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Nonlinear Adaptive Control of Spacecraft Maneuvers

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A novel method is presented to carry out near-minimum-time maneuvers of a spacecraft of unknown inertia. The formulation assumes the presence of three orthogonal reaction wheels located near the system mass center and oriented arbitrarily with respect to the spacecraft principal axes. Modified Rodrigues parameters along with their shadow set are used as the primary attitude coordinates. Open-loop maneuver laws are designed while solving the equations of motion by inverse dynamics approach. This approach permits the approximate imposition of the maximum saturation torque constraint. The torque profiles are near-bang-bang, with the instantaneous switches replaced by cubic splines of specified duration. An adaptive tracking control law is developed to determine perturbations to the nominal open-loop torque commands that will ensure the actual motion to follow the nominal motion in the presence of uncertainty in the inertia matrix and errors in the initial attitude. Global stability of the overall closed-loop controller is proved analytically and demonstrated by numerical simulations.

I. Introduction

THE general problem of large-angle attitude maneuvers of an orbiting spacecraft has been a subject of considerable study. Design of feedback control for such problems using Lyapunov stability theory was shown by Vadali and Junkins[1] and Wie and Barba.[2] For example, during the Clementine mission, Creamer et al.[3] presented a Lyapunov stability theory-derived control law for a reaction-wheel controller that gave excellent on-orbit results.

The task of choosing an appropriate attitude coordinate system for expressing rotation of a body-fixed reference frame with respect to a suitable inertial frame is implicit in all large-angle maneuver problems. A very recent survey paper presents the properties of various kinematic coordinate sets.[4] It is well known that the use of a minimal representation by the Euler angles yields singular orientations in which the solution remains undetermined. With the addition of a degree of redundancy, it can be shown that the problems encountered with Euler angles can be avoided completely. This method of quaternions (Euler parameters) is based on the fundamental rotation theorem[5,6] proved by Euler that any rotation of a reference frame with respect to another can be represented by a rotation about a properly chosen eigenaxis. Stated in another fashion, the rotation matrix, apart from the trivial case of identity, has a unique eigenvector corresponding to the eigenvalue at unity. Control systems based on the quaternion approach have been applied in a variety of problems, a specific instance being Ref. 7. In another example, Cristi et al.[8] show an adaptive quaternion feedback law for eigenaxis maneuvers.

The modified Rodrigues parameters (MRP), following developments by Marandi and Modi,[9] allow a ±360-deg range of nonsingular rotations. In combination with the corresponding set of shadow parameters, they lead to a globally regular and nonsingular three-parameter attitude representation system.[10] Although accepting the fact that the choice of kinematic coordinates still remains a matter of personal choice, we opt for the nonsingular minimal coordinate description of rotational dynamics afforded by the MRP along with their shadow set.

Application of optimal control theory to the kinematic and dynamic equations of motion leads to a nonlinear two-point boundary-value problem that is solvable by iterative numerical methods, but typically is not compatible with real-time computing constraints. A significant collection of alternative approaches based on Lyapunov stability theory is given in Refs. 11 and 12. These methods lead to efficiently computable suboptimal designs that are compatible with real-time computational constraints and offer advantages such as stability and robustness. Based on these approaches, a procedure to establish an easily obtainable open-loop reference trajectory is presented. The bang-bang optimal control torque, which could excite significant vibrational modes,[13,14] is smoothed by a cubic spline, resulting in an excellent family of large-angle approximately near-minimum-time (suboptimal) rest-to-rest maneuvers.

In space applications, one can expect the system inertia parameters to be subject to uncertainty. In particular problems, these variations could be due to several reasons, such as changes in the overall system configuration. Such situations call for self-tuning (adaptive) control schemes.[15] Creamer et al.[3] demonstrate a spacecraft inertia estimator based on the Kalman filter algorithm that yielded very good results. We present a globally stable nonlinear adaptive control algorithm that tracks the open-loop reference trajectory in the presence of inertia uncertainties. A previous approach by Slotine and DiBenedetto[16] focuses on the development of an adaptive controller involving the Gibbs representation suitable for rotations through angles less than 180 deg. The use of MRP along with their corresponding shadow set in our paper permits maneuvers through the complete range of 360 deg.

The kinematics and dynamics of a rigid spacecraft containing reaction wheels are presented in Sec. II. An inverse dynamics approach to the design of nominal near-minimum-time open-loop maneuvers is given in Sec. III, followed by the nonlinear adaptive control technique in Sec. IV. Numerical simulations and conclusions are given in Secs. V and VI, respectively.

II. Spacecraft Kinematics and Dynamics

The orientation of an arbitrary body-fixed frame $\{\hat{b}\}$ to an arbitrary inertial frame $\{\hat{n}\}$ at any instant of time t is governed via the direction cosine matrix $[BN(t)]$ as

$$\{\hat{b}(t)\} = [BN(t)]\{\hat{n}\} \tag{1}$$

This direction cosine matrix is parameterized by the MRP vector $\boldsymbol{\sigma} = [\sigma_1(t), \sigma_2(t), \sigma_3(t)]^T$. Then,

$$[BN(t)] \triangleq [C(\boldsymbol{\sigma})] = I_3 - \frac{4(1-\boldsymbol{\sigma}^T\boldsymbol{\sigma})}{(1+\boldsymbol{\sigma}^T\boldsymbol{\sigma})^2}[\tilde{\sigma}] + \frac{8}{(1+\boldsymbol{\sigma}^T\boldsymbol{\sigma})^2}[\tilde{\sigma}]^2 \tag{2}$$

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where I_3 is the 3×3 identity matrix and the skew-symmetric tilde matrix $[\tilde{\cdot}]$ is defined as follows:

$$[\tilde{\sigma}] = \begin{bmatrix} 0 & \sigma_3 & -\sigma_2 \\ -\sigma_3 & 0 & \sigma_1 \\ \sigma_2 & -\sigma_1 & 0 \end{bmatrix} \tag{3}$$

The MRP set of coordinates is related to the Euler parameters (quaternions) by the functional relation

$$\sigma_i = (\beta_i / 1 + \beta_o), \qquad i = 1, 2, 3 \tag{4}$$

Note that the Euler parameters themselves can be obtained from the unit vector $\boldsymbol{e}$ along the Euler's principal rotation axis with components $[e_1, e_2, e_3]^T$ and the principal rotation angle Φ as

$$\beta_o = \cos(\Phi/2), \qquad \beta_i = e_i \sin(\Phi/2), \qquad i = 1, 2, 3 \tag{5}$$

