Planck's Constant Lab Report

Experiment Performed by Kyle Drury —— Designed by Dr. Fitzgreen

January 2023

1 Introduction

Planck's constant is a well-known physical constant from the study of quantum mechanics, which helps physicists describe the relationship between a photon's frequency and its energy. This constant can be experimentally determined by bombarding a metal with a known work function to eject photoelectrons, and then monitoring how the flow of these photoelectrons changes as an applied stopping potential difference is slowly raised. When a spike in the current is observed, this corresponds to the exact moment that the kinetic energy of the photoelectrons is significant enough such that they have just enough energy to overcome this known stopping potential.

1.1 The Equation

We can relate the retarding voltage of the apparatus to the measured current between the anode and the cathode with the following equation (with e being the elementary charge and ϕ being the work function):

$$Energy = eV_r = hv - e\phi \rightarrow V_r = (\frac{h}{e})v - \phi = (\frac{hc}{\lambda e}) - \phi \tag{1}$$

When both sides of this equation become equal, this corresponds to the point at which photoelectrons start to rapidly flow across electrodes (note here that we have expressed the equation in terms of wavelength and c because this was the measurement that was given.

2 Experimental Results

Below (figure 1) are the plots of current vs. retarding voltage for the trials of the experiment where the lens was half closed, allowing fewer photons in as a result.

Figure 2 displays the same plots, but with a fully opened lens, thus allowing more photons to the cathode.

3 Data Analysis

3.1 Retarding Potential at Onset Vs. Frequency

The first step in our data analysis is to estimate the point at which the current begins to increase exponentially for each plot, and plot this against the frequency of the light being used. Table 1 summarizes data from Figure 1, and table 2 summarizes the data in Figure 2.

We will summarize these results in a plot that compares frequency (independent variable) and stopping potential (dependent variable). See figure 3.

We see that the slope of our trendline is -0.4202, but we must divide by 1×10^{14} because of how we scaled our plot. Thus, the slope is actually -4.23×10^{-15}.

