Estimating Dark Matter Distributions in Spiral Galaxies from Rotational Velocity Curves

by

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Abstract

By using the observed rotation curve of the Milky Way Galaxy (MW) [7] and known luminous matter distributions [5], a technique for estimating the density profile of the dark matter halo (DMH) of the MW is used. By observing differences in the predicted rotation velocity and observed, the dark matter density can be calculated and fitted to the Pseudo-Isothermal Halo (PIH) and the Navarro-Frenk-White (NFW) models [1].

Previous measurements of the MW's DMH found its mass to be in the neighborhood of 0.70×10^{12} $M_\odot$ with a 50% Bayesian credible interval of $(0.62, 0.81) \times 10^{12}$ [8]. Using the data obtained in this study, two measurements of the MW DMH are yielded, using the PIH and NFW models, respectively; a mass of **2.2(8)e13** $M_\odot$, which falls **2.66**σ away from the expected result of **0.7e12** $M_\odot$ [8]; and a measurement of **7.95(4)e12** $M_\odot$, which falls **244**σ away.

1 Introduction

One of the most reliable ways physicists have been able to infer the existence of dark matter indirectly is by analyzing the rotation curves of galaxies. This is done by plotting the orbital velocity of bodies in a galaxy as a function of their distance from the center. In most cases, galaxies house supermassive black holes in their cores that act as a sort of cosmic engine and a gravitational anchor for all matter in that galaxy.

In spite of the strength of the supermassive black hole at the center of the Milky Way (and all other galaxies), it has been made clear that there is not enough visible matter to hold rapidly rotating galaxies together.

1.1 Rotation Curves & Dark Matter

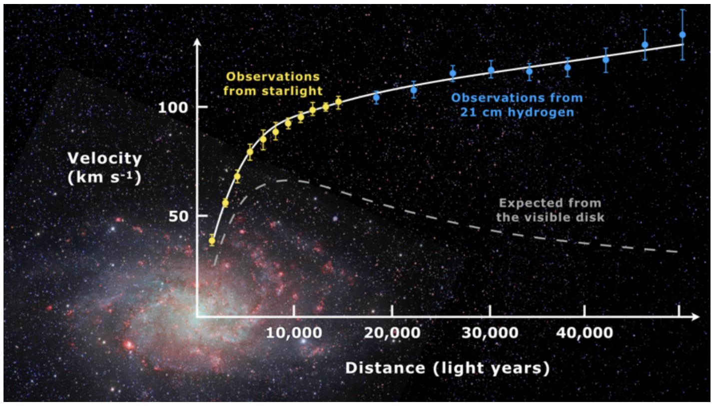

Figure 1: Rotation curve of the Messier 33 as expected from visible disk compared to what is observed [2]

As can be seen in the above plot, the rotational velocity of bodies orbiting further away from the galactic center is much greater than what we would reasonably expect. Through this, the existence of dark matter can be inferred (without dark matter, it would be reasonable to expect that further-out objects orbit slower).

From the latest measurements of WMAP satellite, we know that the Universe consists of 4% baryons, 22% cold dark matter, and 74% dark Energy [3]. Dark energy is thought to be a fundamental property of the cosmic vacuum and is uniformly distributed in spacetime, and has no local gravitational effects (ie. not on the scale of a single galaxy) [4].

Dark matter, which is not evenly distributed, has gravitational influences on the structures of galaxies and is the primary reason why the rotation curves of galaxies do not line up with models that only account for visible matter.

1.2 Matter Distribution in Galaxies

Because dark energy (DE) is distributed throughout the interstellar medium between galaxies as much as it is *within* galaxies, DE may be excluded from galactic compositions.

The structure of a galaxy can essentially be described in five parts; the black hole (BH) at the center, the inner bulge (or core), the bulge, the disk, and the DMH [5]. The plot on the left is from [5], and the plot on the right depicts the digitized curves that will be used in the analysis.