Thus, the Euler parameters satisfy the holonomic constraint

$$\beta_o^2 + \beta_1^2 + \beta_2^2 + \beta_3^2 = 1 \tag{6}$$

Using Eq. (5) in Eq. (4), we can show that the MRP vector can be written as

$$\boldsymbol{\sigma} = \boldsymbol{e} \tan(\Phi/4) \tag{7}$$

Neither the Euler parameters nor the MRP vector are unique. The shadow set associated with the MRP can be found by using $-\beta_i(t)$ instead of $\beta_i(t)$ in Eq. (4). The mapping from the original set to the shadow set is

$$\sigma_i^S = -\sigma_i \big/ \sigma^T \sigma, \qquad i = 1, 2, 3 \tag{8}$$

The kinematic differential equations in terms of the MRP are given by

$$\dot{\boldsymbol{\sigma}} = [D(\boldsymbol{\sigma})]\boldsymbol{\omega} \tag{9}$$

where $\boldsymbol{\omega} = [\omega_1, \omega_2, \omega_3]^T$ are the $\{\hat{b}\}$ components of the angular velocity of the body frame with respect to the inertial frame $\{\hat{n}\}$ and

$$D(\boldsymbol{\sigma}) = \frac{1}{2}\left[\left(\frac{1 - \boldsymbol{\sigma}^T\boldsymbol{\sigma}}{2}\right) I_3 + [\tilde{\sigma}] + \boldsymbol{\sigma}\boldsymbol{\sigma}^T\right] \tag{10}$$

The shadow coordinates are denoted with a superscript S, merely to distinguish them from the original coordinates σ_i. Note that both $\boldsymbol{\sigma}$ and $\boldsymbol{\sigma}^S$ represent the same physical orientation, similar and related to the case of the two possible sets of Euler parameters and the principal rotation vector. However, the modified Rodrigues shadow vector $\boldsymbol{\sigma}^S(t)$ has the opposite singular behavior to that of the original vector $\boldsymbol{\sigma}(t)$. We note that both $\boldsymbol{\sigma}$ and $\boldsymbol{\sigma}^S$ satisfy exactly the same differential equation of Eq. (9) and, individually, both are continuous functions of time.

The switching from the original MRP components to their shadow counterparts using Eq. (8) may be done conveniently whenever the condition $\boldsymbol{\sigma}^T\boldsymbol{\sigma} = 1$ is met. Note that this switching represents a jump discontinuity from the regular MRP to their shadow counterparts but precludes any possibility of kinematic singularities. Such a switching condition also ensures that the magnitude of the MRP vector during the switching condition is continuous and the MRP components before and after switching are always bounded within the unit sphere.

Euler's equations for the rotational motion of a rigid spacecraft with three reaction wheels about the principal axes can be written as

$$\dot{\boldsymbol{\omega}} = (I - J)^{-1}[\tilde{\omega}\boldsymbol{H} - \boldsymbol{u}] \tag{11}$$

$$\boldsymbol{H} = I\boldsymbol{\omega} + J\boldsymbol{\Omega} \tag{12}$$

$$\dot{\boldsymbol{\Omega}} = J^{-1}\boldsymbol{u} - \dot{\boldsymbol{\omega}} \tag{13}$$

where $\boldsymbol{\Omega}$ is the wheel-speed vector in the wheel frame, $\boldsymbol{u}$ is the torque vector applied to the wheels in the wheel frame, I is the composite spacecraft inertia matrix, and $\boldsymbol{H}$ is the angular momentum vector. J is the diagonal wheel inertia matrix in the case of reaction wheels about spacecraft principal axes. For the more general situation in which the reaction wheels are about nonprincipal axes, we have the following structure for J:

$$J = [J_1\{v_1\} \mid J_2\{v_2\} \mid J_3\{v_3\}]$$

where J_i is the 3×3 diagonal inertia matrix of the ith wheel and v_i are the $\{\hat{b}\}$ components of the unit vector $\hat{\boldsymbol{v}}_i$ along the spin axis of the ith wheel. These equations of motion for the most general case are developed in Ref. 6, starting at p. 128. The skew-symmetric body-rate matrix $\tilde{\omega}$ follows from the definition of the cross-product operator in Eq. (3). For a system not subject to external disturbances, the wheel-speed histories during the course of the maneuver can be determined without separate integration of Eq. (13) from the conservation of the angular momentum [defined in Eq. (12)], resulting in

$$\boldsymbol{\Omega}(t) = J^{-1}[\boldsymbol{H}(t) - I\boldsymbol{\omega}(t)] = J^{-1}\{[BB(t, t_o)]\boldsymbol{H}(t_o) - I\boldsymbol{\omega}(t)\} \tag{14}$$

where

$$\boldsymbol{H}(t_o) = I\boldsymbol{\omega}(t_o) + J\boldsymbol{\Omega}(t_o)$$

The matrix $BB(t, t_o)$ satisfies

$$[BB(t, t_o)] = [BN(t)][BN(t_o)]^T \tag{15}$$

This operator can be seen as the transformation matrix from any instantaneous body frame to the initial body frame governed by the relation

$$\{\hat{b}(t)\} = [BB(t, t_o)]\{\hat{b}(t_o)\} \tag{16}$$

III. Design of Reference Maneuver Trajectory

In this section, a method to design a near-minimum-time reference maneuver trajectory is established. We begin by introducing three reference frames and an unambiguous notation for the direction cosine matrices defining the relative orientation between these reference frames. The spacecraft fixed-body axes are $\{\hat{b}\} = \{\hat{b}_1, \hat{b}_2, \hat{b}_3\}^T$, whereas the reference axes at time t along the reference maneuver trajectory are $\{\hat{r}\} = \{\hat{r}_1, \hat{r}_2, \hat{r}_3\}^T$ so that $\{\hat{r}(t)\}$ reaches $\{\hat{b}(t_f)\}$ as $t \to t_f$. The inertially fixed axes are denoted as $\{\hat{n}\} = \{\hat{n}_1, \hat{n}_2, \hat{n}_3\}^T$. The following notation is adopted for the direction matrices:

$$\begin{gathered} \{\hat{b}(t)\} = [BN(t)]\{\hat{n}\}, \qquad \{\hat{r}(t)\} = [RN(t)]\{\hat{n}\} \\ \{\hat{b}(t)\} = [BR(t)]\{\hat{r}(t)\}, \qquad [BR(t)] = [BN(t)][RN(t)]^T \\ [BB(t_1, t_2)] = [BN(t_1)][BN(t_2)]^T \\ [RR(t_1, t_2)] = [RN(t_1)][RN(t_2)]^T \end{gathered} \tag{17}$$

Three sets of MRP vectors $\boldsymbol{\sigma}$, $\boldsymbol{\sigma}_r$, and $\boldsymbol{s}$ are introduced to parameterize the three direction cosine matrices $[BN(t)]$, $[RN(t)]$, and $[BR(t)]$, respectively. The MRP vectors $(\boldsymbol{\sigma}, \boldsymbol{\sigma}_r, \boldsymbol{s})$ satisfy definitions similar to Eq. (2). Note that vector $\boldsymbol{s}$ parameterizes the tracking errors, and for small tracking errors, we can show that $\boldsymbol{s} \approx \boldsymbol{\sigma} - \boldsymbol{\sigma}_r$.

