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Automatic Spacecraft Detumbling by Internal Mass Motion

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In the operation of future manned space vehicles there will always be a finite probability that an accident will occur which results in uncontrolled tumbling of a craft. Hard docking by a manned rescue vehicle is not acceptable because of the hazardous environment to which rescue crewmen would be exposed and excessive maneuvering accelerations during docking operations. A movable-mass control concept, which is activated upon initiation of tumbling and is autonomous, can convert tumbling motion into simple spin. Such a device would greatly facilitate crew evacuation and final despinning by external means. Motion of a control mass, according to a selected control law, will either increase or decrease kinetic energy. The complete equations of motion for an asymmetric rigid spacecraft containing a movable mass are presented, and appropriate control law and system parameters are selected to minimize kinetic energy, resulting in simple spin about the major principal axis. Simulations indicate that for a large space station experiencing a collision, which results in tumbling, a 1% movable mass is capable of stabilizing motion in 2 hr.

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Nomenclature

a, b = mass track offset distances
$\mathbf{a}$ = inertial acceleration of reference point
A_n, B_n = Fourier series coefficients ($n = 1, 2, \ldots$)
c_1, c_2 = control law parameters
$\mathbf{f}$ = force applied to the control mass and reacted to by the vehicle $= \mu\ddot{\mathbf{r}}$
F = forcing function of mass equation of motion
$\mathbf{H}$ = total angular momentum of system relative to main body center of mass
I_1, I_2, I_3 = principal moments of inertia of main vehicle
$\bar{\bar{I}}$ = inertia tensor of main vehicle
k = modulus of Jacobian elliptic functions
m = mass of control mass
M = mass of main vehicle
$\mathbf{M}$ = external moment acting on system
P = precessional frequency
$\mathbf{r}$ = position vector of control mass relative to main vehicle center of mass
$\mathbf{S}$ = first moment of mass of the system
T_{rot}, T = rotational kinetic energy of the system
$\dot{T}_{sec}$ = secular part of the rate of change of rotational kinetic energy
x, y, z = components of $\mathbf{r}$ in body-fixed reference frame
X_1, X_2, X_3 = principal vehicle axes
z_{max} = maximum amplitude of control mass oscillation
α, β, γ = amplitudes of angular rotation rates
μ = reduced mass $= mM/(m+M)$
τ = period of forcing function of equation of mass motion
τ_c = time constant associated with stabilizing performance of the control system
$\boldsymbol{\omega}$ = angular velocity vector of main body with components $\omega_1, \omega_2, \omega_3$

Introduction

UNCONTROLLED tumbling of a manned space vehicle may result from an accident such as a collision, malfunction, or other disabling event associated with the application of a large disturbing torque. Since there is no preferred axis of rotation associated with such motion, a hazardous environment is created for rescue crewmen. Thus, a hard docking by a manned vehicle is unacceptable, because it would be dangerous and require excessive maneuvering. Detumbling can be accomplished by eliminating all angular rates or by stabilizaing motion about one axis. This latter technique results in simple spin about the axis of maximum inertia. There are several suggested internal and external devices for controlling tumble. Mass expulsion devices require onboard storage of propellants over long periods. Several thrusters would have to be dedicated to this type of system. Thus, such schemes could be complex and massive. Momentum exchange devices could quickly saturate and are difficult to start in a tumbling situation. Passive energy dissipation devices are reliable and simple, but they require long periods of time to reduce kinetic energy if initial rates are low. Of particular concern here is an autonomous internal system which uses a movable mass to quickly stabilize motion about the major principal axis. Such a system would be activated upon initiation of tumble and would greatly facilitate crew evacuation and final despinning by external means.

The complete equations of motion of a rigid spacecraft with attached control mass are presented making no assumptions concerning vehicle symmetry or magnitude of the transverse angular rates. A control law relating control mass motions to vehicle motions is selected based on Liapunov stability theory. A method of determining control system parameter values, based on an estimate of the worst case tumble state, is also presented. For a large space station and realistic initial conditions, it is shown that the movable mass control system is capable of quickly decreasing the kinetic energy of the system to its minimum state, establishing a simple spin about the maximum inertia axis. Similar control systems may be useful with unmanned spacecraft if initial tumble states can be anticipated in order to preselect parameter values.

Early studies related to effects of internal moving parts in a space vehicle by Roberson[1] and Grubin[2] led to development of the equations of motion which model a rigid main body carrying an arbitrary number of moving components. These investigations were concerned with the destabilizing influence of internal mass motion. Roberson chose the composite center of mass of the system as the reference point, leading to time varying moments of inertia and a moving reference point. Grubin circumvented this problem by choosing the vehicle center of mass as the reference point. This seems more natural, since motion of the moving mass may be referred directly to the vehicle axes. Kane and Sobala[3] investigated the possibility of attitude stabilization through internal mass motion. A symmetric satellite with two particles oscillating along the axis of symmetry was

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considered. Kane and Scher[4] have discussed the possibility of moving the parts of a connected system relative to each other to convert undesirable tumbling or nutation into simple spin. In general, the scheme is to move parts relative to each other in such a way that the rotational kinetic energy changes to its maximum or minimum value. The system would then be in a simple spin state about either its minor or major principal axis, respectively. Thus, previous works have proposed movable masses for a variety of applications. However, most of these have been developed for vehicles which are symmetric or experience only small transverse rotation rates which permit some degree of linearization. A movable mass control system has not previously been developed for the general case of an asymmetric vehicle with arbitrarily large rotation rates about each axis.

