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Fast adaptive pose tracking control for satellites via dual quaternion upon non-certainty equivalence principle

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ABSTRACT

This paper presents the implementation of the high performance adaptive control algorithm for 6-DOF systems described by dual quaternions. The fast adaptation is achieved from the introduced stable adaptation dynamics in the closed-loop system which cannot be expected in the conventional adaptive controller designed upon the certainty equivalence framework. The development of the proposed adaptive control algorithm relies on the two newly introduced algebraic operations. The one is a dual filter for a dual relative error signal. The dual filter essentially has a linear low-pass filter structure and enables us to recast the system dynamics into a linear parameter-affine system dynamics in dual quaternions combined with dual control inputs. The other is an isomorphism between a dual quaternion and a vector for the error measurement. Using this isomorphism, the stability proof of the closed-loop dual quaternion system based on the proposed Lyapunov-like function is rewritten in the form of familiar matrix/vector algebra. In order to demonstrate the performance improvement, numerical simulation results are given, which compare the proposed adaptive control method with the conventional one from the certainty-equivalence principle.

1. Introduction

Majority of the spacecraft control problems are addressed in terms of two separate subproblems: the position control and the attitude control problem. The former is about the translational motion and the latter is about the rotational motion. However, the growing demands on the controller design for the cooperative/proximity missions between the spacecraft necessitate the new control framework based on the unified pose dynamics (position/attitude or 6-DOF). Due to the inherent nonlinearity in the rotational dynamics, the controller design based on the unified representation of the system dynamics involving both translational and rotational dynamics is not a trivial task. Among the various attempts for the pose system description, a dual quaternion is emerged again because of its compactness. A unit dual quaternion consists of a unit quaternion and a vector quaternion representing a translation via Plücker coordinate. Thus, it inherits the benefit of a unit quaternion such as the nonsingular implementation of attitude dynamics in 3-D (three dimensional) space. Further, it is also able to describe the translational dynamics simultaneously, which makes the unit dual quaternion a viable alternative for the representation of 6-DOF (six degree-of-freedom) dynamics among various transformations (e.g. homogeneous transformation matrix, conventional vector/quaternion pair, etc.). The algebraic properties of dual quaternions are mainly originated from the geometric nature of the 6-DOF transformation [13]. It is shown that the unit dual quaternion is the most compact and computationally efficient tool among others [1,5,6]. Due to such efficiency of the unit dual quaternion in 6-DOF rigid body dynamics, it is

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recently adopted to the controller design problems in [3,4,7,9,12,14,15] to name a few.

For the spacecraft pose control systems, several algorithms with dual quaternion representation appeared in [3,4,12]. All of them take the advantage of the dual quaternion algebra and succeed in stabilizing the system for both rotational and translational motions simultaneously. In [4], the authors emphasize the effectiveness of the dual quaternion in proximity operations between satellites and propose an adaptive controller using the certainty equivalence (CE) principle. The CE framework for the adaptive controller design has a well-known major drawback. Since the adaptation law is designed to cancel the uncertain parameter effects in the closed-loop dynamics, the transient performance of the controller is inevitably degraded compared to the deterministic controller. In an effort to avoid the performance degradation in the CE-based adaptive attitude tracking controller, the non-certainty equivalent (NCE) adaptive attitude tracking controller is proposed in [11] by bypassing the integrability condition required in the immersion and invariance (I&I) type controller [2,8]. Although the controller in [11] dramatically improved the transient/steady-state performance of the closed-loop system, there is no result on the NCE-based adaptive controller design for the pose tracking problems.

The main contribution of the paper is the extension of the adaptive controller in [11] for the pose tracking problems. The proposed control scheme inherits the benefits (compact pose tracking dynamics and fast adaptive control) from both dual quaternion representation and NCE-based adaptive control method. During the development of the controller, we introduce two new concepts: a dual filter and an unary operator, vectorizer. The new dual filter is a linear low-pass filter for a dual relative error quaternion. The dynamics of the filter are expressed using dual numbers and quaternions. By using a dual filter, the closed-loop system dynamics are transformed into filtered dynamics. However, because of the translational component in the dual quaternion representation, the system dynamics have no parameter-affine expression which is different from [11] and should be taken care of separately through the dual control input. The vectorizer, newly introduced operator, is an isomorphism between a dual quaternion space and a vector space to simplify the calculation of the measure function (equivalently, norm) in a dual quaternion space. By virtue of the introduced dual filter and operator, we can design an adaptive controller with the non-certainty equivalence principle. As a result of the non-certainty equivalence adaptation algorithm, the proposed adaptive controller shows the improved transient performance in 6-DOF dynamics and inherits all the benefits of the non-certainty equivalence framework such as faster adaptation and no escape from correct unknown parameter values with the price of increased system dimensions including filter states.