The maneuver initial and final conditions are completely determined using the initial and final MRP vectors and angular velocity vectors $\{\boldsymbol{\sigma}(t_o), \boldsymbol{\omega}(t_o)\}$ and $\{\boldsymbol{\sigma}(t_f), \boldsymbol{\omega}(t_f)\}$. Furthermore, any admissible reference trajectory must match the desired actual boundary conditions and $\{\boldsymbol{\sigma}_r(t_f) = \boldsymbol{\sigma}(t_f), \boldsymbol{\omega}_r(t_f) = \boldsymbol{\omega}(t_f)\}$.

An excellent family of large-angle maneuvers can be generated by forcing the nominal open-loop maneuver to occur about the principal axis of rotation between the initial and target final state. As defined in Eq. (17), the rotation from the initial position to the final position of the reference axes is established by a direction cosine matrix $[RR(t_f, t_o)]$, where

$$\begin{gathered} \{\hat{r}(t_f)\} = [RR(t_f, t_o)]\{\hat{r}(t_o)\} \\ [RR(t_f, t_o)] = [RN(t_f)][RN(t_o)]^T \end{gathered} \tag{18}$$

Notice that $[RR(t, t_o)]$ represents the instantaneous projection of $\hat{r}(t)$ onto the initial attitude $\hat{r}(t_o)$ and, for small motions, is a near-identity matrix. A unit vector $\boldsymbol{l}$ along the principal axis of rotation from the initial to the final desired attitude can be obtained by solving for the eigenvector of $[RR(t_f, t_o)]$ corresponding to the eigenvalue $+1$ as follows:

$$[RR(t_f, t_o)]\boldsymbol{l} = (1.0)\boldsymbol{l}, \qquad \boldsymbol{l} = [l_1, l_2, l_3]^T, \qquad \boldsymbol{l}^T\boldsymbol{l} = 1 \tag{19}$$

Using the notation that RR_{ij} denotes the ijth element of the matrix $[RR(t_f, t_o)]$, the principal rotation angle is

$$\Theta = \tan^{-1}\left[\frac{l_1(RR_{23} - RR_{32}) + l_2(RR_{31} - RR_{13}) + l_3(RR_{12} - RR_{21})}{RR_{11} + RR_{22} + RR_{33} - 1}\right] \tag{20}$$

The reference maneuver trajectory can be developed by first considering the single-axis maneuver of a rigid spacecraft about the principal axis of rotation $\boldsymbol{l}$ subject to an external torque $u_s^*(t)$. Because there is uncertainty in the inertia matrix, we assume the knowledge of a (nominal) estimate of mass moment of inertia of the spacecraft about the principal axis of rotation $\boldsymbol{l}$. For the single-axis rotation, the nonlinear equations of motion can be approximated by

$$I_s\ddot{\theta} = u_s^*, \qquad \left|u_s^*(t)\right| \le u_{\max}^* \tag{21}$$

Note that this equation is exact when the single-axis rotation about the axis $\boldsymbol{l}$ coincides with any of the principal axes of the spacecraft. Empirical evidence, depending on the maneuver rate, suggests that the contribution of the neglected gyroscopic terms in Eq. (21) varies between 2 and 5%. However, these errors are very likely to be smaller than the inertia errors (e.g., in the value of I_s) themselves. Thus, justification for the approximation in Eq. (21) follows from the observation that it is not very useful to use a open-loop reference maneuver law based on erroneous inertia values in an attempt to compensate for nonlinear terms very likely to be smaller than the inertia errors themselves.

For the single-axis maneuver governed by Eq. (21), it can be shown that the minimum time-optimal control subject to the saturation torque constraint $u_{\max}^*$ is of the bang-bang type. Then, for a rest-to-rest maneuver through an angle Θ, the bang-bang control $u_s^*(t)$ has the form

$$u_s^*(t) = u_{\max}^* \operatorname{sgn}\left(t - \frac{t_f}{2}\right), \qquad t_f = \sqrt{\frac{4I_s\Theta}{u_{\max}^*}} \tag{22}$$

Even though we do not explicitly consider flexible body dynamics, the bang-bang control law in Eq. (22) will excite significant vibration of the flexible degrees of freedom. It is easy to introduce a smoothed bang-bang control law defined as follows:

$$\frac{u_s^*(t)}{u_{\max}^*} = \begin{cases} \left(\dfrac{t}{\alpha t_f}\right)^2\left[3 - 2\left(\dfrac{t}{\alpha t_f}\right)\right], & t_o \le t \le \alpha t_f \\ 1, & \alpha t_f \le t \le \dfrac{t_f}{2} - \alpha t_f \equiv t_1 \\ 1 - 2\left(\dfrac{t - t_1}{2\alpha t_f}\right)^2\left[3 - 2\left(\dfrac{t - t_1}{2\alpha t_f}\right)\right], & t_1 \le t \le \dfrac{t_f}{2} + \alpha t_f \equiv t_2 \\ -1, & t_2 \le t \le t_f - \alpha t_f \equiv t_3 \\ -1 + \left(\dfrac{t - t_3}{2\alpha t_f}\right)^2\left[3 - 2\left(\dfrac{t - t_3}{\alpha t_f}\right)\right], & t_3 \le t \le t_f \end{cases} \tag{23}$$

where α controls the sharpness of the switches, with $\alpha = 0$ generating the bang-bang instantaneous torque switches given by Eq. (22) and $\alpha = 0.25$ generating the smoothest member of the family of torque histories. Using this type of a torque law, the time for maneuver t_f can be obtained as

$$t_f = \sqrt{\frac{4I_s\Theta}{u_{\max}^*\left(1 - 2\alpha + \frac{2}{5}\alpha^2\right)}} \tag{24}$$