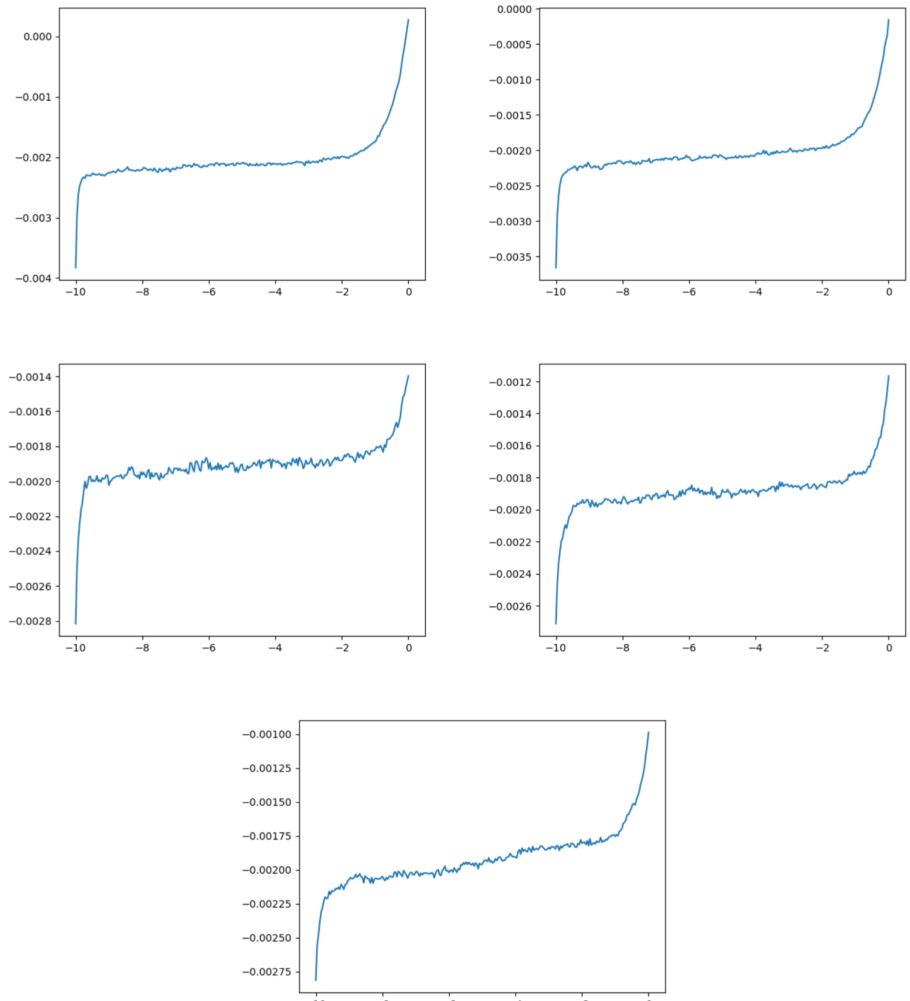

Figure 1: First 5 Trials: Semi-Closed Lens

Test Number	Stopping Potential at Onset (V)	Frequency $\frac{c}{\lambda}(Hz \times 10^{14})$	Estimated Uncertainty in Voltage
1	-1.8	7.412	± 0.1
2	-1.5	6.884	± 0.1
3	-1.3	6.103	± 0.1
4	-1.0	5.493	± 0.1
5	-0.9	5.199	± 0.1

Table 1: Stopping Potential Vs. Frequency for First 5 Trials

Test Number	Stopping Potential at Onset (V)	Frequency $\frac{c}{\lambda}(Hz \times 10^{14})$	Estimated Uncertainty in Voltage
6	-1.8	7.4122	± 0.1
7	-1.5	6.8842	± 0.1
8	-1.2	6.1032	± 0.1
9	-0.9	5.4932	± 0.1
10	-0.8	5.1992	± 0.1