The paper is organized as follows. First, we summarize the quaternion/dual quaternion operations used in the subsequent sections. Then, the dual quaternion dynamics for 6-DOF systems are derived with the definition of a dual relative error. In Section 3. the non-certainty equivalent adaptive controller is derived and the stability proof is given with detailed analysis. The effectiveness of the proposed method is demonstrated through numerical simulations in Section 4. Finally, we conclude with a summary and potential research directions.

2. Dual quaternion dynamics

2.1. Dual quaternion algebra

Here, we introduce necessary dual quaternion operators for the algebraic manipulation. Since the dual quaternion consists of a unit quaternion and a vector quaternion, we briefly summarize the quaternion operators before the dual quaternion operators. We treat a quaternion as an ordered pair $q=(\overline{q}, q_4)$, where $\overline{q}=[q_1, q_2, q_3]^T \in \mathbb{R}^3$ is the vector part and $q_4 \in \mathbb{R}$ is the scalar part of the quaternion. The vector quaternion refers to the quaternion with zero scalar part. Correspondingly, the scalar quaternion refers to the quaternion with zero vector part. In addition to the standard quaternion addition, $+$, and multiplication, $*_q$, we use the binary operators defined in Table 1 to manipulate vectors in the context of quaternion algebra.

Since the scalar quaternion has only one component, we identify scalar quaternions with real numbers (this identification is always possible since they are naturally isomorphic to each other). We also use the conventional matrix–vector multiplication for the product between a 4×4 matrix and a quaternion. Finally, the unit quaternion condition is simply expressed as $q \cdot q = 1_q$ for any quaternion q wherein the identity quaternion 1_q is defined as $[0,0,0,1]^T \in \mathbb{R}^4$. Accordingly, $0_q=[0,0,0,0]^T \in \mathbb{R}^4$ is a zero quaternion.

A dual quaternion consists of a quaternion and a vector quaternion. We denote a dual quaternion with a hat symbol like $\hat{q}=q_r+\epsilon q_d$ where q_r, q_d are quaternions and referred as the real and dual part of $\hat{q}$, respectively. ϵ is the dual unit defined as $\epsilon^2=0$ and $\epsilon \neq 0$ (i.e., nilpotent). The operators are defined in Table 2 [4]. We denote a dual quaternion space with $\mathbb{H}_d$ borrowed from [4]. Thus, the set of unit dual quaternions $\mathbb{H}_d^u$ is the subset of $\mathbb{H}_d$ and defined by

$$\mathbb{H}_d^u=\left\{\hat{q} \in \mathbb{H}_d \mid \hat{q} \cdot \hat{q}=\hat{q} *_d \hat{q}^*=\hat{q}^* *_d \hat{q}=\hat{1}\right\}$$

where $\hat{1}=1_q+\epsilon 0_q$ is the identity dual quaternion. Similarly, $\hat{0}=0_q+\epsilon 0_q$ is the zero dual quaternion. In addition to the binary operations in Table 2, we introduce a new unary operator $(\cdot)^v$, vectorizer, as follows:

$$(\cdot)^v: \quad \mathbb{H}_d \to \mathbb{R}^8 \tag{1a}$$

$$q_r+\epsilon q_d \mapsto [q_r^T, q_d^T]^T \tag{1b}$$

Table 1
Basic binary operators for quaternion algebra. a, b are quaternions.

Conjugate	$a^*=(-\overline{a}, a_4)$
Dot product	$a \cdot b=(\overline{0}, \overline{a} \cdot \overline{b}+a_4 b_4)$
Cross product	$a \times b=(b_4\overline{a}+a_4\overline{b}+\overline{a} \times \overline{b}, 0)$
Norm	$\lVert a \rVert^2=aa^*=a^*a=a \cdot a$

Table 2
Binary operations for dual quaternion algebra. $\hat{a}, \hat{b}$ are dual quaternions and $\lambda \in \mathbb{R}$.