Equations of Motion

The generalized vector equation of motion for a system of connected rigid bodies is given by[2]

$$\mathbf{M} = \dot{\mathbf{H}} + \mathbf{S} \times \mathbf{a} \tag{1}$$

with respect to an arbitrary reference point whose inertial acceleration is $\mathbf{a}$. In this analysis $\dot{\mathbf{H}}$ implies differentiation with respect to an inertial frame. Equation (1) reduces to the standard form, $\mathbf{M} = \dot{\mathbf{H}}$ when the reference point is fixed or is the system center of mass. The vehicle considered here consists of a rigid main body and attached control mass, as shown in Fig. 1. An appropriate reference point is the main body center of mass. Expanding Eq. (1) yields a set of three coupled nonlinear differential equations for the vehicle dynamics in terms of the vehicle angular rates, and the movable mass position, velocity, and acceleration. All of these quantities are measured with respect to the body fixed principal axes. Thus, the equations for torque free motion become

$$\begin{aligned}&[I_1+\mu(y^2+z^2)]\dot{\omega}_1+[I_3-I_2+\mu(y^2-z^2)]\omega_2\omega_3+\\&\quad\mu[-xy\dot{\omega}_2-xz\dot{\omega}_3+(2y\dot{y}+2z\dot{z})\omega_1+yz(\omega_3{}^2-\omega_2{}^2)-\\&\quad 2\dot{x}y\omega_2-2\dot{x}z\omega_3-xz\omega_1\omega_2+xy\omega_1\omega_3+y\ddot{z}-z\ddot{y}]=0\end{aligned} \tag{2}$$

$$\begin{aligned}&[I_2+\mu(z^2+x^2)]\dot{\omega}_2+[I_1-I_3+\mu(z^2-x^2)]\omega_3\omega_1+\\&\quad\mu[-yz\dot{\omega}_3-yx\dot{\omega}_1+(2z\dot{z}+2x\dot{x})\omega_2+zx(\omega_1{}^2-\omega_3{}^2)-\\&\quad 2\dot{y}z\omega_3-2\dot{y}x\omega_1-yx\omega_2\omega_3+yz\omega_2\omega_1+z\ddot{x}-x\ddot{z}]=0\end{aligned} \tag{3}$$

$$\begin{aligned}&[I_3+\mu(x^2+y^2)]\dot{\omega}_3+[I_2-I_1+\mu(x^2-y^2)]\omega_1\omega_2+\\&\quad\mu[-zx\dot{\omega}_1-zy\dot{\omega}_2+(2x\dot{x}+2y\dot{y})\omega_3+xy(\omega_2{}^2-\omega_1{}^2)-\\&\quad 2\dot{z}x\omega_1-2\dot{z}y\omega_2-zy\omega_3\omega_1+zx\omega_3\omega_2+x\ddot{y}-y\ddot{x}]=0\end{aligned} \tag{4}$$

where $\mu = mM/(m+M)$, the system reduced mass. These are valid irrespective of the physical mechanism driving the control mass.

Equation (1) may also be written as

$$\dot{\mathbf{H}}_b + \mu\mathbf{r} \times \ddot{\mathbf{r}} = 0$$

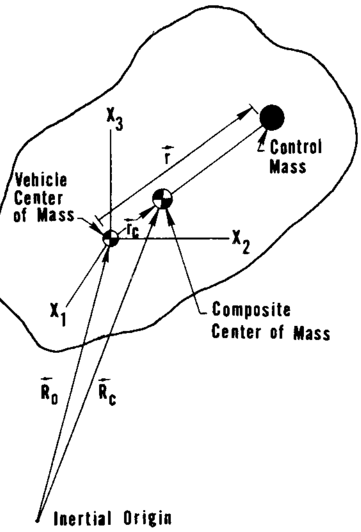

Fig. 1 Main body and attached mass system geometry.

where $\dot{\mathbf{H}}_b$ is the main body angular momentum about its center of mass. Thus, a force which acts on the control mass is defined as $\mathbf{f} = \mu\ddot{\mathbf{r}}$. This can be expressed in component form as

$$\begin{aligned}f_1 = \mu[&\ddot{x}-2\dot{y}\omega_3+2\dot{z}\omega_2-y\dot{\omega}_3+z\dot{\omega}_2+y\omega_1\omega_2+\\&z\omega_1\omega_3-x(\omega_2{}^2+\omega_3{}^2)]\end{aligned} \tag{5}$$

$$\begin{aligned}f_2 = \mu[&\ddot{y}-2\dot{z}\omega_1+2\dot{x}\omega_3-z\dot{\omega}_1+x\dot{\omega}_3+z\omega_2\omega_3+\\&x\omega_2\omega_1-y(\omega_3{}^2+\omega_1{}^2)]\end{aligned} \tag{6}$$

$$\begin{aligned}f_3 = \mu[&\ddot{z}-2\dot{x}\omega_2+2\dot{y}\omega_1-x\dot{\omega}_2+y\dot{\omega}_1+x\omega_3\omega_1+\\&y\omega_3\omega_2-z(\omega_1{}^2+\omega_2{}^2)]\end{aligned} \tag{7}$$

The rotational kinetic energy of the system about the composite center of mass may be generally expressed as

$$T_{\text{rot}} = [\boldsymbol{\omega}\cdot\bar{\bar{I}}\cdot\boldsymbol{\omega}+M\dot{\mathbf{r}}_c\cdot\dot{\mathbf{r}}_c+m\dot{\mathbf{r}}_m\cdot\dot{\mathbf{r}}_m]/2 \tag{8}$$

By the definition of center of mass

$$\dot{\mathbf{r}}_c = m\dot{\mathbf{r}}/(m+M), \quad \dot{\mathbf{r}}_m = M\dot{\mathbf{r}}/(m+M)$$

so that Eq. (8) becomes

$$T_{\text{rot}} = (1/2)\boldsymbol{\omega}\cdot\bar{\bar{I}}\cdot\boldsymbol{\omega}+(1/2)\mu\dot{\mathbf{r}}\cdot\dot{\mathbf{r}}$$

The rate of energy change is obtained directly by differentiation

$$\dot{T}_{\text{rot}} = \boldsymbol{\omega}\cdot\bar{\bar{I}}\cdot\dot{\boldsymbol{\omega}}+\dot{\mathbf{r}}\cdot\mathbf{f}$$

Also note that

$$\bar{\bar{I}}\cdot\dot{\boldsymbol{\omega}}+\boldsymbol{\omega}\times\bar{\bar{I}}\cdot\boldsymbol{\omega} = -\mathbf{r}\times\mathbf{f}$$

leads to

$$\boldsymbol{\omega}\cdot\bar{\bar{I}}\cdot\dot{\boldsymbol{\omega}} = -\boldsymbol{\omega}\cdot\boldsymbol{\omega}\times\bar{\bar{I}}\cdot\boldsymbol{\omega}-\boldsymbol{\omega}\cdot\mathbf{r}\times\mathbf{f}$$

