive. Additionally, without a large set of blackjack data, we were unable to statistically confirm the predictive power of the model and accurate modeling of real world games. Ideally, next steps for validation would involve acquiring data from many real world hands of blackjack from casinos, with which we could compare sensitivities and assess how well the model's output matches the acquired data under similar circumstances to the data's collection.

Data Analysis

The data was analyzed using a hypothesis test and ANOVA. In the raw data, the percentage of games won varied widely based on betting strategy, ranging from 0.159 to 0.526. Data analysis was performed in an attempt to explain these trends. These methods concluded with statistical significance that both parameters of max bet and count threshold have an effect on win percentage. It was also concluded that there is a greater win percentage when the count was in between 10-20 than there was when the count was between 0-10.

Two Factor ANOVA

We performed a two factor ANOVA to test the hypothesis that the effects of changing the threshold and the maximum bet were both equal to zero. For threshold, the p-value was 8.72E-06, and for the maximum bet, the p-value was 1.11E-05. From the p-values produced, we can reject this null hypothesis and conclude that both the threshold and the maximum bet have a significant effect on whether or not the player will "win" the game using each betting strategy.

Hypothesis Test

The following hypothesis test compares the win percentage when the count is between 11-21 and when the count is between 0-10. These ranges were chosen because there was enough samples from each range. Also, these ranges give a good representation of a 'low count' versus a 'high count'. If the null hypothesis is rejected it may be possible to generalize the result to conclude that the higher the count gets, the higher odds the player has to win.

$$\mu_{10} = \textit{The win percentage when the count is between } 11 \textit{ and } 21$$
$$\mu_{0} = \textit{The win percentage when the count is between } 0 \textit{ and } 10$$

$$\mu = \mu_{10} - \mu_0$$

The null hypothesis is that there is no difference in win percentage when the count is between 11 and 21, compared to the count being 0 and 10. The alternative hypothesis is that the win percentage is higher when the count is between 11 and 21, compared to the count being 0 and 10; otherwise stated as the difference between the two means being greater than 0.

$$H_0: \mu = 0$$
$$H_A: \mu > 0$$

$n = 100$
$\alpha = 5\% = 0.05$
$\overline{X} = 0.18\%$
$S = .01085$

$Z_\alpha = 2.326$

$$Z_{score} = \frac{\overline{X} - \mu}{S/\sqrt{n}} = \frac{0.0018 - 0}{.01085/\sqrt{100}} = 1.659$$

$p - value = P(Z > Z_{score}) = 1 - P(Z \leq 1.659) = 1 - 0.9514$
$\boldsymbol{p - value = 0.0486}$

One-hundred samples were used from our simulation in each range. An alpha level of 0.05 was chosen for the significance level. The calculated p-value for this test was lower than the alpha value; therefore we were able to reject the null hypothesis and conclude that the difference in the mean win percentage when the count is between 11-21 and 0-10 is greater than 0. In other words, the mean win percentage is higher when the count was 11-21 compared to the 0-10 range.

<u>Line Graph</u>
Figure 1 shows a *very* small upward linear trend in winning percentage as the count increases. The slope is approximately 0.02%. Also as the count increased, there were less data points at each level since it would get up to that count less often. That is why there is increased variability in the trend towards the higher numbers.

