Roots of Polynomials and Computational Speed of Resulting Algorithms

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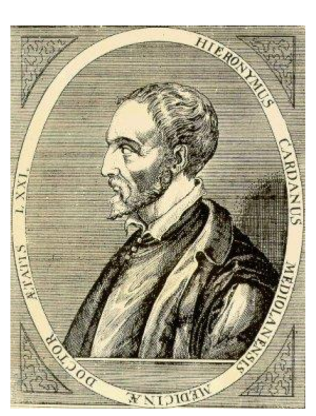
Background

Brahmagupta was an Indian mathematician and astronomer who lived in the seventh century. Brahmagupta also was one of the first mathematicians to recognize negative numbers He was also credited as being the first person to discover a general solution to a quadratic equation.

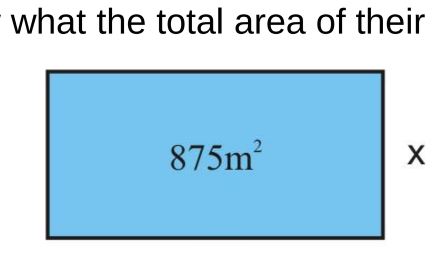
René Descartes is a French mathematician from the seventeenth century. Descartes published the well-known quadratic formula and was the first mathematician to represent the horizontal direction on a plane as x and the vertical direction on a plane as y.



Gerolamo Cardano was credited with the discovery of the Cubic Formula and the Quartic Formula both published in Ars Magna, however he was not the discoverer of either of the formulas/ The solution found its way through word of mouth through several other mathematicians before Cardano, who eventually published the solution.

History

Quadratics are some of the most common equations that are found in real world settings. Quadratics were originally thought to be first solved around 2000 B.C. by the Egyptian, Chinese and Babylonian engineers. The story goes that these engineers when considering what they were building they knew what the total area of their design needed to be.



They also know what proportions their side lengths needed to be as well; however, they did not know hat size of the lengths of the side would be required to satisfy these designs. Therefore, the need to find the lengths of sides to satisfy certain design was thought to be the first example of a quadratic function.

Definition of a Polynomial

A root or solution to a polynomial is a number x which solves the polynomial equation

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0 = 0$$

or more simply put the x value for which the equation is equal to zero.