It can be shown from the binomial expansion of Eq. (24) that the fractional increase of the maneuver time from the corresponding optimal value is approximately proportional to α. The maximum increase in the maneuver time [for $\alpha = 0.25$, which is the smoothest member of the family of Eq. (23)] is less than 38% compared to the bang-bang ($\alpha = 0$) optimal control case of Eq. (22). Qualitatively, a sufficiently smooth and low amplitude torque history and well-tuned tracking control laws will make the most flexible structure behave more like a rigid structure. For well-chosen reference maneuvers, the maneuver times for flexible spacecraft can usually be kept within 10 to 20% of the theoretical rigid-body minimum-time maneuver so that the reference trajectory begins with the initial attitude and ends with the desired final attitude, it is necessary that $\theta(t)$ satisfy the following boundary conditions:

$$\theta(t_o) = 0, \qquad \theta(t_f) = \Theta \tag{25}$$

In addition to Eq. (25), in the case of a rest-to-rest maneuver, we have the boundary conditions

$$\dot{\theta}(t_o) = 0, \qquad \dot{\theta}(t_f) = 0 \tag{26}$$

Then, using Eq. (23) and the boundary conditions (26), we can integrate Eq. (21) for $\dot{\theta}(t)$ as follows:

$$\frac{I_s\dot{\theta}(t)}{u_{\max}^*} = \begin{cases} \dfrac{1}{\alpha^2 t_f^2}\left(t^3 - \dfrac{t^4}{2\alpha t_f}\right), & t_o \le t \le \alpha t_f \\ t - \dfrac{\alpha t_f}{2}, & \alpha t_f \le t \le t_1 \\ t - 4\alpha t_f\left(\dfrac{t - t_1}{2\alpha t_f}\right)^3 + 2\alpha t_f\left(\dfrac{t - t_1}{2\alpha t_f}\right)^4 \\ \quad - \dfrac{\alpha t_f}{2}, & t_1 \le t \le \dfrac{t_f}{2} \end{cases} \tag{27}$$

$$\dot{\theta}(t) = \dot{\theta}(t_f - t), \qquad \forall\, \frac{t_f}{2} \le t \le t_f$$

Further integrating Eq. (21) and using the boundary condition (25), we obtain

$$\frac{I_s\theta(t)}{u_{\max}^*} = \begin{cases} \dfrac{1}{\alpha^2 t_f^2}\left(\dfrac{t^4}{4} - \dfrac{t^5}{10\alpha t_f}\right), & t_o \le t \le \alpha t_f \\ \dfrac{t^2}{2} - \left(\dfrac{\alpha t_f}{2}\right)t + \dfrac{3\alpha^2 t_f^2}{20}, & \alpha t_f \le t \le t_1 \\ \dfrac{t^2}{2} - 2\alpha^2 t_f^2\left(\dfrac{t - t_1}{2\alpha t_f}\right)^4 + \dfrac{4}{5}\alpha^2 t_f^2\left(\dfrac{t - t_1}{2\alpha t_f}\right)^5 \\ \quad -\left(\dfrac{\alpha t_f}{2}\right)t + \dfrac{3\alpha^2 t_f^2}{2}, & t_1 \le t \le \dfrac{t_f}{2} \end{cases} \tag{28}$$

$$\theta(t) = \theta(t_f) - \theta(t_f - t), \qquad \forall\, \frac{t_f}{2} \le t \le t_f$$

Taking an inverse kinematics approach, for the smoothed bang-bang law in Eq. (23) and the corresponding values of $\dot{\theta}(t)$ and $\theta(t)$ from Eqs. (27) and (28), respectively, we can write

$$\mu_{ir}(t) = l_i \tan[\theta(t)/4], \qquad \omega_{ir}(t) = l_i\dot{\theta}(t), \qquad \dot{\omega}_{ir} = l_i\ddot{\theta}(t)$$
$$i = 1, 2, 3 \tag{29}$$

Note that $\boldsymbol{\mu}_r$ in this equation is the MRP vector parameterizing the direction cosine matrix $[RR(t, t_o)]$. One may obtain the MRP vector

$\boldsymbol{\sigma}_r$ that parameterizes the reference motion direction cosine matrix $[RN(t)]$ by the identity

$$[C(\boldsymbol{\mu}(t))] = [C(\boldsymbol{\sigma}_r(t))][C(\boldsymbol{\sigma}_r(t_o))]^T \tag{30}$$

In Eq. (30), the matrix operator $C[\cdot]$ is already defined in the sense of Eq. (2). The inverse dynamics solution for the corresponding required instantaneous control torque applied to the reaction wheels to carry out the reference maneuver is obtained by rewriting Eq. (11) as follows:

$$u_r = \tilde{\omega}_r \boldsymbol{H}_r - (I_n - J)\dot{\boldsymbol{\omega}}_r \tag{31}$$

where

$$\boldsymbol{H}_r(t) = [RR(t,t_o)]\boldsymbol{H}_r(t_o), \qquad \boldsymbol{H}_r(t_o) = I_n\boldsymbol{\omega}_r(t_o) + J\boldsymbol{\Omega}_r(t_o) \tag{32}$$

Here I_n is some nominal inertia matrix of the spacecraft that is assumed to be known. Upon computing the reference torques u_r from Eq. (31) and comparing them with the saturation torques, it will be evident that the maneuver time [Eq. (24)] assumed in computing the reference trajectory needs to be scaled up or down until the worst-case maximum commanded torque [from Eq. (31)] is comfortably under the saturation limit. The absolute value of the maximum of each component of the reference torque may be taken about 5% less than the saturation limit to permit additional torque capability while superimposing the tracking-law feedback torque perturbations without fear of saturating any of the actuators. Because the nonlinear differential equations are solved algebraically, these computations can be done quickly (in near real time). The maneuver determined by this process will correspond to a near-minimum-time maneuver. Note, however, that if the nominal inertia matrix I_n used in determining the approximate open-loop maneuver law is very different from the true inertia matrix (more than 15% error), then no claims can be made regarding the time optimality of the maneuver.

IV. Adaptive Perturbation Feedback Control

Here, we present an adaptive controller for tracking the body frame $\{\hat{b}\}$ along the smooth reference maneuver $\{\hat{r}\}$ given any uncertainty in the system inertia matrix and departures in the initial orientation so that fine pointing is accomplished by the final time t_f. Using the definition for the angular momentum vector in Eq. (12), we rewrite Euler's equation of motion (11) as follows:

$$(I - J)\dot{\boldsymbol{\omega}} = \tilde{\omega}[BB(t,t_o)]\{I\boldsymbol{\omega}(t_o) + J\boldsymbol{\Omega}(t_o)\} - u \tag{33}$$

The reference motion already defined in Eq. (31) satisfies the following relation:

$$(I_n - J)\dot{\boldsymbol{\omega}}_r = \tilde{\omega}_r[RR(t,t_o)]\{I_n\boldsymbol{\omega}_r(t_o) + J\boldsymbol{\Omega}_r(t_o)\} - u_r \tag{34}$$

