Charge-to-Mass Ratio of the Electron: Data Analysis

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Abstract

The charge-to-mass ratio of the electron was first measured to be -1.8e11 ± 7.1e9 C/kg, which falls within 1 standard deviation of the CODATA value. This result was attained using a precisely controlled magnetic field produced by a solenoid, whose current was measured directly, to bend a beam of electrons in a circular path. By equating the centripetal force with the magnetic force, a precise calculation of the ratio between the electron's ratio of mass (kg) and charge (C) can be produced. The solenoid voltage was then measured, and using a precisely known shunt resistor, the charge-to-mass ratio was calculated to be 1.6084e11 ± 2.3853e9 C/kg, which is a statistically insignificant result.

1 Data Collection

Initially, the current required to deflect the electron beam toward various targets was measured directly, and the following data was obtained (units are in amps):

Circle	Bar 1	Bar 2	Bar 3	Bar 4	Bar 5
2.7	2.498	2.198	1.896	1.696	1.504
2.9	2.841	2.399	2.098	1.896	1.696
3.2	3.202	2.699	2.3	1.999	1.798
3.602	3.401	2.901	2.5	2.2	1.999
3.802	3.601	3.001	2.599	2.299	2.098
3.903	3.702	3.202	2.8	2.4	2.198
4.203	3.903	3.299	2.901	2.498	2.999

Figure 1: Initial Solenoid Current Measurements (A)

Following this, a shunt resistor was introduced. Using the precisely known resistor of the shunt resistor and taking note of the operating voltage of the solenoid, theoretically, a more accurate measurement should be attainable. The below tables show the solenoid voltage data and the calculated solenoid current using the shunt resistor. Units of voltage are mV, so a quantity in mΩ is used to calculate the current, presented in units of amps.

Circle	Bar 1	Bar 2	Bar 3	Bar 4	Bar 5
27.529	26.52	22.501	19.482	17.466	15.534
30.547	28.593	24.502	21.493	19.48	17.466
33.526	31.556	26.521	23.512	20.489	18.472
36.555	33.526	28.539	25.512	21.494	19.481
38.572	35.547	30.55	26.522	23.512	20.49
40.592	37.564	32.568	27.53	24.504	22.502
42.599	39.583	33.531	29.554	25.517	23.513

Circle	Bar 1	Bar 2	Bar 3	Bar 4	Bar 5
2.7529	2.652	2.2501	1.9482	1.7466	1.5534
3.0547	2.8593	2.4502	2.1493	1.948	1.7466
3.3526	3.1556	2.6521	2.3512	2.0489	1.8472
3.6555	3.3526	2.8539	2.5512	2.1494	1.9481
3.8572	3.5547	3.055	2.6522	2.3512	2.049
4.0592	3.7564	3.2568	2.753	2.4504	2.2502
4.0592	3.9583	3.3531	2.9554	2.5517	2.3513

Figure 2: Solenoid Voltage (Top); Current calculated with Shunt Resistance (Bottom)

It is important to note that each row represents another dataset using a different anode potential. The top row is with 20 V, and with each successive row, the anode voltage is incremented by +5 V up to 50 V.

The distance to the targets is provided in the lab manual, and the radii of the electron paths are just distances (diameters) divided by two:

Bar Number	Distance/Diameter (m)	Radius r (m)
1	0.0648	0.0324
2	0.0775	0.0388
3	0.0902	0.0451
4	0.1030	0.0515
5	0.1154	0.0577
6	0.0600	0.0300

Table 1: Distance of Bars from Filament

Note that the 6th 'bar' doesn't actually exist, but is a crude approximation of the distance to a point just in front of the first bar.

2 Relevant Derivations

2.1 The Equation

To determine the charge-to-mass ratio, the data must be represented such that a linear fit can be used to describe a constant of proportionality between the x and y variables that is connected to the charge-to-mass ratio. From the manual, we have the following equation:

$$\frac{e}{m} = 2.47 \times 10^{12} (\frac{a^2}{N^2})(\frac{V}{I^2 r^2}) \tag{1}$$

Let $2.47 \times 10^{12}(\frac{a^2}{N^2}) = c$, because this is a constant that does not change. The above equation can be rearranged to give the following relationship:

$$\frac{I^2 c}{V} = \frac{1}{r^2}\frac{m}{e} \tag{2}$$

2.2 Error Propagation

Let $y = \frac{I^2 c}{V}$. Then...

$$\alpha_y = \frac{\partial y}{\partial V}\alpha_V + \frac{\partial y}{\partial I}\alpha_I = \frac{I^2 c}{V^2}\alpha_V + \frac{2Ic}{V}\alpha_I \tag{3}$$

The uncertainty in the current (for the first measurements), as stated above is ± 0.00005 A. The uncertainty in the voltage is ± 0.5 V (because the measurements are sensitive to 1V).

When it comes to the current measurement obtained with the *shunt resistor*, error propagation is more involved. Because I is no longer being measured directly, but is being calculated using the solenoid current and shunt resistance, separate error calculations must go into each value of α_I.

$$I = \frac{V}{R} \rightarrow \alpha_I = \frac{\partial I}{\partial V}\alpha_V + \frac{\partial I}{\partial R}\alpha_R = \frac{\alpha_V}{R} + \frac{V\alpha_R}{R^2} \tag{4}$$

Here, $\alpha_V = 0.0005$mV (half the smallest digit in the measurements), and $\alpha_R = 0.01$ mΩ.