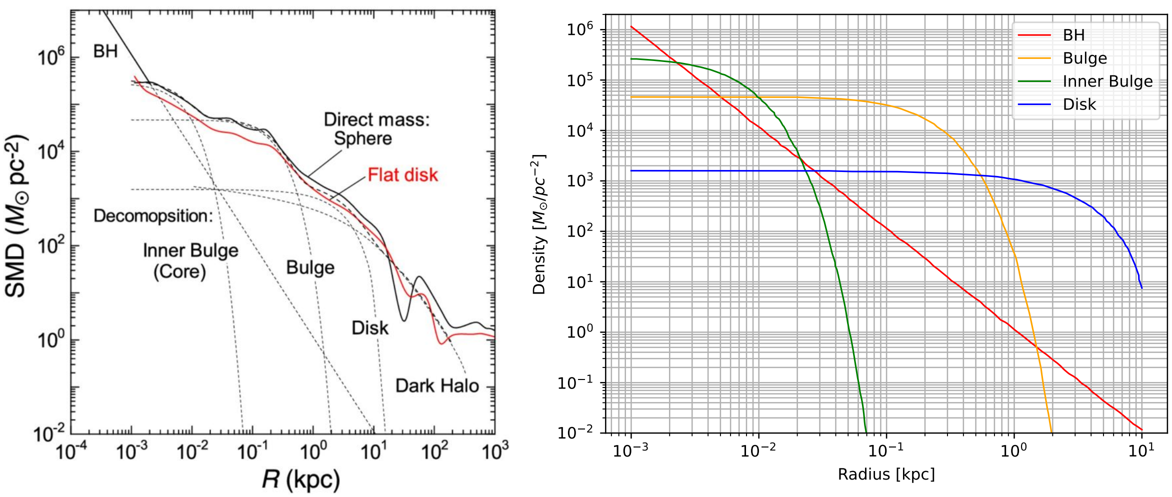

Figure 2: Surface Mass Density Distribution in the Milky Way, approximated by a disk

1.3 Dark Matter Halo Profiles

The most simplistic model for DMHs in galaxies is the pseudo-isothermal Halo (PIH) model:

$$\rho_{DM}(r) = \rho_0 \left[1 + \left(\frac{r}{r_c}\right)^2\right]^{-1} \tag{1}$$

where r_c is the so-called core radius, and ρ_0 is the finite central density [6].

The Navarro–Frenk–White (NFW) profile is a distribution of dark matter fitted to DMHs identified in N-body simulations by Julio Navarro, Carlos Frenk, and Simon White. Is the most common profile used to describe DMHs [6].

$$\rho_{DM}(r) = \rho_{crit} \frac{\delta_0}{r/r_s(1 + r/r_s)^2} \tag{2}$$

where ρ_{crit} is the critical density of the universe (1.4771×10^{-7} $M_\odot/pc^3$ [9]), r_s is the so-called break radius, and δ_0 is the dimensionless density parameter

$$\delta_0 = \frac{200}{3} \frac{c^3}{\ln(1+c) - c/(1+c)} \tag{3}$$

Here, we define c, R_{200}, and M_{200}:

$$c = \frac{R_{200}}{r_c} \qquad\qquad R_{200} = \left(M_{200}/\frac{4}{3}\pi 200 \rho_{crit}\right)^{1/3} \tag{4}$$

where M_{200} is the mass of entire halo [1].

2 Computational Approach

The fundament of our strategy lies with the rotation curve of a galaxy, such as that of the Milky Way:

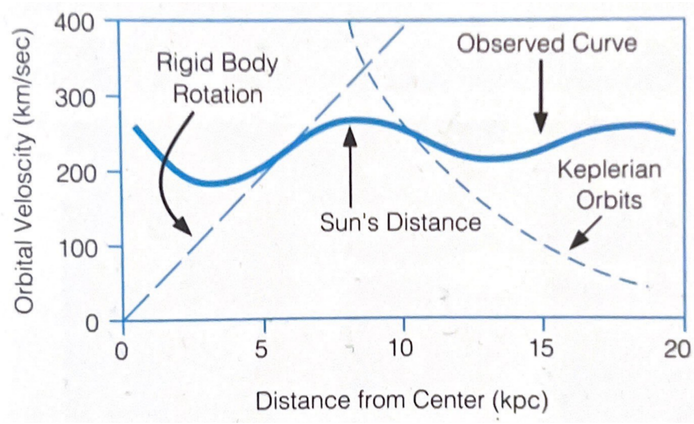

Figure 3: Rotational velocity of the MW from 0 to 20 kpc [7]