The difference between Eqs. (33) and (34) leads us to

$$\begin{aligned}\dot{x} &= (I-J)^{-1}(I_n - I)\dot{\boldsymbol{\omega}}_r + (I-J)^{-1}\tilde{\omega}[BB(t,t_o)]I\boldsymbol{\omega}(t_o) \\ &\quad + (I-J)^{-1}\big[\tilde{\omega}[BB(t,t_o)]J\boldsymbol{\Omega}(t_o) \\ &\quad - \tilde{\omega}_r[RR(t,t_o)]\{I_n\boldsymbol{\omega}_r(t_o) + J\boldsymbol{\Omega}_r(t_o)\} - \delta u\big]\end{aligned} \tag{35}$$

where $x = \boldsymbol{\omega} - \boldsymbol{\omega}_r$ and $\delta u = u - u_r$. Recognizing that the unknown inertia matrix I is symmetric, Eq. (35), after some algebraic manipulations, can be rewritten as follows:

$$\begin{aligned}\dot{x} &= (I-J)^{-1}[(I_n - I)\dot{\boldsymbol{\omega}}_r + q(s,x,t) - \delta u] \\ &\quad + (I-J)^{-1}\Big[\Big\{I_{11}\tilde{B}^{11} + I_{22}\tilde{B}^{22} + I_{33}\tilde{B}^{33} \\ &\quad + I_{12}\tilde{B}^{12} + I_{13}\tilde{B}^{13} + I_{23}\tilde{B}^{23}\Big\}\boldsymbol{\omega}(t_o)\Big]\end{aligned} \tag{36}$$

where

$$\begin{aligned}q(s,x,t) &= \tilde{\omega}[BB(t,t_o)]J\boldsymbol{\Omega}(t_o) \\ &\quad - \tilde{\omega}_r[RR(t,t_o)]\{I_n\boldsymbol{\omega}_r(t_o) + J\boldsymbol{\Omega}_r(t_o)\}\end{aligned}$$

and

$$\tilde{B}^{ij} = \tilde{\omega}[BB(t,t_o)]\Delta^{ij}, \qquad i,j = 1,2,3$$

and

$$\Delta^{ij}_{kl} = \begin{cases}1 & \text{if} \quad (i = k \text{ and } j = l) \text{ or } (i = l \text{ and } j = k) \\ 0 & \text{otherwise}\end{cases} \qquad k,l = 1,2,3 \tag{37}$$

Thus for each i and j, matrices B^{ij} and Δ^{ij} are order 3×3. Following these definitions, Eq. (36) can be written compactly as follows:

$$\dot{x} = (I-J)^{-1}[(I_n - I)\dot{\boldsymbol{\omega}}_r + M^*p(s,x,t) + q(s,x,t) - \delta u] \tag{38}$$

where

$$M^* = [I_{11}I_3 \mid I_{22}I_3 \mid I_{33}I_3 \mid I_{12}I_3 \mid I_{13}I_3 \mid I_{23}I_3], \qquad M^* \in \mathcal{R}^{3\times 18} \tag{39}$$

$$p(s,x,t) = [\tilde{B}^{11} \mid \tilde{B}^{22} \mid \tilde{B}^{33} \mid \tilde{B}^{12} \mid \tilde{B}^{13} \mid \tilde{B}^{23}]^T\boldsymbol{\omega}(t_o) \tag{40}$$

Note that I_3 in Eq. (39) is the identity matrix of order 3 and I_{ij} are the ijth elements of the 3×3 unknown symmetric inertia matrix I.

Similarly, the attitude departure motion s, which parameterizes the direction cosine matrix $[BR(t)]$, is governed by

$$\dot{s} = D(s)x - D(s)[C(s) - I_3]\boldsymbol{\omega}_r \tag{41}$$

where the definitions of $C(s)$ and $D(s)$ follow from Eqs. (2) and (10), respectively.

In the case in which the inertia matrix I is completely known, we may attempt to find the perturbation feedback control law using a positive-definite Lyapunov function defined as follows:

$$V_1 = \tfrac{1}{2}\big[x^TW_1x + s^TW_2s\big], \qquad W_i = W_i^T > 0\,\big(\in \mathcal{R}^{3\times 3}\big) \qquad i = 1,2 \tag{42}$$

The time derivative of V_1 can be obtained as

$$\dot{V}_1 = x^TW_1\dot{x} + \dot{s}^TW_2s \tag{43}$$

Using Eqs. (38) and (41), we can rewrite $\dot{V}_1$ as

$$\begin{aligned}\dot{V}_1 &= -x^TW_1(I-J)^{-1}[\delta u - (I_n - I)\dot{\boldsymbol{\omega}}_r - M^*p(s,x,t) \\ &\quad - q(s,x,t)] + x^TD^TW_2s - \boldsymbol{\omega}_r^T[C(s) - I_3]^TD^TW_2s\end{aligned} \tag{44}$$

Choosing the feedback control law δu so as to make $\dot{V}_1 \le 0$ is not very convenient because of the presence of the last term in Eq. (44). Hence, we present a transformation matrix T that maps $x(t)$ to $\boldsymbol{\omega}_r(t)$:

$$\boldsymbol{\omega}_r(t) = T(t)x(t) \tag{45}$$

It can be recognized that there is no unique matrix that accomplishes this transformation. We choose the following as matrix T:

$$T_{ji} = \begin{cases}\dfrac{x_i\omega_{rj}}{x^Tx} & \text{if} \quad \|x\| > 10^{-12} \\ 0 & \text{otherwise}\end{cases} \qquad i,j = 1,2,3 \tag{46}$$

which has the property that it minimizes the cost function

$$\sum_{i=1}^{3}\sum_{j=1}^{3}T_{ij}^2$$

Using Eq. (46), we rewrite Eq. (41) more compactly as

$$\dot{x} = (I-J)^{-1}[(I_n - I)\dot{\boldsymbol{\omega}}_r + M^*p[(s,x,t) + q(s,x,t) - \delta u] \tag{47}$$

$$\dot{s} = G(s,x,t)x, \qquad G(s,x,t) = D(s)\{I_3 - [C(s) - I_3]T\} \tag{48}$$

Following these developments, Eq. (44) becomes

$$\dot{V}_1 = -x^T W_1 (I - J)^{-1}[\delta u - (I_n - I)\dot{\boldsymbol{\omega}}_r - M^* p(s, x, t) - q(s, x, t)] + x^T G^T W_2 s \tag{49}$$

Now we can choose a stabilizing tracking control law δu to be

$$\delta u(t) = (I - J)\left[x + W_1^{-1} G^T W_2 s\right] + (I_n - I)\dot{\boldsymbol{\omega}}_r + M^* p(s, x, t) + q(s, x, t) \tag{50}$$

and we have $\dot{V}_1 = -x^T W_1 x \le 0$. However, the 3×3 matrices $(I - I_n)$ and $(I - J)$ are unknown and hence the control law stated in Eq. (50) cannot be employed. We outline a direct adaptive control method for this problem in the developments that follow.