Addition	$\hat{a}+\hat{b}=(a_r+b_r)+\epsilon(a_d+b_d)$
Scalar multiplication	$\lambda\hat{a}=\lambda a_r+\epsilon(\lambda a_d)$
Product	$\hat{a}*_d\hat{b}=(a_r*_q b_r)+\epsilon(a_r*_q b_d+a_d*_q b_r)$
Swap	$\hat{a}^{\dagger}=a_d+\epsilon a_r$
Conjugate	$\hat{a}^{*}=a_r^{*}+\epsilon a_d^{*}$
Dot product	$\hat{a}\cdot\hat{b}=(a_r\cdot b_r)+\epsilon(a_r\cdot b_d+a_d\cdot b_r)$
Cross product	$\hat{a}\times\hat{b}=a_r\times b_r+\epsilon(a_r\times b_d+a_d\times b_r)$

which is an isomorphism between $\mathbb{H}_d$ and $\mathbb{R}^8$. It is followed by the definition of the inverse operator $(\cdot)^p$ meaning that $((\cdot)^v)^p$: $\mathbb{H}_d \to \mathbb{H}_d$ and $((\cdot)^p)^v$: $\mathbb{R}^8 \to \mathbb{R}^8$ are identity functions. By virtue of (1), the dual quaternion norm [16] and the product a dual quaternion with any 8×8 matrix M [4] are converted into a vector norm and a matrix/vector product, respectively. The following identities are deduced from Table 2 and (1a):

$$(\hat{a})^v\cdot\left(\hat{b}*_d\hat{c}\right)^v=\left(\hat{b}^{\dagger}\right)^v\cdot\left(\hat{a}^{\dagger}*_d\hat{c}^{*}\right)=\left(\hat{c}^{\dagger}\right)^v\cdot\left(\hat{b}^{*}*_d\hat{a}^{\dagger}\right) \tag{2a}$$

$$(\hat{a})^v\cdot\left(\hat{b}\times\hat{c}\right)^v=\left(\hat{b}^{\dagger}\right)^v\cdot\left(\hat{c}\times\hat{a}^{\dagger}\right)^v=\left(\hat{c}^{\dagger}\right)^v\cdot\left(\hat{a}^{\dagger}\times\hat{b}\right) \tag{2b}$$

2.2. Relative error dynamics in a dual quaternion form

In the following, we define three frame indices (B, D, I) for the analysis. The body-fixed frame, reference/desired frame, and inertial frame are represented by B, D, and I, respectively. The meaning of a vector quaternion $a^X_{Y/Z}$ with super/subscript frame indices X, Y, and Z is the vector $\overline{a}$ of Y relative to Z expressed in X. Based on this, the dual quaternion representation involving a rotation $q_{B/I}$ and a translation $\overline{r}^B_{B/I}$ is given by $\hat{q}_{B/I}=q_{B/I}+\epsilon\frac{1}{2}q_{B/I}*_q r^B_{B/I}$ [16]. $\hat{q}_{D/I}$ is also defined in the same way. Then, the definition of the 6-DOF relative error between the body frame and the reference/desired frame is defined by [16]

$$\hat{q}_{B/D}=\hat{q}^{*}_{D/I}*_d\hat{q}_{B/I}=q_{B/D}+\epsilon\tfrac{1}{2}q_{B/D}*_q r^B_{B/D} \tag{3}$$

where $q_{B/D}$ is the unit quaternion between B and D and $r^B_{B/D}=r^B_{B/I}-r^B_{D/I}$ is the relative position vector quaternion between the origins of B and D expressed in B. We assume that D's states (desired pose information) are known and their derivatives are bounded. By differentiating (3), we obtain the following kinematic equations in a dual quaternion form [12]:

$$\dot{\hat{q}}_{B/D}=\tfrac{1}{2}\hat{q}_{B/D}*_d\hat{\omega}^B_{B/D} \tag{4}$$

where $\hat{\omega}^B_{B/D}=\hat{\omega}^B_{B/I}-\hat{\omega}^B_{D/I}$ is the dual relative velocity and $\hat{\omega}^B_{B/I}=\omega^B_{B/I}+\epsilon\left(v^B_{B/I}+\omega^B_{B/I}\times r^B_{B/B}\right)$ with the angular velocity ω and the translational velocity v (we add $\omega^B_{B/I}\times r^B_{B/B}$ for the completeness of the dual quaternion algebra although $r^B_{B/B}=0_q$). The coordinate transformation using dual quaternions is similar to that of quaternions so that $\hat{\omega}^B_{B/I}=\hat{q}^{*}_{B/I}*_d\hat{\omega}^I_{B/I}*_d\hat{q}_{B/I}$. By taking the time derivative of $\hat{\omega}^B_{B/D}$ (an equivalent procedure is given in [12]),we have