Given a rotation velocity and by invoking Gauss's law for gravitation, the total mass enclosed at a certain point r from the galactic center can be calculated. The nice thing about the PIH and NFW models is that the DMH and visible matter disk have circular symmetry, so the situation can be treated as if all the mass were concentrated at a point a distance r away.

$$v(r) = \sqrt{\frac{GM}{r}} \Longrightarrow M(r, v) = \frac{v^2 r}{G} = M_L + M_D \tag{5}$$

$$G \approx 4.3009 \times 10^{-3} \, \mathrm{pc} \, (\mathrm{km/s})^2 \, \mathrm{M}_\odot^{-1} \tag{6}$$

Here, subscripts L and D denote luminous and dark matter respectively. M_L is confined to a disk, and the luminous mass density function ρ_L is known (Fig 2). Therefore,

$$M_L(r) = \int_0^r \rho_L(r')(2\pi r')dr' \tag{7}$$

In the case of the PIH model, there are two fit parameters: r_c and ρ_0, which will have uncertainties. Likewise, r_c and δ_0 will have uncertainties for the NFW model (refer to equation 3 for δ_0; but M_{200} is unknown, and this parameter cannot be known beforehand. However, it can be compared to the calculated value after our simulation).

These measurements will be used to calculate the total mass of the MW's DMH. The necessary equations to express the uncertainty in the fit DMH mass as a function of the uncertainty of the fit parameters will need to be derived.

Note that since the DMH is spherical, so its mass is given by the following equation:

$$M_D = \int_0^{R_g} \rho_D(r)(4\pi r^2)dr \tag{8}$$

For a given model parameter ϕ of $\rho_D(r)$, the uncertainty on M_D, denoted α_{M_D} is the following:

$$\alpha_{M_D} = \alpha_\phi \left[\frac{\partial}{\partial \phi} \int_0^{R_g} \rho_D(r)(4\pi r^2)dr \right] \tag{9}$$

Both profiles are continuous and differentiable with respect to all parameters in their expected neighborhoods. For multiple parameters, we would differentiate with respect to those parameters too, and add their respective uncertainties. We will refer to Eadie & Juric [8], who found the mass of the MW DMH to be 0.70×10^{12} $M_\odot$ with a 50% Bayesian credible interval of $(0.62, 0.81) \times 10^{12}$ $M_\odot$.

3 Objectives

1. Fit the ρ_D data with the PIH and NFW models. Which fit is better? What are the fit parameters?
2. What is the mass of the MW DMH according to both models? How does it compare to the value of 0.70×10^{12} $M_\odot$? Does it fall within the 50% Bayesian credible interval? [8]
3. What is ρ_0 in the PIH model? What is δ_0 in the NFW model? Do δ_0 and r_c have the relationship described in (3) and (4)?

4 Preliminary Data Collection

By summing the densities from Figure 2, the total density curve can be attained, as well as the total mass enclosed as a function of radius:

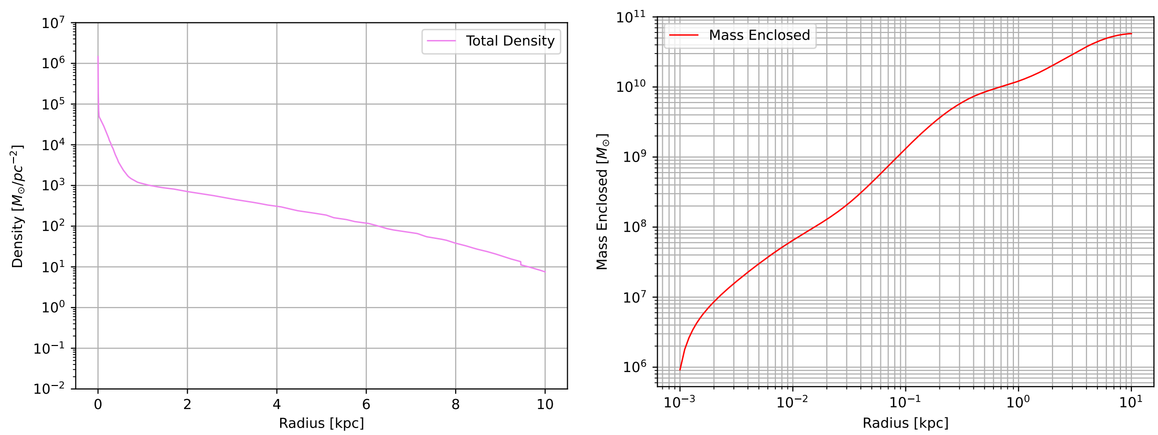

Figure 4: Total luminous density and mass enclosed

From [7] and using equation (5), the total mass enclosed is calculated (blue):

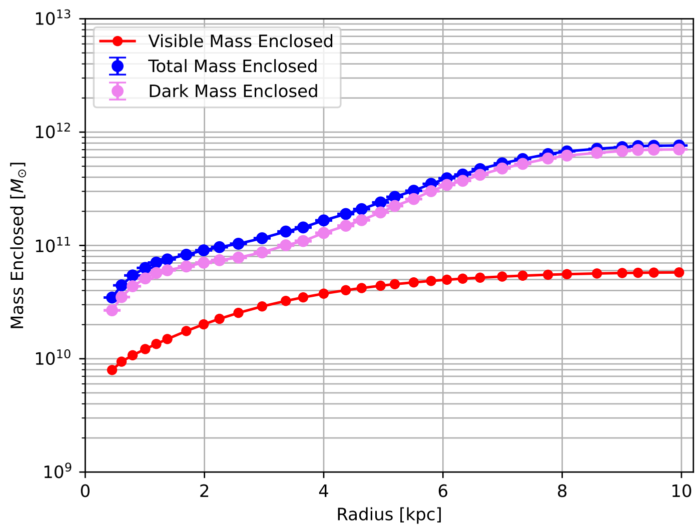

Figure 5: Total, luminous, and dark mass enclosed

These are very nice results; the total mass enclosed is greater than the visible mass enclosed (red) by a reasonable factor. The difference between points represents the mass of the total enclosed dark matter (purple).

5 Analysis

Knowing the total mass of dark matter enclosed as a function of radius, the mass of dark matter contained in the i^{th} shell (the difference in dark matter mass contained in spheres with radii r_i and r_{i-1}) can easily be calculated:

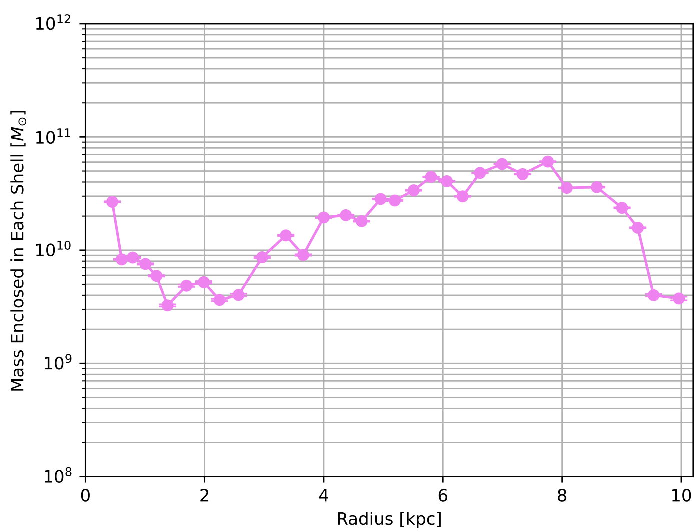

Figure 6: Mass of dark matter contained in each "shell"

Similarly, the volume of each can be calculated, thereby yielding the *dark matter density* as a function of the radius. From there, the data can be fitted with the NFW and PIH models to determine the parameters of the MW's DMH.