It is assumed that $(I - J)$ is nonsingular and positive definite, which is true in virtually all practical situations. The control objective is to find a bounded adaptive control input $\delta u(t) \in \mathcal{R}^3$ such that

$$\text{as} \qquad t \to \infty, \qquad \begin{cases} x(t) \to 0 \\ s(t) \to 0 \end{cases} \tag{51}$$

In fact, we need to achieve good tracking even before $t = t_f$. To achieve the objective stated in Eq. (51), we define a model (reference) plant with state $x_m(t) \in \mathcal{R}^3$ satisfying the following differential equation:

$$\dot{x}_m = A_m x_m + d(s, x, t) \tag{52}$$

The term $d(s, x, t) \in \mathcal{R}^3$ has to be bounded and stabilizing. We will outline a procedure to specify it later. The constant matrix A_m is strictly stable, i.e., all of its eigenvalues are on the left half of the complex plane.

One intuitively reasonable approach to choosing A_m is to linearize the dynamics about the final target state to obtain an approximate linear system of the form $\dot{x} = Ax + Bu$. Then, choosing u of the feedback form $u = -Gx$ leads to the approximate closed-loop system $\dot{x} = (A - BG)x$. We can use any convenient linear control design method to design the feedback gain matrix G; then, $A_m = A - BG$. However, A_m is chosen to be negative definite and it may need further modification in view of the matching conditions stated later.

The rate of the transient response of x can be controlled by appropriately choosing the matrix A_m. In the situation in which the parameter I is completely known, the control law $\delta u(t)$ that forces Eq. (47) to reduce to Eq. (52) can be found by

$$\delta u(t) = -K^*[A_m x + d(s, x, t)] + L^* \dot{\boldsymbol{\omega}}_r + M^* p(s, x, t) + q(s, x, t) \tag{53}$$

by satisfying the following matching equations:

$$K^* = (I - J), \qquad L^* = (I_n - I) \tag{54}$$

Note that we cannot employ control law (53) because we lack knowledge of K^*, L^*, and M^* and thus propose the adaptive control law

$$\delta u(t) = -K(t)[A_m x + d(s, x, t)] + L(t)\dot{\boldsymbol{\omega}}_r + M(t) p(s, x, t) + q(s, x, t) \tag{55}$$

where $K(t)$, $L(t)$, and $M(t)$ represent time-varying adaptive estimates of the unknown parameters K^*, L^*, and M^*, respectively. The vector function $\boldsymbol{d}(\boldsymbol{s}, \boldsymbol{x}, t)$ is determined so that the adaptive control law meets the control objective. Using Eqs. (53–55) in Eq. (47), we can write

$$\dot{x} = A_m x + \boldsymbol{d}(\boldsymbol{s}, \boldsymbol{x}, t) + (I - J)^{-1}\{-K^*[A_m x + \boldsymbol{d}(\boldsymbol{s}, \boldsymbol{x}, t)] + L^* \dot{\boldsymbol{\omega}}_r + M^* p(s, x, t) + q(s, x, t) - \delta u\} \tag{56}$$

Note that, if we adopt the expression in Eq. (53) for $\delta u(t)$, then the state dynamics [Eq. (47)] will simply be

$$\dot{x} = A_m x + d(s, x, t) \tag{57}$$

However, we use the adaptive control of Eq. (55) because we do not know K^*, L^*, and M^*. Invoking the series-parallel model[15] to generate the estimate $\hat{x}(t)$ of $x(t)$, we adopt the following structure for the estimator dynamics:

$$\dot{\hat{x}} = A_m \hat{x} + d(s, x, t) + (I - J)^{-1}\{-K(t)[A_m x + d(s, x, t)] + L(t)\dot{\boldsymbol{\omega}}_r + M(t) p(s, x, t) + q(s, x, t) - \delta u\} \tag{58}$$

By virtue of the control law of Eq. (55), it is obvious that the expression within the braces vanishes and the estimated state dynamics simplify to

$$\dot{\hat{x}} = A_m \hat{x} + d(s, x, t) \tag{59}$$

From Eqs. (52) and (59), it is recognized that if $x_m(0) = \hat{x}(0)$, then $x_m(t) \equiv \hat{x}(t)$ for all times $t \ge 0$. Moreover, if we define the estimation error as

$$e(t) \stackrel{\Delta}{=} x(t) - \hat{x}(t) \tag{60}$$

then the difference of Eqs. (56) and (58) gives the error dynamics

$$\dot{e} = A_m e + (I - J)^{-1}\{\tilde{K}[A_m x + d(s, x, t)] - \tilde{L}\dot{\boldsymbol{\omega}}_r - \tilde{M} p(s, x, t)\} \tag{61}$$

where

$$\tilde{K}(t) = K(t) - K^*, \qquad \tilde{L}(t) = L(t) - L^*, \qquad \tilde{M}(t) = M(t) - M^* \tag{62}$$

Here, we define a positive-definite matrix Γ such that $\Gamma^{-1} = K^*$. Γ is positive definite because $K^* = (I - J)$ is positive definite by our assumption. We proceed to obtain adaptive update laws for the estimated parameters. A positive-definite and radially unbounded function is defined as

$$V = e^T P e + \mathrm{tr}[\tilde{K}^T \Gamma \tilde{K} + \tilde{L}^T \Gamma \tilde{L} + \tilde{M}^T \Gamma \tilde{M}] + (x_m + s)^T P (x_m + s) \tag{63}$$

where $P = P^T > 0$ is the solution of the Lyapunov equation:

$$P A_m + A_m^T P = -Q \tag{64}$$

for some $Q = Q^T > 0$. The existence of $P > 0$ is guaranteed by the fact that A_m is a stable matrix. The time derivative of V can be obtained from Eq. (63):

$$\begin{aligned} \dot{V} &= e^T P A_m e + e^T A_m^T P e + 2 e^T P K^{*-1}\Big\{\tilde{K}[A_m x + d(s, x, t)] \\ &\quad - \tilde{L}\dot{\boldsymbol{\omega}}_r - \tilde{M} p(s, x, t)\Big] + 2\,\mathrm{tr}[\tilde{K}^T \Gamma \dot{\tilde{K}} + \tilde{L}^T \Gamma \dot{\tilde{L}} + \tilde{M}^T \Gamma \dot{\tilde{M}}] \\ &\quad + (x_m + s)^T P [A_m x_m + d(s, x, t) + G(s, x, t) x] \\ &\quad + [A_m x_m + d(s, x, t) + G(s, x, t) x\Big\}^T P (x_m + s) \end{aligned} \tag{65}$$