Table 2: Stopping Potential Vs. Frequency for Last 5 Trials

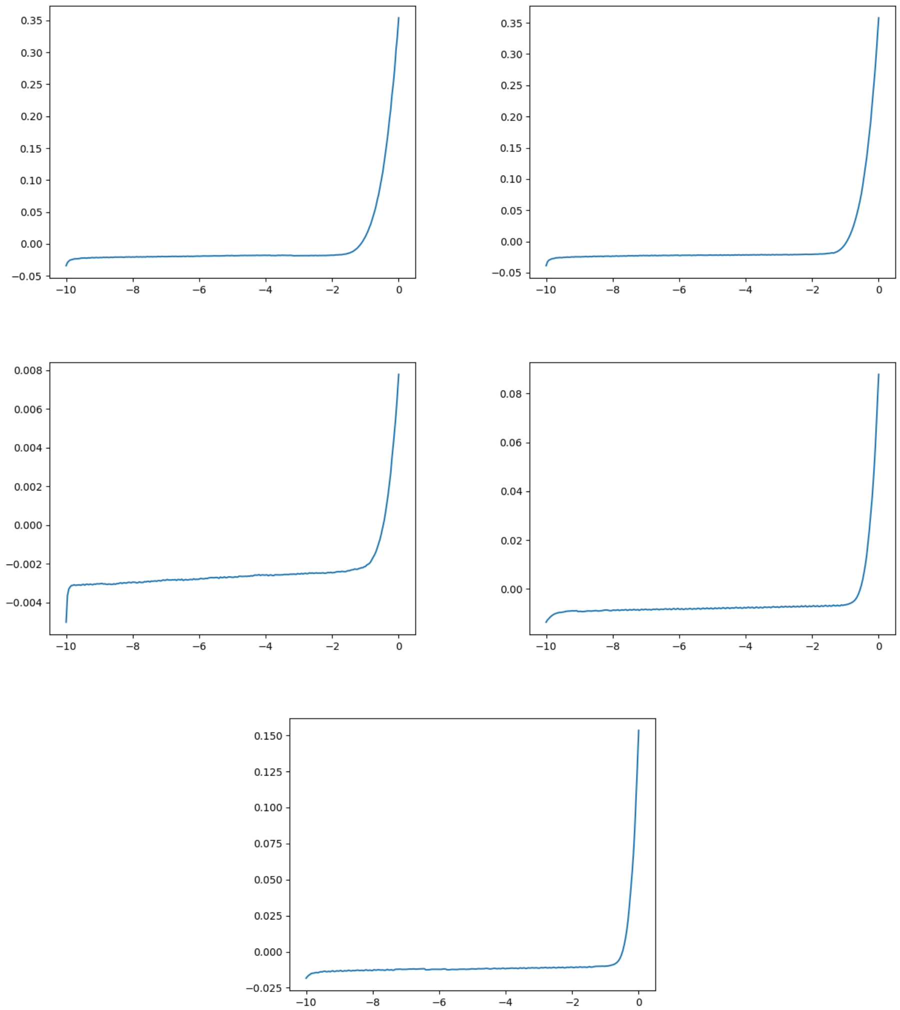

Figure 2: Last 5 Trials: Fully Opened Lens

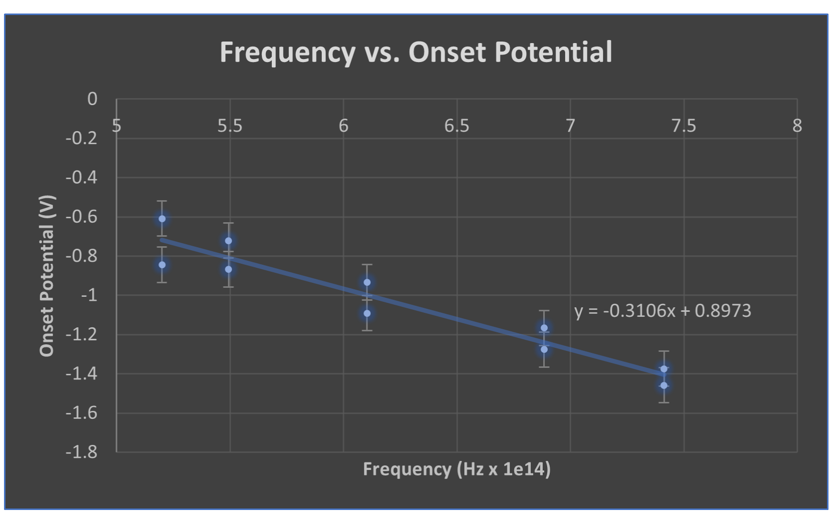

Figure 3: Frequency vs. Onset Potential (All 10 Trials)

3.2 Calculating our Experimental Value of Planck's Constant

Looking at equation (1), we deduce that the slope of our trendline from figure 3 must be equal to the quantity h/e, where $e = 1.602176634 \times 10^{-19}$ C. We can now calculate h:

$$-4.23 \times 10^{-15} = m = \frac{h}{e} \rightarrow h = e(-4.23 \times 10^{-15}) = \mathbf{6.78 \times 10^{-34}} \tag{2}$$

Thus, our experimental value of Planck's constant is 6.78 $\times 10^{-34}$. Compared to the widely accepted value of 6.602 $\times 10^{-15}$, our value has an error of about 2.63%

3.3 A More Methodical Approach

Here, we will calculate Planck's constant more precisely. First, we shall plot all of our trials again, except this time, we will add a trend line to the 'mostly linear' part of the plot, and another one to the 'mostly-exponential' part. Given the equations of the lines, we can calculate the POI. This intersection will be our more-accurate onset voltage, which we will plot against frequency later.

Table 3 shows the slope and y-intercept for each of the trendlines that describe the linear exponential segments of each trial. Uncertainties *are* calculated but are not listed here. Table 4 shows the points of intersection (with uncertainty).

Test	Linear Slope	Linear Intercept	Exponential Slope	Exponential Intercept
1	2.872e-05	-0.001976	0.001112	-0.0003965
2	3.513e-05	-0.001906	0.001016	-0.0006538
3	1.420e-05	-0.001843	0.0003038	-0.001527
4	1.734e-05	-0.001803	0.0005315	-0.001357
5	4.255e-05	-0.001722	0.0007186	-0.001151
6	0.0004827	-0.01633	0.1832	0.2348
7	0.0004971	-0.01951	0.2238	0.2410
8	9.071e-05	-0.002236	0.007718	0.004889
9	0.0002882	-0.006480	0.09587	0.06106
10	0.0003495	-0.01004	0.1907	0.1059

Table 3: Equations of Trendlines

Test	Frequency $\frac{c}{\lambda}(Hz \times 10^{14})$	Intersection	Uncertainty
1	7.412	-0.6857	0.01214
2	6.884	-0.7838	0.01562
3	6.103	-0.9148	0.01856
4	5.493	-1.153	0.02947
5	5.199	-1.184	0.02692
6	7.412	-0.7274	0.01346
7	6.884	-0.8571	0.01726
8	6.103	-1.071	0.02421
9	5.493	-1.415	0.03711
10	5.199	-1.641	0.04796

Table 4: Intersections of Trendlines

Figure 4 shows the datasets from figure 2, except they are fitted with trendlines and only show data from -9V to 0V (this was so that the linear and exponential parts of the data could easily be isolated). Figure 5 depicts each corresponding residual plot from each plot in figure 4. Note that plots appear in order from tests 1-10 moving from left to right, and from top to bottom.