The first term on the right-hand side of this equation is zero, thus giving

$$\dot{T}_{\text{rot}} = -\boldsymbol{\omega}\cdot\mathbf{r}\times\mathbf{f}+\dot{\mathbf{r}}\cdot\mathbf{f}$$

Since

$$\dot{\mathbf{r}} = [\dot{\mathbf{r}}]+\boldsymbol{\omega}\times\mathbf{r}$$

this equation simplifies to

$$\dot{T}_{\text{rot}} = [\dot{\mathbf{r}}]\cdot\mathbf{f} \tag{9}$$

Thus, the rate of change of kinetic energy is found to be independent of the vehicle inertia properties and dependent only on the relative velocity of the control mass and the force applied to the mass. Motion of the control mass is specified by a control law, which was selected using this relation.

Control Law Selection

Equations (2–4) determine the attitude response of the spacecraft for specified motion of the control mass. It is the function of the control law to relate motion of this mass to measurable vehicle parameters such that kinetic energy decreases. A satisfactory control law should not be unnecessarily complicated and should not have excessive power or sensor requirements. It should, however, require determination of only measurable vehicle parameters, produce stable responses, and result in a final state of simple spin about axis of maximum inertia. For the work presented here, the vehicle is assumed to have three distinct moments of inertia, I_1, I_2, and I_3, such that $I_3 > I_2 > I_1$. Since initial tumble rates may be large about all three axes, the equations of motion cannot be simplified by linearization. However, a limited number of simple cases were identified which permitted development of the control law selected here.

Liapunov stability theory states that a system of differential equations and mass equation of motion produced by the control law will be completely stable and approach its minimum state if there exists a scalar function $V(u)$ with properties[5]

$$V(u) > 0 \quad \text{for all} \quad u \neq 0 \tag{10a}$$

$$\dot{V}(u) \leqq 0 \quad \text{for all} \quad u \tag{10b}$$

$$V(u) \to \infty \quad \text{as} \quad \|u\| \to \infty \tag{10c}$$

where u is the state vector and $\|u\|$ is the Euclidean norm of the state vector. For this case a convenient scalar to use as a Liapunov function is the system kinetic energy. Due to the nature of this function, conditions (10a) and (10c) are auto-

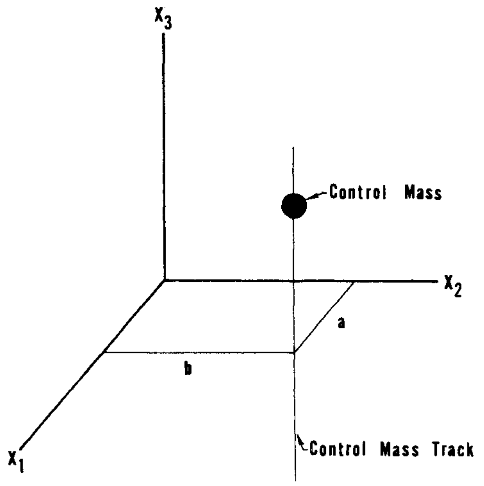

Fig. 2 Orientation of control mass path.

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matically satisfied. Thus, if a control law is chosen such that condition (10b) is satisfied, the system will be completely stable and approach its minimum state. Consider the case where the control mass is restricted to move along a track parallel to the X_3 axis, and offset from this axis as shown in Fig. 2. For this case, the rate of change of rotational kinetic energy given by Eq. (9) simplifies to

$$\dot{T}_{\text{rot}} = \dot{z} f_3$$

Thus, if the force applied to the control mass is selected as

$$f_3 = -\mu c \dot{z} \tag{11}$$

then the energy rate becomes

$$\dot{T}_{\text{rot}} = -\mu c \dot{z}^2$$

which satisfies condition (10b). Using Eqs. (7) and (11) gives the mass equation of motion

$$\ddot{z} + c\dot{z} - (\omega_1{}^2 + \omega_2{}^2) z = a\dot{\omega}_2 - b\dot{\omega}_1 - a\omega_3\omega_1 - b\omega_2\omega_3$$

This form suggests a second-order system being driven by motion of the tumbling vehicle. However, the vehicle dynamics which produce this forcing function are determined by differential equations which are themselves functions of control mass motion. If the effect of mass motion over one cycle is small, the vehicle is in nearly uncontrolled tumbling. This assumption permits the forcing function to be dominated by vehicle motions over any given cycle. The resulting form of mass equation of motion may then be used to determine effects of control law on mass dynamics.

The control law given by Eq. (11) satisfies condition (10b) and decreases the rotational kinetic energy of the system. Also, the forcing function vanishes when a final spin about the X_3 axis is established. However, due to the negative, decaying coefficient of the z term, the mass would not necessarily return to its initial zero position upon reaching a simple spin state. A "reset" to zero would restore full control capability of the system should another tumbling situation arise. This is achieved by modifying the force applied to the control mass

$$f_3 = -\mu c_1 \dot{z} - \mu (c_2 + \omega_1{}^2 + \omega_2{}^2) z \tag{12}$$

The eqı ation for mass motion becomes

$$\ddot{z} + c_1 \dot{z} + c_2 z = a\dot{\omega}_2 - b\dot{\omega}_1 - a\omega_3\omega_1 - b\omega_2\omega_3 \tag{13}$$

This is a conventional second-order differential equation with damping and return to zero position. The associated rate of energy dissipation is

$$\dot{T}_{\text{rot}} = -\mu c_1 \dot{z}^2 - \mu z \dot{z} (c_2 + \omega_1{}^2 + \omega_2{}^2) \tag{14}$$

This formulation departs from the Liapunov method since $\dot{T}_{\text{rot}}$ is not necessarily negative semidefinite. The first term will decrease the rotational kinetic energy of the system while the second term will be oscillatory in nature, increasing and decreasing the energy. If the control system constants c_1 and c_2 are chosen properly the secular negative semidefinite term will dominate over the complete mass cycle. If every mass cycle has a net negative value for $\dot{T}_{\text{rot}}$, the system will approach its minimum energy state and simple spin. Physical significance of the selected control law will be discussed in the next section.

