```
1. We start by setting $\omega=1$. Do the energies you determine correspond
   to what you expect for odd solutions? What about for $\omega=2$? Verify
   the even solutions as well, you only need a few examples here. Recall
   that you can get even solutions by changing the array value which
   *Wave_function* returns.
```

Here are the allowed (odd) energies:

```
Energies for the bound states are:
  1.5000000
  3.5000000
  5.5000000
  7.5000000
  9.5000000
 11.5000001
 13.5000000
 15.5000000
 17.5000000
 19.5000000
 21.5000001
```

For the quantum harmonic oscillator, the allowed energies are given by

$$E_n = h\omega(n + 1/2)$$

If we let h = w = 1, then the allowed energies are 1.5, 3.5, 5.5… (for n odd)... so we have agreement.

And for n even, the energies outputted are 0.5, 2.5, 4.5… (for n even), which agree with the above equation.

```
Energies for the bound states are:
  0.5000000
  2.5000000
  4.5000000
  6.5000000
  8.5000000
 10.5000000
 12.5000000
 14.5000000
 16.5000000
 18.5000000
 20.5000000
```

If we have w = 2, then our equation looks like this:

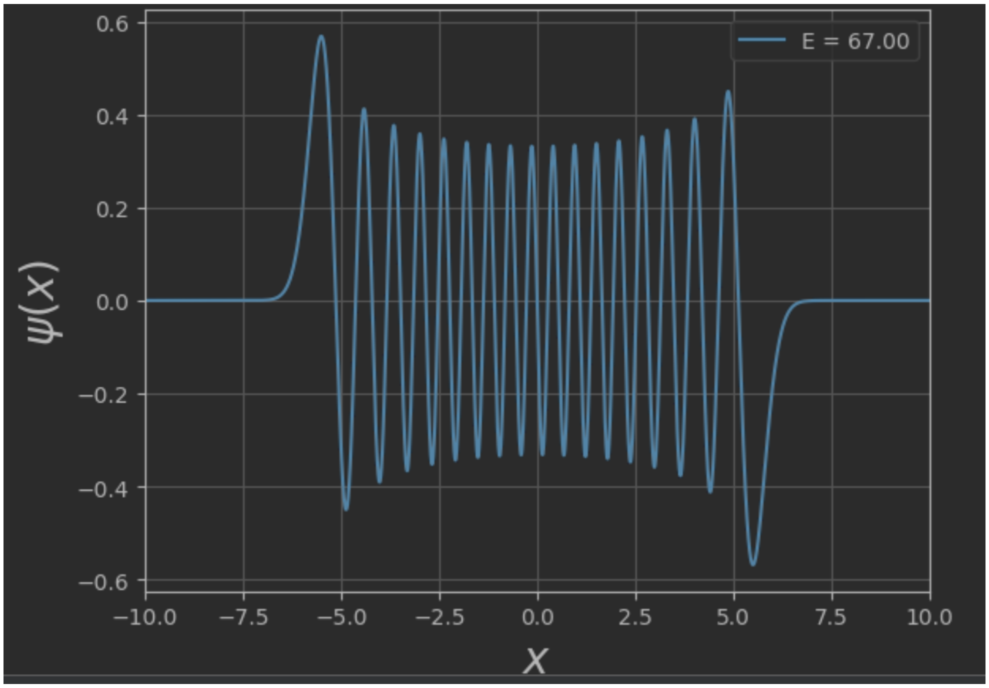
$$E_n = 2(n + 1/2) = 2n + 1$$

Odd energies would be 3, 7, 11, 15; even energies would be 1, 5, 9, 13, 17…

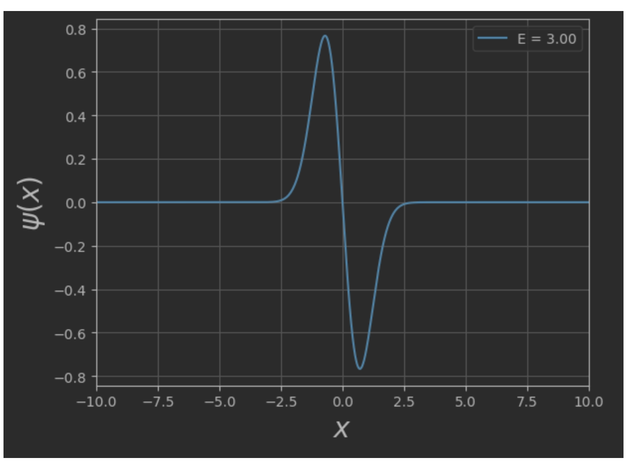
```
Energies for the bound states are:
  3.0000000
  7.0000000
 11.0000000
 15.0000000
 19.0000000
 23.0000001
 27.0000000
 31.0000001
 35.0000001
 39.0000001
 43.0000001
```

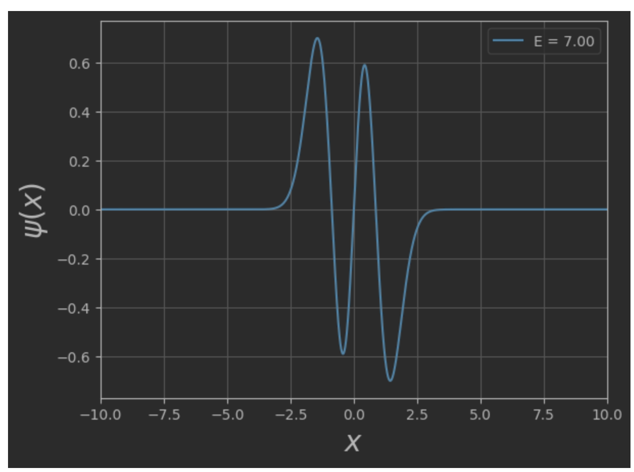
```
Energies for the bound states are:
  1.0000000
  5.0000000
  9.0000000
 13.0000000
 17.0000000
 21.0000000
 25.0000001
 29.0000001
 33.0000001
 37.0000001
 41.0000001
```

```
Consider the 17th odd state with $\omega=2$. How many nodes (zeroes) does the
wavefunction have? Explain your finding.
```




Consider the first two odd wavefunctions.




The first odd function (which is the first excited state), has 3 nodes (the middle one and the two at either end). The second odd function (which is the third excited state) has 5 nodes.
The relationship between the **nth odd state** and the number of nodes = 2n +1. From this, it follows that the 17th odd state will have 35 nodes.

Now let us study the 17th odd state with $\omega=2$ in a bit more detail. As before, we might expect significant 'leaking' of the wavefunction outside the well. However, in the case of the SHO it is not as clear what 'outside the well' means. We determine this by using the classical turning point where $E=V(x)$. Find the classical turning point for the 17th odd state with $\omega=2$. What is the probability of finding the particle outside the well?

(Hint: the 17th Energy value can be found in the array E_zeroes. Also, remember that Python starts indexing from 0 not 1)

```python
xturn = np.sqrt(2*67)/omega
print('Classical turning point:  %11.7f' % xturn)
psioutside=np.where(abs(xall)>xturn,psilr,0)

sum = 0
for x in psioutside: sum = sum + (x**2) * h
proba = sum

print('Probability outside well: %11.7f' % proba)
```

Executed at 2023.11.29 16:06:07 in 32ms

```
Classical turning point:    5.7879185
Probability outside well:   0.0413093
```