$$\begin{aligned}\left(\dot{\hat{\omega}}^B_{B/D}\right)^{\dagger v}=\left(M^B\right)^{-1}\Bigg[\hat{f}^B&-\hat{\omega}^B_{B/I}\times\left(M^B\left(\hat{\omega}^B_{B/I}\right)^{\dagger v}\right)^p\\&-\left(M^B\left(\hat{q}^{*}_{B/D}*_d\dot{\hat{\omega}}^D_{D/I}*_d\hat{q}_{B/D}\right)^{\dagger v}\right)^p\\&-\left(M^B\left(\hat{\omega}^B_{D/I}\times\hat{\omega}^B_{B/D}\right)^{\dagger v}\right)^p\Bigg]^v\end{aligned} \tag{5}$$

where $(\cdot)^{\dagger v}=\left((\cdot)^{\dagger}\right)^v$; $\hat{f}^B=f^B+\epsilon\tau^B$ with vector quaternions f^B and τ^B; $M^B\in\mathbb{R}^{8\times 8}$, dual inertia matrix, is given by

$$M^B=\begin{bmatrix} mI_{3\times 3} & 0_{3\times 1} & 0_{3\times 3} & 0_{3\times 1} & \\ 0_{1\times 3} & 1 & 0_{1\times 3} & 0 & 0_{3\times 1}0_{1\times 3}00_{1\times 3}1 \\ 0_{3\times 3} & 0_{3\times 1} & \overline{I}^B & & \end{bmatrix}, \tag{6}$$

where m is the mass of the body; $I_{3\times 3}$ is the 3-by-3 identity matrix; and $\overline{I}^B\in\mathbb{R}^{3\times 3}$ is the mass moment-of-inertia matrix with respect to the body-fixed frame centered at the mass center of the body. From now on, we mean the multiple superscript operators by the operator composition in same order as in (5). For satellite missions in Earth orbit, we explicitly consider the gravitational force and disturbances from Earth as follows:

$$\hat{f}^B=\hat{f}^B_g+\hat{f}^B_{gg}+\hat{f}^B_J+\hat{f}^B_c \tag{7}$$

where the subscript g is for the gravitational force; gg for the gravity-gradient effect; J for J_2; and c for the actual control input. Each term in (7) is defined by [10,12]

$$\hat{f}^B_g=m\left(a^B_g+\epsilon 0_q\right),\quad \overline{a}^B_g=-\mu\frac{\overline{r}^B_{B/I}}{\|\overline{r}^B_{B/I}\|^3} \tag{8a}$$

$$\hat{f}^B_{gg}=0_q+\epsilon\tau^B_{gg},\quad \overline{\tau}^B_{gg}=3\mu\frac{\overline{r}^B_{B/I}\times\left(\overline{I}^B\overline{r}^B_{B/I}\right)}{\|\overline{r}^B_{B/I}\|^5} \tag{8b}$$

$$\hat{f}^I_J=m\left(a^I_J+\epsilon 0_q\right),\quad \overline{a}^I_J=-\frac{3}{2}\frac{\mu J_2 R_E^2}{\|\overline{r}^I_{B/I}\|^4}\begin{bmatrix}(1-5c^2)\frac{x^I_{B/I}}{\|\overline{r}^I_{B/I}\|}\\(1-5c^2)\frac{y^I_{B/I}}{\|\overline{r}^I_{B/I}\|}\\(3-5c^2)\frac{z^I_{B/I}}{\|\overline{r}^I_{B/I}\|}\end{bmatrix} \tag{8c}$$

where $c=z^B_{B/I}/\|\overline{r}^I_{B/I}\|$; $\mu=398\,600.4418\ \text{km}^3/\text{s}^2$; $J_2=0.0010827$; and $R_E=6378.14$ km. $\hat{f}^B_J$ is calculated through a coordinate transformation given by $\hat{f}^B_J=\hat{q}^{*}_{B/I}*_d\hat{f}^I_J*_d\hat{q}_{B/I}$. By using dual quaternions, the system equations are given by Eqs. (4) and (5) with body force equations (7) and (8). The control objective is to guarantee that $\lim_{t\to\infty}\left\{\hat{q}_{B/D},\hat{\omega}^B_{B/D}\right\}=\left\{\hat{1},\hat{0}\right\}$ with no information on M^B.