Selection of Control System Parameters

For the selected control law, expression (12), the rate of change of rotational kinetic energy is given by Eq. (14), as a function of two control system parameters, c_1 and c_2. Values should be chosen to yield a large dissipation rate and, hence, a fast approach to simple spin, subject to mass amplitude and power constraints. Guidelines for the selection of these parameter values are developed here.

Some qualitative observations can be made immediately concerning effects of c_1 and c_2 on control system performance. The right-hand side of Eq. (13) is again considered as the forcing function of mass dynamics so that

$$\ddot{z} + c_1 \dot{z} + c_2 z = F$$

where the forcing function due to vehicle motion is

$$F = a\dot{\omega}_2 - b\dot{\omega}_1 - a\omega_3\omega_1 - b\omega_2\omega_3 \tag{15}$$

An analogy is now established between mass dynamics and a simple spring-mass-damper system, in which c_1 corresponds to a damping constant and c_2 corresponds to a spring constant. Considering expression (14), for $\dot{T}_{\text{rot}}$, it would appear that the value of c_1 should be large. However, the associated term is also a function of mass relative velocity, which is influenced by c_1. In a spring-mass-damper system, a large value of c_1 corresponds to a strong damper, which would limit the velocity of the mass. Thus, an increase in c_1 may result in a net decrease in the magnitude of this term. A similar tradeoff results for parameter c_2. The second term in Eq. (14) is oscillatory in nature and results in energy addition over part of the mass cycle. This term insures the return of the control mass to its zero position once a simple spin state is reached. To limit energy addition during parts of the mass cycle it may appear that c_2 should be small. However, again considering the spring-mass-damper analogy, a small value of c_2 corresponds to a weak spring which would allow a relatively large amplitude of mass oscillation. These general considerations suggest that an optimum set of control system parameters may exist which maximize the energy dissipate rate while limiting the mass amplitude to a specified value.

An analytic solution of the mass equation of motion would provide control mass position and velocity histories as functions of the parameters c_1 and c_2. Using these solutions, the rate of change of rotational kinetic energy could be maximized subject to the condition of a selected maximum mass amplitude. This optimization could be performed using a Lagrange multiplier technique. Clearly, this is not possible since the forcing function, F, contains angular rates and accelerations of the vehicle. The solution of the mass equation of motion would, therefore, require the simultaneous solution of the vehicle equations of motion. This system of differential equations does not readily lend itself to an analytic solution due to coupling and nonlinearities. On the other hand, a completely numerical solution of the system of equations provides no insight concerning effects of control system parameter selections. Therefore, a partially analytical, partially numerical method was adopted, based on the assumption that the net change in rotational kinetic energy over one mass cycle is small. For a selected tumbling state the free dynamics may be solved numerically, and these results may then be used to calculate the forcing function profile from Eq. (15). Although F will change as tumbling diminishes, the initial tumble state will provide a design point which may be used to size control system parameters.

The nature of the forcing function may be investigated by considering the analytic solution of free vehicle dynamics. Angular rates may be expressed in terms of Jacobian elliptic functions[6]

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$$\omega_1 = \gamma\, cn[p(t-t_o)]$$
$$\omega_2 = \beta\, sn[p(t-t_o)]$$
$$\omega_3 = \alpha\, dn[p(t-t_o)]$$

for the condition $H^2 > 2I_2T$, which corresponds to motion about the maximum inertia axis, and

$$\omega_1 = \gamma'\, dn[p'(t-t_o)]$$
$$\omega_2 = \beta'\, sn[p'(t-t_o)]$$
$$\omega_3 = \alpha'\, sn[p'(t-t_o)]$$

when $H^2 < 2I_2T$, corresponding to the minimum inertia axis. Amplitudes α, β, and γ, and precession frequency p, for $H^2 > 2I_2T$ and corresponding quantities for $H^2 < 2I_2T$ may be found in the literature. Expressions for these quantities are not of direct concern here since only the form of the forcing function is of interest here. This form will be one of the following:

$$F = a(p\beta - \alpha\gamma)\, cn[p(t-t_o)]\, dn[p(t-t_o)] + b(p\gamma - \alpha\beta)\, sn[p(t-t_o)]\, dn[p(t-t_o)] \qquad (16)$$

for $H^2 > 2I_2T$, or

$$F = a(p'\beta' - \alpha'\gamma')\, cn[p'(t-t_o)]\, dn[p'(t-t_o)] + b(p'\gamma'k^2 - \alpha'\beta')\, sn[p'(t-t_o)]\, cn[p'(t-t_o)] \qquad (17)$$

for $H^2 < 2I_2T$. Parameter k is the modulus of the elliptic functions, and $sn\,u$ and $cn\,u$ oscillate between 1 and -1 at the precession frequency p. Function $dn\,u$ oscillates between 1 and $(1-k^2)^{1/2}$ at twice the precession frequency. Thus, F will be oscillatory in form. Using a tabulated form of F obtained from the uncontrolled case results, a Fourier series may be fit to the data points to provide an analytic expression for solving the mass equation of motion. This solution will provide information concerning the relationship between control parameters and system performance.