Using the matrix trace identities in Ref. 17, observe that

$$\begin{aligned} e^T P K^{*-1} \tilde{K}[A_m x + d(s, x, t)] &= e^T P \Gamma \tilde{K}[A_m x + d(s, x, t)] \\ &= \mathrm{tr}\Big\{e^T P \Gamma \tilde{K}[A_m x + d(s, x, t)]\Big\} \\ &= \mathrm{tr}\Big\{[A_m x + d(s, x, t)]^T \tilde{K}^T \Gamma P e\Big\} \\ &= \mathrm{tr}\Big\{\tilde{K}^T \Gamma P e [A_m x + d(s, x, t)]^T\Big\} \end{aligned}$$

To guarantee $\dot{V} \le 0$, simple algebraic manipulations lead us to the following adaptive update laws:

$$\dot{\tilde{K}} = \dot{K} = -P e [A_m x + d(s, x, t)]^T \tag{66}$$

$$\dot{\tilde{L}} = \dot{L} = P e \dot{\boldsymbol{\omega}}_r^T \tag{67}$$

$$\dot{\tilde{M}} = \dot{M} = P e p^T(s, x, t) \tag{68}$$

and we choose

$$d(s, x, t) = A_m s - G(s, x, t)x \tag{69}$$

Substituting these adaptive laws in Eq. (66), we get

$$\dot{V} = -e^T Q e - (x_m + s)^T Q (x_m + s) \tag{70}$$

Developments thus far in this section outline a procedure to get the feedback control torques required to achieve stable tracking. However, the boundedness of the various parameters governing the adaptive control law is a very relevant issue and we address this question in the remainder of this section.

From Eq. (63), $V \geq 0$, and from Eq. (70), $\dot{V} \leq 0$. That means

$$\lim_{t \to \infty} V(t) \triangleq V_\infty \text{ exists} \tag{71}$$

and we can conclude that $V \in \mathcal{L}_\infty$, where the $\mathcal{L}_\infty$ norm is defined as

$$\|y\|_\infty \triangleq \sup_{t \geq 0} |y(t)| \tag{72}$$

If y is a vector function of time in $\mathcal{R}^n$, then $|\cdot|$ denotes the norm in $\mathcal{R}^n$ and we say that $y \in \mathcal{L}_\infty$ when $\|y\|_\infty$ exists. Similarly, we can define the $\mathcal{L}_p$ norm as

$$\|y\|_p \triangleq \left[\int_0^\infty |y(\tau)|^p \, \mathrm{d}\tau\right]^{1/p} \tag{73}$$

for $p \in [1, \infty]$ and we say that $y \in \mathcal{L}_p$ when $\|y\|_p$ exists, i.e., when $\|y\|_p$ is finite. The norm definitions in Eqs. (72) and (73) hold for general $m \times n$ matrix functions of time as well if we interpret $|\cdot|$ as the corresponding induced matrix norm on $\mathcal{R}^{m \times n}$.

It then follows from the result that $V \in \mathcal{L}_\infty$ and Eqs. (62) and (63) that

$$e \in \mathcal{L}_\infty, \qquad \begin{matrix} \tilde{K} \in \mathcal{L}_\infty, & \tilde{L} \in \mathcal{L}_\infty, & \tilde{M} \in \mathcal{L}_\infty \\ K \in \mathcal{L}_\infty, & L \in \mathcal{L}_\infty, & M \in \mathcal{L}_\infty \end{matrix} \tag{74}$$

$$(x_m + s) \in \mathcal{L}_\infty \Rightarrow x_m \in \mathcal{L}_\infty, \qquad s \in \mathcal{L}_\infty \tag{75}$$

We then see from Eqs. (55) and (69) that

$$d \in \mathcal{L}_\infty, \qquad \delta u \in \mathcal{L}_\infty \tag{76}$$

Moreover, from Eq. (71), we deduce that $e \in \mathcal{L}_2$ and $(x_m + s) \in \mathcal{L}_2$. Also notice that $\dot{e} \in \mathcal{L}_\infty$ [from Eq. (61)] and $(\dot{x}_m + \dot{s}) \in \mathcal{L}_\infty$ [from Eqs. (48) and (59)]. Using Barbalat's lemma[18] we conclude that $e(t) \to 0$ as $t \to \infty$ and $[x_m(t) + s(t)] \to 0$ as $t \to \infty$. That proves the boundedness of various functions in the adaptive loop. To show that $x_m(t), s(t) \to 0$ as $t \to \infty$, we present the following theorem given by Cristi et al.[8]

Theorem 1. Consider a class of sliding surfaces in the state space $[x_m, s]$ in $\mathcal{R}^6$. Define $\Phi(t) \triangleq x_m(t) + s(t)$. If $x_m, \dot{x}_m \in \mathcal{L}_\infty$ and

$$\lim_{t \to \infty} \Phi(t) = 0 \tag{77}$$

then

$$\lim_{t \to \infty} x_m(t) = 0, \qquad \lim_{t \to \infty} s(t) = 0 \tag{78}$$

A rigorous proof to this theorem is given in Ref. 19. An equivalent interpretation can be obtained by using an interesting result by Vadali,[20] in which it is shown that the sliding surface $x_m + s = 0$ is optimal in the sense that it minimizes the cost function

$$J = \int_0^\infty \left[s^T s + x_m^T x_m\right] \mathrm{d}t \tag{79}$$

This implies that both s and x_m are $\mathcal{L}_2$ functions. As a consequence of this theorem, we obtain that $s(t), x_m(t) \to 0$ as $t \to \infty$. Further, because $e(t) = x(t) - \hat{x}(t) \to 0$ as $t \to \infty$ and $x_m(t) \equiv \hat{x}(t)$, we conclude that $x(t) \to 0$ as $t \to \infty$, thus accomplishing our control objective. We emphasize that no claims can be made about the convergence of the adaptive estimates of the parameters $K(t)$, $L(t)$, and $M(t)$ to their respective true values K^*, L^*, and M^* as $t \to \infty$. All that the adaptive feedback control law given in this paper ensures is stable and bounded tracking of the open-loop maneuver trajectory.

V. Simulation Results

In this section, we present simulations of a case in which the true inertia matrix is unknown and the zero angular momentum system performs the desired near-minimum-time, rest-to-rest maneuver in the presence of a maximum saturation torque condition.