In order to fit the DMH density to the NFW profile in a more streamlined manner, a modified NFW model is proposed:

$$\rho_{DM}(r) = \frac{\delta}{r/r_s(1 + r/r_s)^2} \tag{10}$$

where $\delta \equiv \delta_0 \rho_{crit}$.

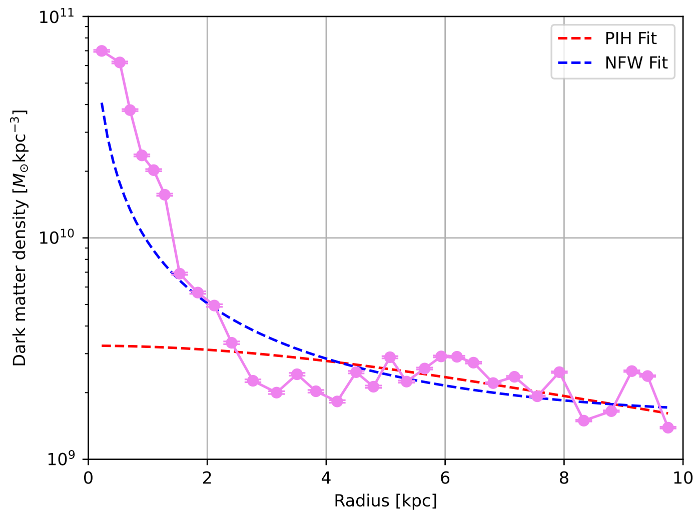

Figure 7: DMH density

Model	**Fit Parameters** [kpc], [$M_\odot$kpc^{-3}]	**Calculated Values**
PIH	r_c, ρ_0	9(3) ; 3.3(7)e9
NFW	r_s, δ	40(10) ; 2.5(5)e8

Table 1: Fit parameters

From δ we can deduce the dimensionless constant $\delta_0 = \delta/\rho_{crit} = 1.7(3)e15$.

The total mass of the MW DMH M_{200} is obtained by integrating these two models over the volume of the halo from 0 to 16.2 kpc (16.2 kpc = 52,820 ly, the radius of the MW). According to PIH model, the DMH mass is 2.2(8)e13 $M_\odot$. With the NFW model, the mass is 1.046(4)e13 $M_\odot$ (see Appendix for error calculations).

6 Discussion

Late estimations of the MW's DMH are in the neighborhood of the results of [8], which are found to be 0.70×10^{12} $M_\odot$ with a 50% Bayesian credible interval of $(0.62, 0.81) \times 10^{12}$ $M_\odot$. The measurement of the DMH using the PIH model, **2.2(8)e13** $M_\odot$, falls **2.66σ** away from the expected result, and **3.17σ** away from the Bayesian interval. Using the NFW model, a measurement of **7.95(4)e12** $M_\odot$ is calculated, which falls **244σ** away from the expected result, and **241.25σ** away from the Bayesian interval.

Using (4), R_{200} is found to be 438836.91(7) kpc, and c 48759.656(8). The uncertainty propagations here are fairly straightforward, but the uncertainty in δ_0, given by (3), is shown in the Appendix. With these calculations, δ_0 is calculated as 789040933(6)e14, which only one order of magnitude away from the fit δ_0. Unfortunately, the standard deviation on this measurement is incredibly small, and the result is very insignificant, which is a further sign of error in this lab.

From Figure 7, it is clear that the NFW model better models the data collected. However, this better-fitting model gives rise to a DMH mass that is far more statistically insignificant than its PIH counterpart, in spite of the fact that the PIH model clearly does a poorer job at modeling the data. This would indicate a large systemic error in this lab. In other words, the means of data collection is flawed. This is unsurprising since the method of obtaining this data was rather crude, and much can be done to improve the accuracy of the MW model used.

The MW galaxy was approximated as a circular-symmetric thin disk with a negligible thickness, enveloped by a spherical DMH. However, in practice, spiral galaxies are not perfect disks. For instance, the MW has an average thickness of about 1000 ly, about 2% the distance of its radius. Furthermore, spiral galaxies are very bugled at the center in the galactic core, so their thickness is not uniform, particularly in toward the center. Of course, the distribution is not circular symmetric; in practice, however, matter is condensed into arm-like structures that swirl the nucleus. The accuracy of [7] is also questionable, considering that the total mass-enclosed data obtained had to be scaled by ~ 5 to stay true to the actual mass of the MW.