The Fourier series form used is

$$F = A_o + \sum_{n=1}^{\infty} \left(A_n \cos\frac{2n\pi t}{\tau} + B_n \sin\frac{2n\pi t}{\tau} \right)$$

where τ is the period of the function F. Since the forcing function, expressed as either Eqs. (16) or (17), contains no secular terms, $A_o = 0$. Also, F is a smooth function, permitting the series to be truncated with sufficient accuracy

$$F = \sum_{n=1}^{m} \left(A_n \cos\frac{2n\pi t}{\tau} + B_n \sin\frac{2n\pi t}{\tau} \right)$$

Coefficients, A_n and B_n, are determined numerically using a standard routine which generates them for a tabulated function. This expansion is rewritten for convenience as

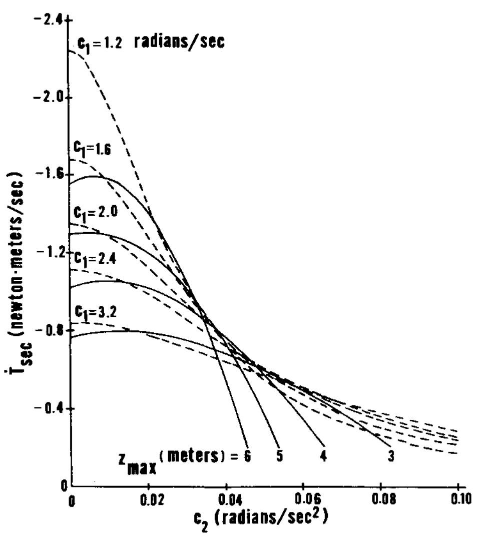

Fig. 3 Control system parameter nomograph.

$$F = \sum_{n=1}^{m} D_n \sin\left(s_n t + \tan^{-1}\frac{A_n}{B_n} \right)$$

where

$$D_n = (A_n^2 + B_n^2)^{1/2}$$

and

$$s_n = 2n\pi/\tau$$

The mass equation of motion then becomes

$$\ddot{z} + c_1\dot{z} + c_2 z = \sum_{n=1}^{m} D_n \sin\left(s_n t + \tan^{-1}\frac{A_n}{B_n} \right)$$

with particular solution

$$z_p = \sum_{n=1}^{m} E_n \sin\phi_n \qquad (18)$$

where

$$E_n = D_n/[(c_2 - s_n^2)^2 + (c_1 s_n)^2]^{1/2}$$

and

$$\phi_n = s_n t + \tan^{-1}\frac{A_n}{B_n} - \tan^{-1}\frac{c_1 s_n}{c_2 - s_n^2}$$

Differentiation yields control mass velocity

$$\dot{z}_p = \sum_{n=1}^{m} E_n s_n \cos\phi_n$$

These solutions for position and velocity histories are used to determine the rate of change of rotational kinetic energy

$$\dot{T}_{rot} = -\mu \sum_{j=1}^{m} \sum_{k=1}^{m} [E_j E_k s_k \{c_1 s_j \cos\phi_j \cos\phi_k + (c_2 + \omega_1^2 + \omega_2^2) \sin\phi_j \cos\phi_k\}]$$

The secular part of this equation is identified as the terms for which $j = k$. Thus,

$$\dot{T}_{sec} = -\frac{\mu}{2} \sum_{n=1}^{m} c_1 E_n^2 s_n^2$$

or

$$\dot{T}_{sec} = -\frac{\mu}{2} \sum_{n=1}^{m} \frac{c_1(A_n^2 + B_n^2)s_n^2}{[(c_2 - s_n^2)^2 + (c_1 s_n)^2]} \qquad (19)$$

This may be used to determine the effect of control system variables on the secular part of the energy dissipation rate. An increase in the weight of the control mass, and hence an increase in the reduced mass, μ, will linearly affect $\dot{T}_{sec}$. Increasing coefficients A_n and B_n will increase $\dot{T}_{sec}$ quadratically. This may be accomplished by increasing the amplitude of the forcing function which corresponds to increasing the mass offset distances a and b. Thus, the control mass track should be placed at the maximum allowable distance from the center of mass of the vehicle.

The effect of c_1 and c_2 is more difficult to obtain, but it is evident that c_2 should be of the order of s_n^2. With this assumption Eqs. (19) and (18) become

$$\dot{T}_{sec} \simeq -\frac{\mu}{2c_1} \sum_{n=1}^{m} (A_n^2 + B_n^2)$$

$$z_p \simeq \frac{1}{c_1} \sum_{n=1}^{m} \frac{A_n^2 + B_n^2}{s_n} \sin\phi_n$$

respectively. The first indicates that c_1 should be selected to be a small value. However, the second equation implies that decreasing c_1 results in increasing the maximum amplitude of the mass motion. These observations agree with those obtained from the spring-mass-damper analogy. It is concluded that c_1 should be the smallest value which limits control mass amplitude to its maximum allowable value.

Equations (18) and (19) may be used to generate a nomograph for selection of control system parameters, c_1 and c_2. Once the control mass weight, track position, and an estimate of initial tumble state are obtained, $\dot{T}_{sec}$ may be calculated for various values of c_1 and c_2. Using Eq. (18) the corresponding maximum mass amplitudes may be determined. Figure 3 shows the nomograph used in the example case discussed later. Each selected maximum mass amplitude has an associate optimum set of c_1 and c_2 values which corresponds to a maximum energy dissipation rate. Thus, once the maximum allowable mass amplitude has been determined, appropriate values of c_1 and c_2 may be selected from the generated nomograph.

The control system parameter selection procedure is simplified if the main vehicle is axially symmetric. Then the modulus of the elliptic functions k, is zero and the Jacobian functions may be replaced by trigonometric functions. The resulting equation of mass motion is

$$\ddot{z}+c_1\dot{z}+c_2 z = D\sin(pt+\delta)$$

where

$$D = [a^2(p\beta-\alpha\gamma)^2+b^2(p\gamma-\alpha\beta)^2]^{1/2}$$

$$\delta = \tan^{-1}\frac{a(p\beta-\alpha\gamma)}{b(p\gamma-\alpha\beta)} - pt_o$$

The particular solution is then obtained as

$$z_p = \frac{D}{[(c_2-p^2)^2+(c_1p)^2]^{1/2}}\sin\left(pt+\delta-\tan^{-1}\frac{c_1p}{c_2-p^2}\right) \quad (20)$$

Comparing this with Eq. (18) leads to the secular part of the rate of energy change, i.e., by setting $m = 1$ and $s_n = p$ in Eq. (19)

$$\dot{T}_{\text{sec}} = -\frac{\mu}{2}\frac{c_1D^2p^2}{[(c_2-p^2)^2+(c_1p)^2]}$$

The maximum mass amplitude is readily obtained from Eq. (20) as

$$z_{\max} = \frac{D}{[(c_2-p^2)^2+(c_1p)^2]^{1/2}}$$

Values of c_1 and c_2 which yield the maximum energy dissipation rate with the side condition of a selected maximum mass amplitude may be determined using a Lagrange multiplier technique.