To illustrate the performance of the direct adaptive controller, the following inertia matrix I and nominal inertia matrix I_n are chosen:

$$I = \begin{bmatrix} 95.0 & -0.69 & 0.18 \\ -0.69 & 190.0 & 0.12 \\ 0.18 & 0.12 & 142.5 \end{bmatrix}, \qquad I_n = \begin{bmatrix} 100.0 & 0.0 & 0.0 \\ 0.0 & 200.0 & 0.0 \\ 0.0 & 0.0 & 150.0 \end{bmatrix} \tag{80}$$

The choice of the nominal inertia matrix I_n reflects approximately 5% errors in the actual system inertia matrix within the dominant diagonal entries. During simulations, it was observed that the adaptive controller could tolerate inertia errors significantly larger than 5% (even of the order 40%). However, such large-scale ignorance of system inertia adversely affects the time optimality of the open-loop maneuver trajectory, and hence we limit our presentation to inertia errors less than 5%. In this example, we assume the reaction wheels to be aligned along the spacecraft principal axes. However, note that the results of this paper are not limited to this special case. The wheel inertia matrix is chosen to be $J = I_3$. The initial conditions for the matrices K, L, and M are all taken as zero. The goal is to drive the spacecraft initially aligned with the inertial frame (Euler 3-1-3 angles [0, 0, 0]) to a final condition given by the Euler angles [70 deg, 30 deg, −25 deg]. The desired maneuver then corresponds to an Euler principal-rotation angle of $\Theta = 53.6$ deg about the axis $\boldsymbol{l}$. We emphasize that the Euler angles listed here are only for visualization, whereas we employed the MRP vector in Eq. (4) as attitude coordinates. Initial random errors in the Euler angle measurements are assumed to have a standard deviation of 5 deg. They are consistent with the worst noises from the sensors because typical sensors have significantly better properties (less than 1-deg errors).

The rest-to-rest open-loop maneuver takes place in near-minimum-time (with a torque smoothing factor $\alpha = 0.25$). The actuators have a saturation constraint given by $|u_i| < 1$. On the basis of several simulations, the parameter matrices A_m and Q are chosen as $Q = -A_m = 0.15\, I_3$. The adaptively controlled trajectories are shown in Fig. 1 for the Euler 3-1-3 angles, Fig. 2 for angular velocities, and Fig. 3 for control torque requirements.

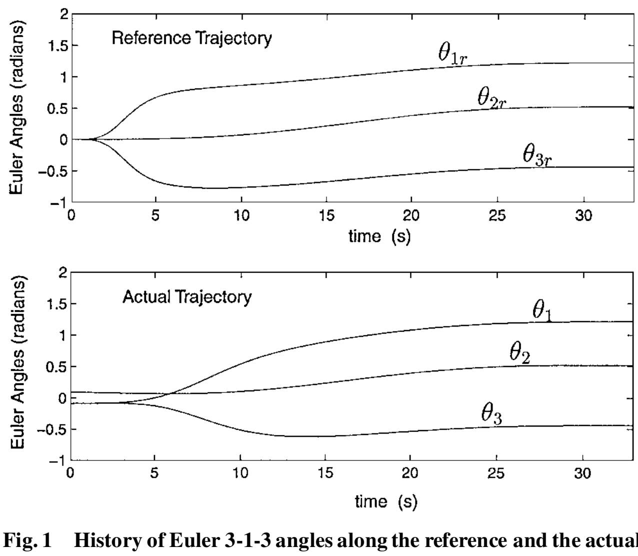

Fig. 1 History of Euler 3-1-3 angles along the reference and the actual maneuver.

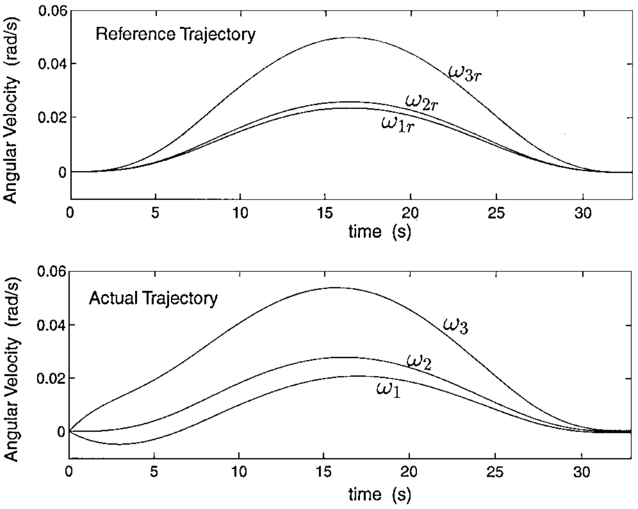

Fig. 2 Angular velocity components along the reference and the actual maneuver.

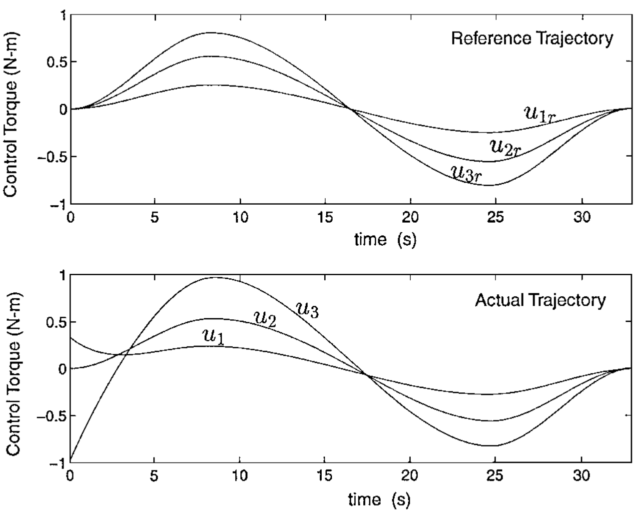

Fig. 3 Control torque components required along the reference and the actual maneuver.

Figures 2 and 3 show that the desired near-minimum-time rest-to-rest maneuver designed by the reference trajectory is accomplished without saturating any of the actuators. It was observed that the parameter matrices $K(t)$, $L(t)$, and $M(t)$ converged with passage of time but the convergence was not to their true values.

VI. Conclusions

A direct adaptive controller is designed to take a spacecraft with three reaction wheels along a near-minimum-time, rest-to-rest maneuver. The true inertia matrix I is assumed to be unknown, as would happen in the case of a space-station-based robotic manipulator used to pick up and deliver loads of various sizes and shapes. However, knowledge of a nominal inertia matrix I_n that is close to the unknown inertia matrix I is assumed. Note that this assumption is not required for the stability of the closed-loop adaptive law. We impose this constraint only for the purpose of designing an approximately time-optimal open-loop maneuver law. The singularity-free MRP are employed for kinematic representation of the system. The method outlined here can be extended easily to accommodate for nonzero initial and final velocities. Because qualitative approximations have been introduced in the context of design of the optimal open-loop trajectory, the extent to which the maneuvers differ from the true minimum-time maneuvers has to be investigated on a case-by-case basis, and that is not the focus of this paper. Global stability and boundedness of the overall direct adaptive tracking control system have been proven analytically and demonstrated by computer simulations, whereas parameter convergence, in general, is not guaranteed.

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