3.4 Calculating Planck's Constant (Again)

With our new data from table 4, we may once again plot onset potential as a function of the frequency of light, and use the slope to calculate Planck's constant (see figure 6).

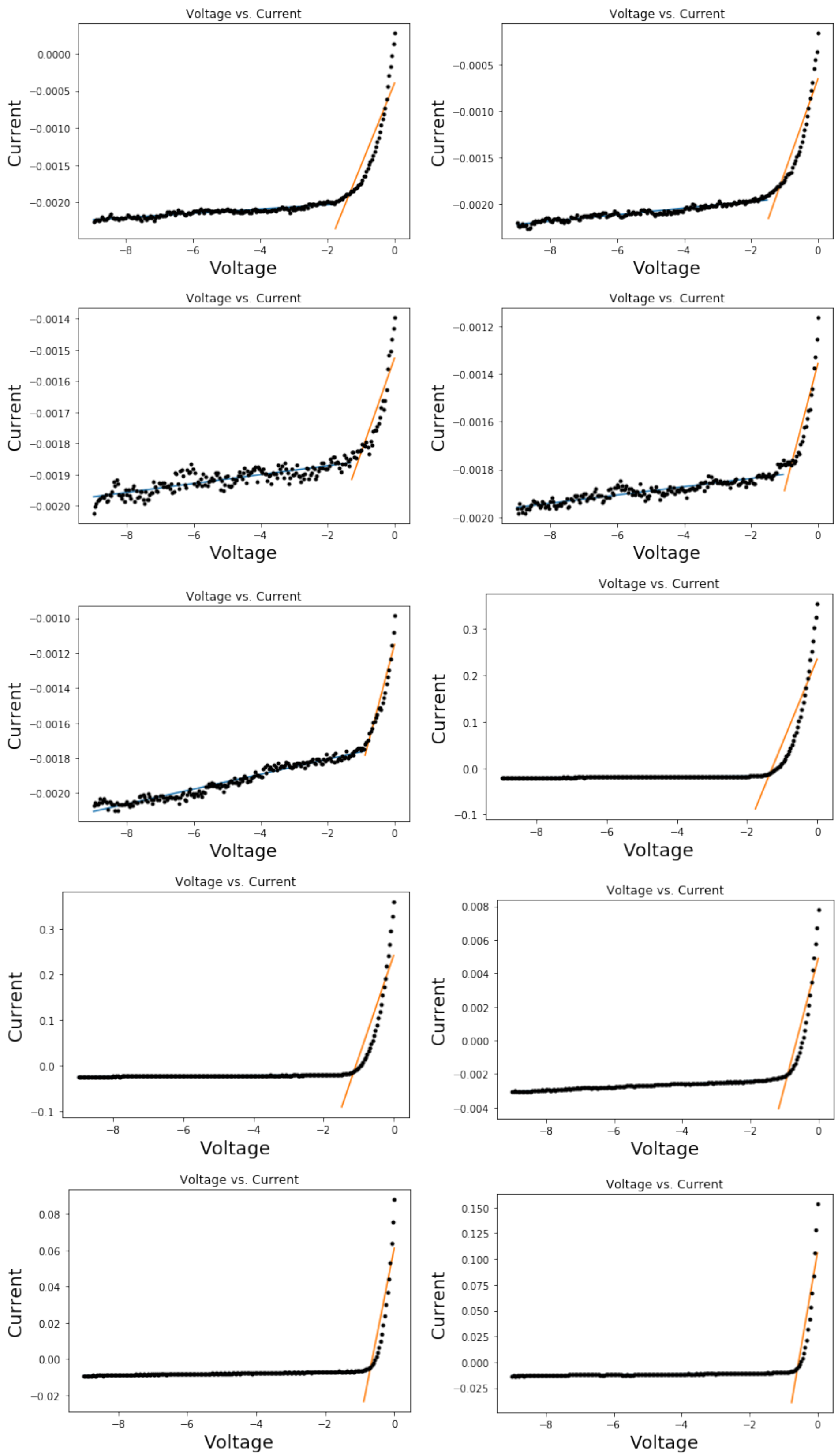

Figure 4: Current Vs. Voltage for all 10 Tests, with Curvefit

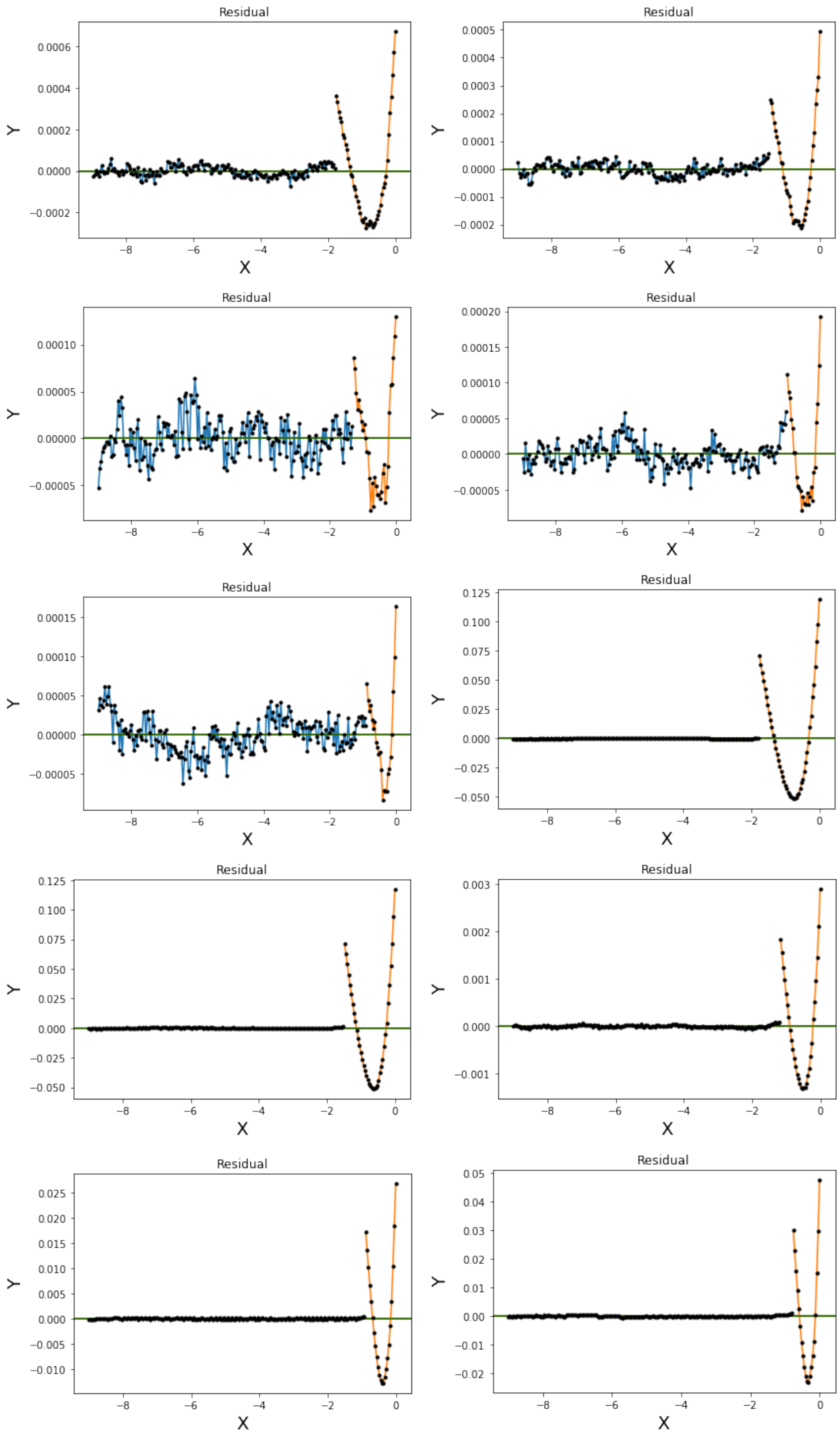

Figure 5: Residuals

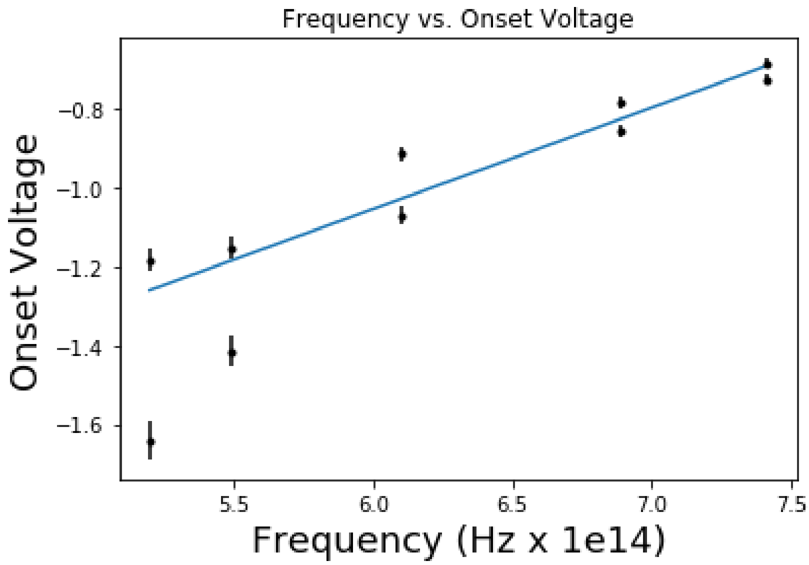

Figure 6: Frequency vs Onset Voltage

The slope of this line is 0.2569 ± 0.03785. Following equation (2), we have:

$$h = e(0.2569 \pm 0.03785) \times 10^{-14} = \mathbf{-4.116 \pm 0.6208 \times 10^{-34}} \tag{3}$$

4 Discussion

Surprisingly, the method of 'eyeballing' actually better approximated the value of Planck's constant than taking an intersection point of trendlines. This is obviously due to the fact that these lines were *linear*, so they were a very rough representation of the actual trend of data. If the data sets had been approximated with a higher-degree polynomial, for instance, the intersection points likely would have been more accurate.

It was quite easy to be able to tell when exactly the current began to spike. By using trendlines, however, the intersection point tends to drift away from the true location of the initial spike. As mentioned before, this problem would be nicely remedied using a polynomial fit with a higher degree.