Although methods presented for selection of control system parameters may be approximations, procedures outlined will provide satisfactory estimates of c_1 and c_2. Final selection of values must be based on actual dynamics of the system obtained by solving the complete vehicle and mass equations of motion.

Sensor and Power Requirements

For the control law expressed as Eq. (12) it is evident that the following quantities must be sensed: f_3, the force applied to the control mass; $\dot{z}$, the control mass velocity relative to the vehicle; z, the control mass position; and the combination $(\omega_1{}^2+\omega_2{}^2)$. Quantity f_3 may be determined using a linear accelerometer mounted on the control mass. If the accelerometer is mounted to detect the $\hat{k}$-component of mass acceleration, the sensed acceleration will be proportional to f_3. Mass position and velocity may be easily sensed using any of a number of simple devices. The combination $(\omega_1{}^2+\omega_2{}^2)$ may be determined with a linear accelerometer mounted on the X_3 axis. The acceleration of a fixed point, P, located by a vector $\mathbf{d}$ from the vehicle center of mass is

$$\mathbf{a}_p = \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{d})+\dot{\boldsymbol{\omega}}\times\mathbf{d}$$

If $\mathbf{d} = d\hat{k}$ the $\hat{k}$-component of $\mathbf{a}_p$ is

$$(a_p)_3 = -d(\omega_1{}^2+\omega_2{}^2)$$

Thus, an accelerometer placed at this point and oriented to sense the $\hat{k}$-component of the acceleration at that point will sense a quantity proportional to $(\omega_1{}^2+\omega_2{}^2)$. Thus, sensor requirements for implementation of the control law appear to be modest.

Force requirements of the control system may be determined by Eq. (7), since this is the force applied to the control mass. Instantaneous power required by the control system is, by definition, the force applied to the mass times the relative velocity. This definition is equivalent to Eq. (9) which gives the rate of change of rotational kinetic energy. As noted previously, $\dot{T}_{\text{rot}}$ will be oscillatory, the negative portions of which result in energy dissipation. Thus, power input to the mass corresponds to the positive positions of $\dot{T}_{\text{rot}}$, and the energy input required is just the integral of these positive portions. The force, power, and energy requirements for a realistic example appear in the next section.

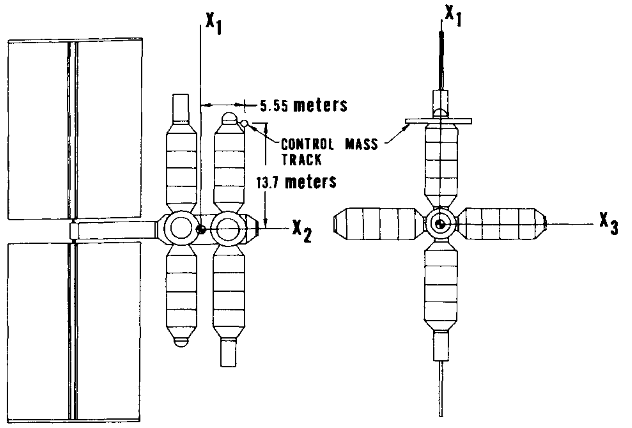

Fig. 4 Modular space station configuration.

Results of Simulations

To demonstrate the feasibility of a movable mass control system using the control law of Eq. (12), the space station shown in Fig. 4 was chosen as an example vehicle. Geometric axes shown are assumed to be principal axes and relevant properties of this spacecraft are given below:

$$I_1 = 5.15\times10^6 \text{ kg-m}^2 \ (3.80\times10^6 \text{ slug-ft}^2)$$
$$I_2 = 6.28\times10^6 \text{ kg-m}^2 \ (4.63\times10^6 \text{ slug-ft}^2)$$
$$I_3 = 6.74\times10^6 \text{ kg-m}^2 \ (4.97\times10^6 \text{ slug-ft}^2)$$
$$M = 9.98\times10^4 \text{ kg} \ (2.20\times10^5 \text{ lbm})$$

This configuration was selected since it is a relatively large vehicle and does not have an artificial-g mode. Thus, a passive damping system such as a viscous ring or pendulum damper would not be practical for this application.

An estimated worst tumble state for this example was obtained from Kaplan[7] and is the result of a collision with a shuttle orbiter. Initial rates are

$$\omega_1(0) = -2.86\times10^{-4} \text{ rad/sec}$$
$$\omega_2(0) = -0.199 \text{ rad/sec}$$
$$\omega_3(0) = 0.103 \text{ rad/sec}$$

For an uncontrolled vehicle, these angular velocity components are oscillatory in nature. Thus, such a tumble state is severe and does not lend itself to the assumption of small transverse rates. The control mass track is placed at the farthest allowable point from the vehicle center of mass and oriented parallel to the maximum inertia axis. For this position the mass track offset distances are $a = 13.7$ m (45 ft), $b = 5.5$ m (18 ft). A control mass of 99.8 kg (220 lbm) which corresponds to 0.1% of the vehicle weight was used to generate the nomograph of Fig. 3. Different values of control mass weight will shift the curves up or down but will not affect the shape of the curves. The dotted lines give the secular rate of change of rotational kinetic energy as a function of control system parameters c_1 and c_2. Smaller values of c_1 give larger values of $\dot{T}_{\text{sec}}$. Also shown by solid lines are curves which represent given maximum mass amplitudes. For each selected amplitude there appears to be an optimum set of c_1 and c_2 which results in the optimum values of $\dot{T}_{\text{sec}}$. Thus, once the maximum mass amplitude has been determined, the proper control system parameters may be chosen using the nomograph.