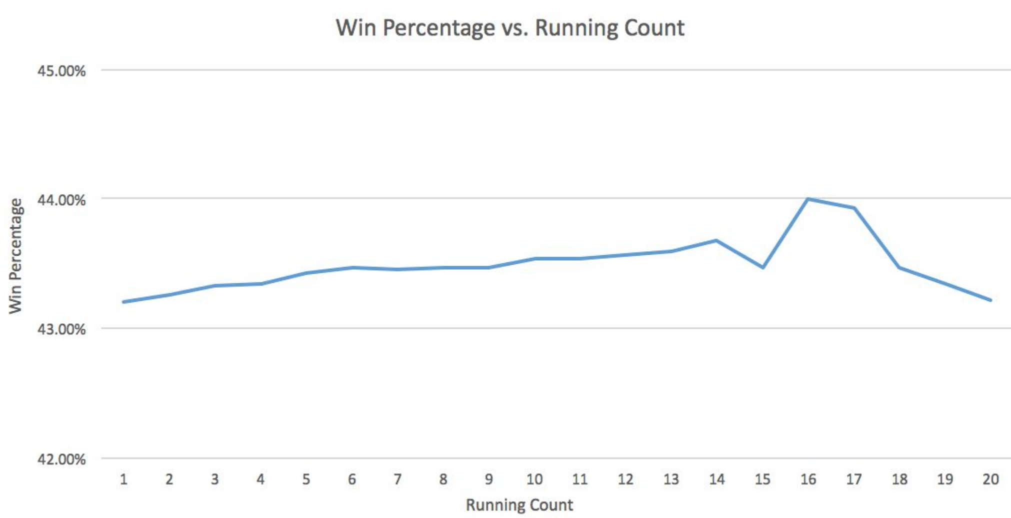

figure 1: line chart of win percentage versus running count

Conclusion
The hypothesis test showed at a significance level of 5% that the probability of winning a hand when the count was in the interval of 11-21 was greater than the probability of winning when the count was between 0-10. This trend could be applied to smaller intervals to conclude that the higher the running count is, the higher the probability of winning each hand. This upward trend was shown in *figure 1.* Even though the odds of winning a single hand increased, that increase

was very small. The extremely small increase in the probability of winning a single hand had no impact in the average number of games won. The probability of winning any hand remained between 43% and 45% regardless of the running count.

The two factor ANOVA showed that both parameters of interest had a significant effect on win percentage. As max bet increased and the count threshold decreased, average wins approached 50%. However, correlation does not imply causation. It must be noted that both of the parameters that showed higher average wins also produced simulations that played many fewer hands, therefore giving the player fewer opportunities to lose. If the max bet continued increasing and the count threshold continued decreasing it would show that the optimal strategy to make a profit is to go all in on one hand. Or, extrapolated even further, to not play at all. The highest expected outcome occurs when zero hands of blackjack are played and the initial $1,000 is kept.

Therefore, the data shows that betting strategy increases the likelihood of winning a game, but not of winning an individual hand. Logically, this means that the hands won must have a larger payout than average, since they occur with the same frequency. The small advantage that card counting gives to players is that the relative amount of tens and aces in the deck is higher when the count is high. Knowing this, one can conclude that there is a better chance the player will get a blackjack, which has a higher payout. Players would then want to bet very high to increase the winnings of the now more likely 3:2 payout. We found that even though this is true, it did not make up for the losses and non-blackjacks that also occurred when the count and wagers were high. Under the conditions tested, a player could increase their odds of winning compared to playing without a betting strategy, but they could not be more likely to win than lose.

Further Research

Our betting strategy consisted of increasing the bet when the running count was high. Though we were able to achieve a win percentage greater than 50% on some simulations, this does not guarantee that this is the best strategy to use while playing. More investigation into strategies that vary the player's bet based on the current count, not betting when the count is low, playing multiple hands against the dealer at a higher count, and cashing out at different current winnings will yield different results. Other counting strategies that divide the running count by an estimate of the decks remaining in the dealer's shoe may yield higher expected winnings. Due to the variability of the cards being played and the game's advantage being directed towards the dealer, it is very difficult to identify and utilize a strategy that involves lower risk for the player. If a player were able to keep track of specific cards that have been dealt out of the deck and make smart bets based on the odds of seeing certain cards on the next hand, they may achieve better results than this simulation. However, this strategy would not be feasible for an average human and would still not guarantee winning. Our data involves a one-on-one game with a dealer that usually does not happen in casinos. Further investigations that involve multiple players at a table betting with different strategies to simulate a more realistic scenario would most likely lead to lower expected winnings. Strategies that involve a team of people that keep the count at different tables and a "high roller" that bets big on tables with higher count would be interesting to investigate. All of these strategies could get you in trouble with the casino if they catch on to what you are doing. Even though these strategies are not illegal, the casino has the right to not allow you to play if they believe you are manipulating the game.