A bizarre consequence of this "drifting" phenomenon is that the slope of our trendline turned out to be positive, whereas the trendline obtained from 'eyeballing' data was negative. Of course, we should expect the slope to be negative, as it makes sense that as the frequency of light increases, the energy should also increase. As such, we would expect the spike to occur sooner (ie. at a lower, more negative potential) because the photoelectrons would have more energy to overcome the retarding potential.

This is what we observe in figure 3. Conversely, as seen in figure 6, as the frequency gets higher (more energetic photons), the potential becomes *less* negative, which makes no sense (as outlined above).

Another source of potential error may be the large amount of noise experienced in the first five trials (figures 1, 4, 5). While this is likely because of the half-closed lens (all of the fully-opened lens tests exhibited much smoother curves), it may be a reason for the skew in our data and the wildly inaccurate second calculation of the Planck Constant.

5 Conclusions

The data collected from the experiment proved to be valid for some tests, and not for others. By estimating the point at which photoelectrons start leaving the cathode at an exponential rate, we were able to get a good idea of the relationship between the frequency of light and the onset potential. This slope allowed us to calculate (to a fairly accurate degree) the value of Planck's constant.

To get a more accurate idea of when these ”turning points” occurred, the linear and exponential parts were represented with trendlines calculated using curvefit in matplotlib. These calculated points of intersection turned out to be a significantly poorer way of visualizing the relationship between frequency and retarding potential, and the results produced yielded an experimental Planck constant that was negative.

It is likely that the inaccurately calculated points of intersection were caused by the copious noise in the first five trials, and the fact that the data were fitted using a trendline, rather than some higher degree polynomial (which would have given a much better picture of the intersection points).

It would be interesting to repeat this experiment with different frequencies of light to see if the same results appear. Using different metals for the anode and cathode would also have some interesting implications.