Although the nomograph shows relatively large energy dissipation rates for $c_2 = 0$, the assumptions under which this was

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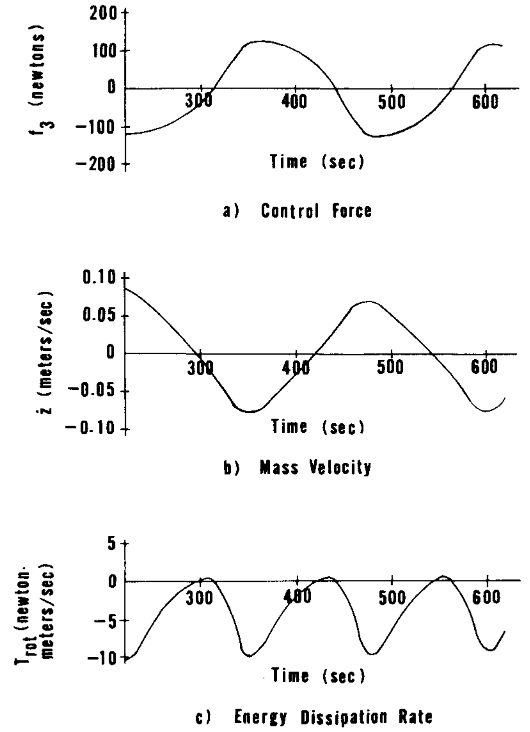

Fig. 5 Typical cycles of control force, mass velocity, and energy dissipation rate.

generated must be kept in mind. It was assumed that the forcing function would be purely oscillatory so that it could be approximated by a Fourier series. For the actual case, however, the system is damped. A value of $c_2 = 0$ corresponds to having no spring in the spring-mass-damper analogy discussed previously. The nonoscillatory nature of the forcing function will cause control mass oscillations to migrate away from the initial position for $c_2 = 0$. Thus, a nonzero value of c_2 is required to insure return to zero. Once the control system parameters have been selected, the mass and vehicle equations of motion may be solved numerically.

Before stating the results, the physical significance of the control law given by Eq. (12) will be discussed briefly. Figures 5a and 5b show typical cycles of control force acting on the control mass, f_3, and velocity relative to the body fixed axes, $\dot{z}$, respectively. Comparing these curves it is evident that the control law causes the control force to be generally opposite to the relative velocity. Figure 5c shows the rate of change of rotational kinetic energy of the system which is the product of total force and relative velocity. It can be seen that $\dot{T}_{rot}$ is generally negative, which corresponds to energy dissipation. The positive portions of $\dot{T}_{rot}$ correspond to energy addition. These portions are due to the second term of Eq. (14) which was selected to insure that the control mass oscillates about its zero position and returns there after simple spin is established. The situation is further clarified by Fig. 6 which shows typical cycles of control mass position over the same time period as the quantities shown in Fig. 5. Superimposed on the mass cycle is a schematic of the control mass with the directions of mass velocity and the force acting on the mass shown. It is evident that the force given by the control law is generally a retarding force. In the first half cycle the mass is moving "up" and the applied control force is directed "down" which produces energy dissipation. Eventually this force will be in the same direction resulting in energy addition. After this portion of the half cycle, the force and velocity will again be directed oppositely producing more energy dissipation.

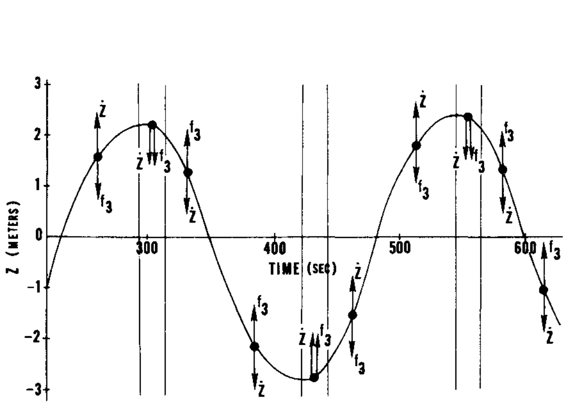

Fig. 6 Typical cycle of control mass position.

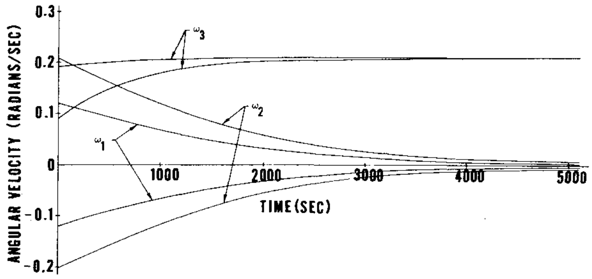

Fig. 7 Envelopes of angular velocity component oscillations.

If a maximum mass amplitude of 3 m is selected, the proper values of c_1 and c_2 are, from Fig. 3

$$c_1 = 3.2 \text{ rad/sec}$$

$$c_2 = 0.02 \text{ rad/sec}^2$$

For a control mass weight of 998 kg (2,200 lbm), the stated initial conditions, and the selected parameter values, the resultant angular velocity histories are shown in Fig. 7. Since the decay of the oscillation amplitudes is important, only envelopes formed by these oscillations are shown. The control system effectively collapses the ω_1 and ω_2 envelopes to zero, as the mean value of the ω_3 increases to its steady spin value to maintain constant angular momentum. Figure 8 shows the envelope formed by the control mass oscillations, indicating that the maximum amplitude slightly exceeds the predicted value of 3 m. This may be attributed to the homogeneous solution of the mass equation of motion which was neglected in determining the nomograph of Fig. 3. However, this overshoot occurs only during the first mass cycle and is small, thus, justifying neglect of this solution. Asymmetry of the envelope is also attributed to the homogeneous solution, since oscillations become symmetric as transients decay.

Figures 7 and 8 indicate that for the stated initial conditions, a maximum mass amplitude of approximately 3m, and a control mass weight of 998 kg (2,200 lbm), a movable mass control system using the control law given by Eq. (12), is capable of converting the tumbling motions of the modular space station into simple spin within 2 hr. To investigate the effect of various parameters on control system performance a time constant, τ_c,

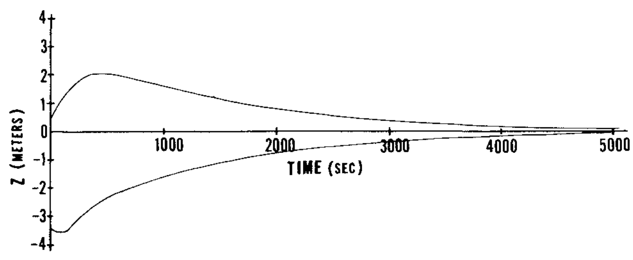

Fig. 8 Envelope of control mass oscillations.