$$f(x) = 0$$

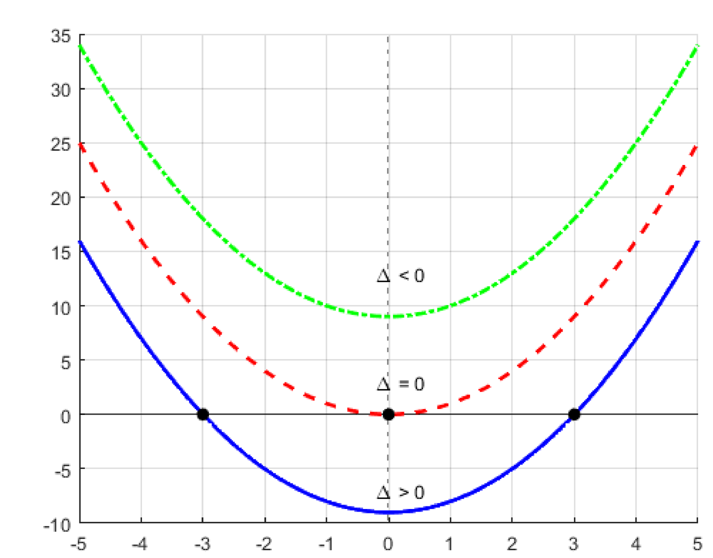
Quadratic Formula

If given a second degree polynomial of the form

$$f(x) = 0 = ax^2 + bx + c, a \neq 0,$$

then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$



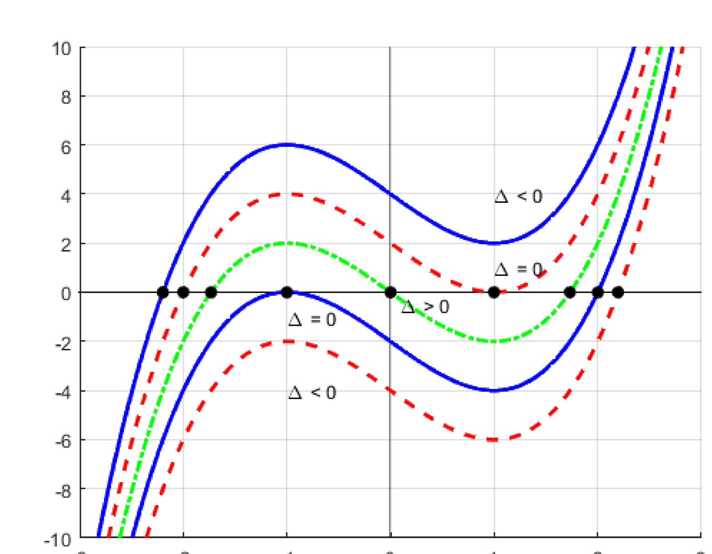
Cardano's Formula

If given a third degree depressed cubic of the form $y^3 + py + q = 0$, then

$$y_1 = \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}$$

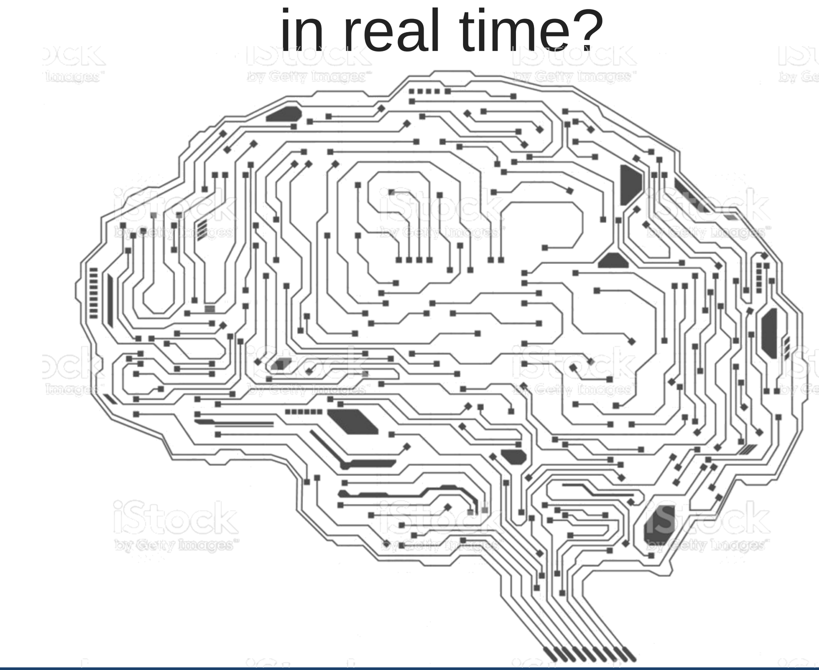
$$y_2 = \left(\frac{-1 + \sqrt{3}i}{2}\right)\left(\sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}\right) + \left(\frac{-1 - \sqrt{3}i}{2}\right)\left(\sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}\right)$$

$$y_3 = \left(\frac{-1 - \sqrt{3}i}{2}\right)\left(\sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}\right) + \left(\frac{-1 + \sqrt{3}i}{2}\right)\left(\sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}\right)$$



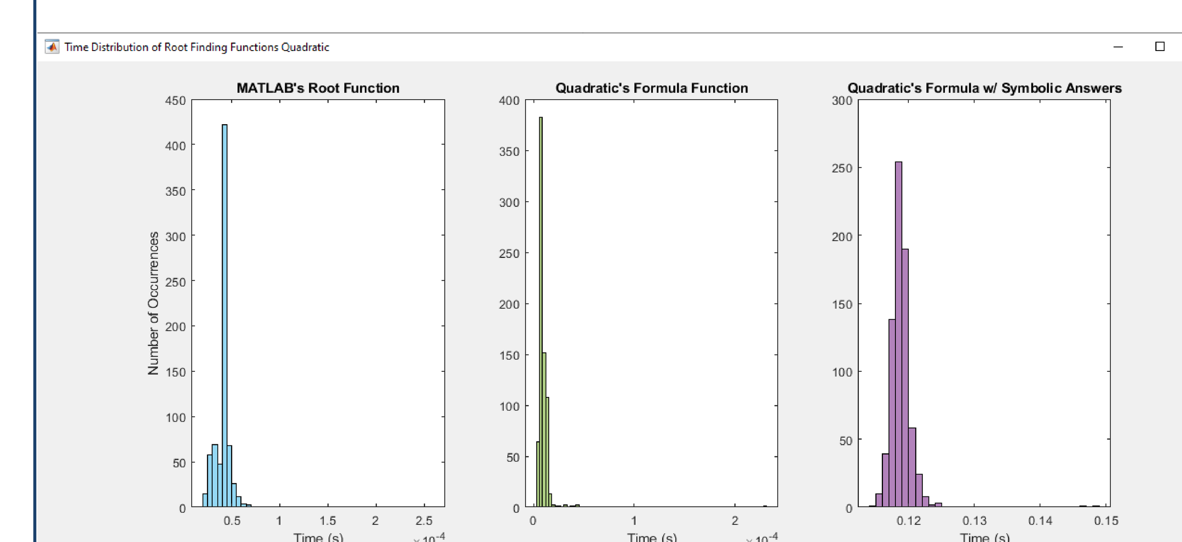
Computational Speed Testing

Comparing MATLAB's internal root finding function to the Quadratic formula and Cardano's Formula. Which function is faster in real time?



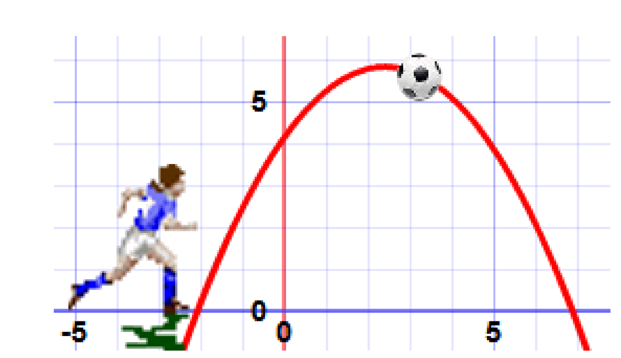
Conclusion

Improvement for Quadratic, but not for Cardanos…



Real World Applications

- Physics
- Engineering
- Architecture



Acknowledgements

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