Appendix A

Assumptions
Many casinos have different rules for different tables. To keep the results consistent and narrow the scope of the simulation to focus on the variables of interest many assumptions were made regarding the specific rules of the blackjack table. These rules were chosen to create the most realistic simulation results without over complicating the model.

- It was assumed that only one person was at the table playing the dealer. This made the situation much easier to simulate since there was no need to implement other people randomly appearing and disappearing from the table. This assumption is justified since other players cards do not affect the outcome of any specific hand, nor do they affect the playing strategy during the hand.
- It was assumed that the dealer did not offer insurance or even money since those are never options chosen to pursue by the player under basic strategy. Those options are offered by the casino since they have a positive expected payoff for the dealer.
- The player can only split their hand once. If the player splits aces, only one card is dealt to each ace. This is a common rule at most casinos. The player is able to double down on each split hand.
- The option to “surrender” is not offered since it is not typically available in casinos.
- The dealer hits on soft 17. And when the player gets a blackjack the payout is 3 to 2. These are very popular rules at many casinos.
- There is no burn card at the beginning of each reshuffled deck. This is a very popular thing to do at most tables. But since the burn card is face down the player does not have any information of which card is burned. Probabilistically this is the same as keeping the card in the deck of cards that have not been played.
- The minimum bet at the table is $5.00. There is no maximum allowed bet since in real life it would be greater than any amount that the player chose to bet.
- The maximum amount of cards allowed in any hand is 10. This assumptions is reasonable because the odds of the happening are very slim. Also, some tables have rules about hands exceeding 5 or 6 cards. This 10 card limit was never exceeded in hundreds of thousands of hands that were simulated
- Running count is used instead of true count. True count is equal to the running count divided by the number of decks left, and is a more accurate representation of the deck. But the true count is harder to use in practice, therefore to make the simulation more realistic the running count is used instead.
- Six decks were used in each shoe for the majority of the data collection. This was chosen because it is the most often used number decks for a variety of casinos.
- The decks are thoroughly shuffled and card distribution is completely random.
- The player starts each game with $1000.00 and will leave the table after a profit or loss of $500.00. These parameters are extremely arbitrary in the real world since all players will buy in with different amounts, and have different thresholds to leave; but they were necessary to determine constant values to have consistency between each game.

Definitions

Insurance - Insurance is a side bet that the dealer has blackjack and is treated independently of the main wager. It pays 2:1 (meaning that the player receives two dollars for every dollar bet) and is available when the dealer's exposed card is an ace.

Even money - If the player is dealt a blackjack and the dealer is showing a 10, the player has an option of asking for "even money" (payoff 1:1) instead of taking the chance of the dealer having an ace in the hole and losing the blackjack payoff

Split - If the player is dealt two cards of the same value, the player has an option of "splitting" the two cards and playing two hands against the dealer with an additional card from the shoe on each of the paired cards.

Soft hand - defined as a hand where one of the cards is an ace. An ace can be treated with a value of 1 or 11 (whichever is more advantageous for the player).

Soft 17 - a name assigned to a dealer's hand that contains an ace that sums to 7/17. The common rule is for dealers to hit on a soft 17.

Hard hand - defined as a hand with no aces.

Upcard - The card that is shown face up in the dealer's hand. The second card is kept face down until all players have busted or stayed with their hands.

Hit - The term used when a player wishes to be dealt another card.

Doubling down - this strategy is used when a player would like to double their bet before hitting on their dealt hand. The player receives only one additional card and then stays with their hand played (assuming they do not exceed 21).

Go all in - The player makes a bet of all their current money.

Surrender a hand - The player gives up on the hand before cards are dealt. This results in the player losing half of the amount bet and no chance to win on that hand.

Hi Lo Card Count System - This is one of many different possible card counting strategies. With this system cards that count as plus one are: 2, 3, 4, 5, and 6. Cards that count as minus one are aces and any ten valued card (10, J, Q, K). Neutral cards are 7, 8, 9. The count of a whole deck sums to zero with this strategy.