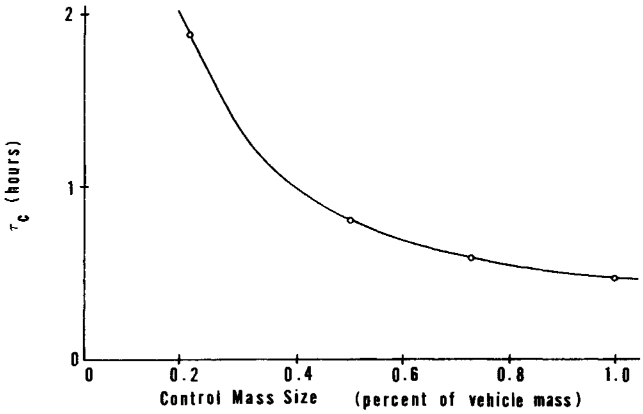

Fig. 9 Variation of time constant with control mass weight.

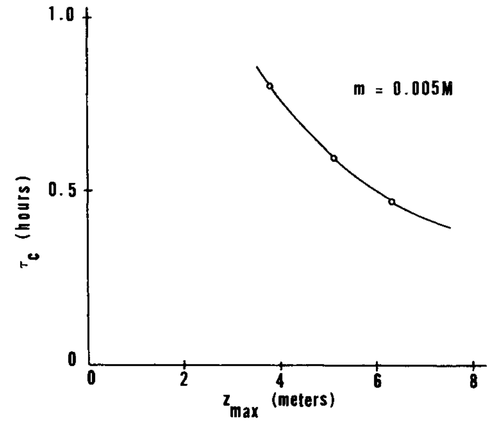

Fig. 10 Variation of time constant with maximum mass amplitude.

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will be defined as the time required for the control system to collapse the ω_1 or ω_2 envelope to $1/e$ times its initial value. Sensitivity of control mass size on system performance is illustrated in Fig. 9. An increase in control mass weight has a marked effect on the system time constant. The shape of the curve is as expected since an extremely small mass produces very little energy dissipation while an extremely large mass produces a large energy dissipation rate and, hence, a small time constant. However, the peak power and force required also increases with increased control mass size. Thus, the control mass should be selected as large as possible consistent with imposed weight, power, and force limitations. Total energy required to operate the control system does not vary appreciably with control mass weight. Energy and peak power requirements are shown in Table 1 for various control mass sizes.

To demonstrate the effect of mass displacement amplitude on control system performance, two other sets of c_1, c_2 values were selected from Fig. 3. The result is shown in Fig. 10, which gives the variation of time constant with maximum mass amplitude for a control mass of 499.0 kg (1,100 lbm). Table 2 gives the values of c_1 and c_2 that were used along with the resulting maximum amplitudes during the first and second mass cycles and predicted values given by Fig. 3.

Table 1 Variation of required energy with control mass weight

Control mass, kg	Peak power required, w	Required energy, w-sec
199.6	0.48	63.9
499.0	1.1	68.7
725.8	1.5	71.6
998.0	1.9	74.6

Table 2 Predicted and actual maximum mass amplitudes for modular space station simulation[a]

c_1 rad/sec	c_2 rad/sec^2	Predicted z_{max}, m	Computed z_{max} m First cycle	Second cycle
3.2	0.020	3.0	3.7	2.8
2.4	0.014	4.0	5.1	3.7
1.9	0.012	5.0	6.3	4.6

[a] Predicted amplitudes fall between computed amplitudes during the first and second mass cycles. Note that larger mass amplitudes result in larger overshoot relative to the predicted value, indicating the increased effect of the homogeneous solution.

Conclusions

A movable mass control system has been conceived to convert the tumbling motions of a disabled vehicle into simple spin. It has been shown that a control law relating mass motion to vehicle motion may be formulated using Liapunov stability theory. This technique is useful for designing control systems where the governing equations of motion cannot be linearized.

For a large space station which is tumbling as a result of a collision with a shuttle orbiter, a movable mass system is capable of detumbling the space station within a period of two hours for the assumptions used. This is accomplished with 1% of vehicle mass and a displacement amplitude of approximately 3 m.

The following conclusions were drawn concerning control system design: 1) The mass track should be placed as far as possible from the vehicle center of mass and oriented parallel to the maximum inertia axis. 2) The control mass size should be as large as possible, while being consistent with peak force and power limitations. 3) The performance of the control system may be improved through larger mass amplitudes. From the cases considered, it can be concluded that parameter selection procedures are valid and provide realistic estimates of system values.

References

[1] Roberson, R. E., "Torques on a Satellite Vehicle from Internal Moving Parts," *Transactions of the ASME: Journal of Applied Mechanics*, Vol. 25, June 1958, pp. 196–200.

[2] Grubin, C., "Dynamics of a Vehicle Containing Moving Parts," *Transactions of the ASME: Journal of Applied Mechanics*, Vol. 29, Sept. 1962, pp. 486–488.

[3] Kane, T. R. and Sobala, D., "A New Method for Attitude Stabilization," *AIAA Journal*, Vol. 1, No. 6, June 1963, pp. 1365–1367.

[4] Kane, T. R. and Scher, M. P., "A Method of Active Attitude Control Based on Energy Considerations," *Journal of Spacecraft and Rockets*, Vol. 6, No. 5, May 1969, pp. 633–636.

[5] La Salle, J. P., "Some Extensions of Liapunov's Second Method," *IRE Transactions on Circuit Theory*, Dec. 1960, pp. 520–527.

[6] Synge, J. L. and Griffith, B. A., *Principles of Mechanics*, McGraw-Hill, New York, 1959, Chaps. 13–14.

[7] Kaplan, M. H., "Despinning and Detumbling Satellites in Rescue Operations," *Fifth International Space Rescue Symposium, 23rd Congress of the International Astronautical Federation*, Vienna, Austria, 1972, pp. 225–243.