3 Data Analysis

3.1 Initial Measurements

$\frac{I^2 c}{V}$ may now be plotted against $\frac{1}{r^2}$ to get a slope of $\frac{m}{e}$ with an uncertainty. Below, this plot is shown.

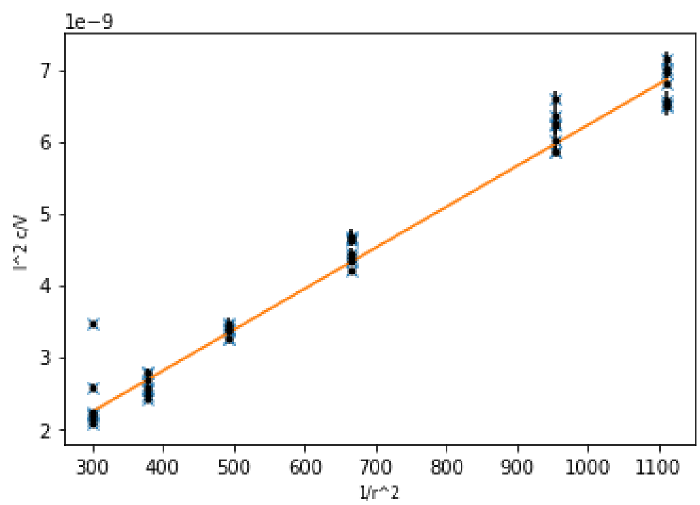

Figure 3: Plot to give $\frac{m}{e}$ as slope

The slope of this line (generated using Python) is 5.71228857483143e-12 ± 2.2694668267850584e-13. Because our plot is limited in accuracy by V (2 significant figures), the measurement of the slope is limited by this. It can be concluded that the slope is 5.7e-12 ± 2.3e-13.

The CODATA value of $\frac{e}{m}$ is -1.75882001076 $\times 10^{11}$ C/kg. It should be expected that the negative inverse of the slope should be equal to this value. The slope ($\frac{m}{e}$) is 5.7e-12 ± 2.3e-13.

Let R be the $\frac{e}{m}$ ratio, and r be the $\frac{m}{e}$ ratio.

$$R = r^{-1} \rightarrow \alpha_R = \frac{\partial R}{\partial r}\alpha_r = \frac{1}{r^2}\alpha_r \tag{5}$$

It follows that the $\frac{e}{m}$ ratio is -1.8e11 ± 7.1e9 C/kg, which falls within 1 standard deviation of the CODATA value listed in the paragraph above.

3.2 Precision Measurements with Shunt Resistor

Using the solenoid current values obtained on the trial with the shunt resistor and the same $\frac{1}{r^2}$, the following plot can be made (uncertainty bars are shown).

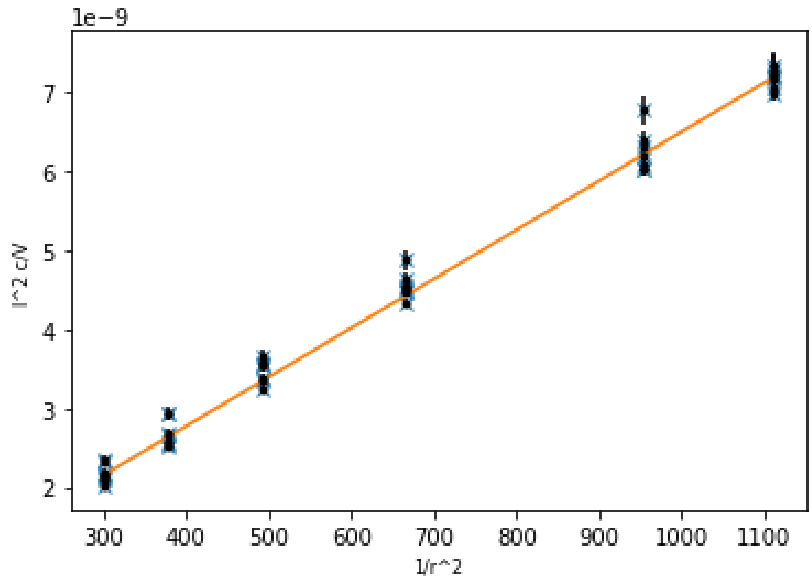

Figure 4: Plot to give $\frac{m}{e}$ as slope (with shunt resistor)

The slope of this line is 6.217202969971812e-12 ± 9.219953203303344e-14. For this data, our precision is limited by the number of significant figures in the solenoid voltage measurements, which is 5. It can be concluded that 6.2172e-12 ± 9.2200e-14.

Refer to equation (3) to convert this measurement into the $\frac{e}{m}$ ratio. This gives the $\frac{e}{m}$ ratio as 1.6084e11 ± 2.3853e9 C/kg. This falls more than three standard errors away from the expected CODATA result and is not statistically significant.

4 Conclusions

Unexpectedly, the method of directly measuring the solenoid current, which yielded an e/m ratio of -1.8e11 ± 7.1e9 C/kg, was a much better method for finding the charge-to-mass ratio than using the precisely described shunt resistor. However, the value obtained with the shunt resistor, -1.6084e11 ± 2.3853e9 C/kg, is still within about 91.44% of the expected CODATA value. Even so, the original measurement was far superior, only falling one standard away from the expected CODATA result. A potential reason for this shortcoming with the shunt resistor is that this method had more calculations involved, and so small noise in the original data has a higher chance to greatly alter the final results.

5 Supporting Documents

Scripts and original data linked at the GitHub repository here.