If this lab was to be repeated, it would be wise to derive some sort of relationship between brightness and mass for different regions of the MW, and then, using this relationship, obtain the mass of the MW using a picture. This distribution can be normalized to match the actual mass of the MW. Along these lines, a circular symmetric model could be discarded in favor of a more realistic spiral structure, where two spatial coordinates x and y are used, rather than just r. Of course, a third spatial component z could be introduced to incorporate variations in galaxy thickness, which would be more realistic.

7 Conclusions

By approximating the MW as a circular symmetric disk, and the DMH as a sphere, the dark matter distribution is estimated using two models: the pseudo-Isothermal Halo, and the Navarro Frenk White models, which have parameters (r_c, ρ_0) and (r_s, δ_0), respectively. r_c and ρ_0 are **9(3) kpc** and **3.3(7)e9 $M_\odot$ kpc^{-3}**, respectively. Likewise, r_s and δ_0 are **40(10) kpc**, and **1.7(3)e15** (δ_0 is dimensionless). Using these fit parameters, two measurements of the MW DMH mass are obtained.

With the PIH model, a mass of **2.2(8)e13** $M_\odot$ is yielded, falls **2.66σ** away from the expected result of **0.7e12** $M_\odot$. Using the NFW model, a measurement of **7.95(4)e12** $M_\odot$ is calculated, which falls **244σ** away from the expected result. Even though the NFW model is a better fit for the data, the results are far less statistically significant than the PIH model, which is in moderate agreement with the expected value.

8 Sources

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9 Appendices

9.1 Error Propagation for DMH Integration

In the case of the PIH model, the following is obtained:

$$\alpha_M = \left[\alpha_{\rho_0}\frac{\partial}{\partial\rho_0} + \alpha_{r_c}\frac{\partial}{\partial r_c}\right]\int_0^{R_g}\rho_0\left[1+\left(\frac{r}{r_c}\right)^2\right]^{-1}4\pi r^2 dr \tag{11}$$

$$\alpha_M = \int_0^{R_g}\left(\alpha_{\rho_0}\left[1+\left(\frac{r}{r_c}\right)^2\right]^{-1} + 2\alpha_{r_c}\rho_0\left[1+\left(\frac{r}{r_c}\right)^2\right]^{-2}\frac{r^2}{r_c^3}\right)4\pi r^2 dr \tag{12}$$

In the case of the NFW model, the following is obtained:

$$\alpha_M = \left[\alpha_{\delta_0}\frac{\partial}{\partial\delta_0} + \alpha_{r_s}\frac{\partial}{\partial r_s}\right]\int_0^{R_g}\rho_{crit}\frac{\delta_0}{r/r_s(1+r/r_s)^2}4\pi r^2 dr \tag{13}$$

$$\alpha_M = \int_0^{R_g}\rho_{crit}\left(\frac{\alpha_{\delta_0}}{r/r_s(1+r/r_s)^2} + \frac{\delta_0\alpha_{r_s}}{r}\left[1+\frac{r}{r_s}\right]^{-2} + \frac{2\delta_0\alpha_{r_s}r}{r_s}\left[1+\frac{r}{r_s}\right]^{-3}\right)4\pi r^2 dr \tag{14}$$

9.2 δ_0 Uncertainty

We have (3), which we differentiate with respect to c:

$$\frac{\partial}{\partial c}\frac{200}{3}\frac{c^3}{\ln(1+c) - c/(1+c)} \tag{15}$$

$$= \frac{3c^2}{\ln(1+c) - c/(1+c)} + \frac{c^3(\ln(1+c) - c/(1+c))^{-2}}{1+c}\left(1+\frac{c}{(1+c)^2}\right) \tag{16}$$

9.3 Code

The link to the Jupyter Notebook used in the analysis and data collection is shown here:
https://github.com/kylethedrury/dark-matter-project