3. Non-certainty equivalent adaptive controller

In this section, we propose a new 6-DOF adaptive controller based on the non-certainty equivalence framework. We borrow the definitions of the vector norm and related boundedness condition such as $\mathcal{L}_\infty$ and $\mathcal{L}_2$ for dual quaternions by using a vectorizer in (1). In addition, we define the combined dual relative error $\hat{s}$ as

$$\hat{s} = \hat{\omega}^B_{B/D} + k_p\left(\hat{q}^*_{B/D} *_d \left(\hat{q}_{B/D} - \hat{1}\right)^\dagger\right)^\dagger$$
$$= \hat{\omega}_e + k_p \hat{q}_e^\dagger \tag{9}$$

where $k_p > 0$ is a control gain. For notational simplicity, we introduce $\hat{\omega}_e = \hat{\omega}^B_{B/D}$ and $\hat{q}_e = \hat{q}^*_{B/D} *_d \left(\hat{q}_{B/D} - \hat{1}\right)^\dagger$ in (9) and omit $*_d$ in the following.

Theorem 1. *Consider the satellite motion in Earth orbit whose system equations are described by Eqs. (4) and (5) without M^B information and suppose the adaptive dual control input $\hat{f}^B_c$ is generated by*

$$\hat{f}^B_c = -\left[W(\overline{\theta}+\overline{\beta})\right]^p - \gamma\left[W_f W_f^T\left[-k_r(\hat{s}_f)^\dagger\right.\right.$$
$$\left.\left. + (\hat{s})^\dagger + k_r \hat{q}_e\right]^v\right]^p - \hat{w}_{ext} \tag{10a}$$

$$\dot{\theta} = -\gamma\left\{\left[W - (k_r + 2k_d)W_f\right]^T (\hat{s}_f)^{\dagger v} - k_r W_f^T (\hat{q}_e)^v\right\} \tag{10b}$$

$$\beta = \gamma W_f^T (\hat{s}_f)^{\dagger v} \tag{10c}$$

where $k_p, k_r, k_d, \gamma > 0$ are any control gains; and the dual quaternion $\hat{w}_{ext}$ and the regressor matrix W are obtained from

$$(W\eta)^p + \hat{w}_{ext} = -\hat{\omega}_e \times \left(M^B (\hat{\omega}_e)^{\dagger v}\right)^p - \left[M^B\left(\hat{q}^*_{B/D} \dot{\hat{\omega}}^D_{D/I} \hat{q}_{B/D}\right)^{\dagger v}\right.$$
$$\left. + M^B\left(\hat{\omega}^B_{D/I} \times \hat{\omega}^B_{B/D}\right)^{\dagger v}\right]^p + \hat{f}^B_g + \hat{f}^B_{gg} + \hat{f}^B_J$$
$$+ k_p M^B \left(\dot{\hat{q}}_e\right)^v$$
$$+ k_d M^B (\hat{s})^{\dagger v} + (k_d + k_r) k_r M^B (\hat{q}_e)^v + k_r M^B \dot{\hat{q}}_e \tag{11}$$

where $\eta = [m, \overline{I}^B_{11}, \overline{I}^B_{12}, \overline{I}^B_{13}, \overline{I}^B_{22}, \overline{I}^B_{23}, \overline{I}^B_{33}]^T \in \mathbb{R}^7$ is an unknown parameter vector from M^B; W is the 8×7 regressor matrix containing all the inertia related terms; and the dual quaternion $\hat{w}_{ext}$ is the remaining extra terms that are not involved with inertia terms. W_f and $\hat{s}_f$ in Eq. (10) are calculated by the following stable linear filter equations:

$$\dot{W}_f = -(k_d + k_r) W_f + W \tag{12a}$$

$$\dot{\hat{s}}_f = -(k_d + k_r)\hat{s}_f + \hat{s} \tag{12b}$$

with any initial conditions $W_f(0) \in \mathbb{R}^{8\times 7}, \hat{s}_f(0) \in \mathbb{H}_d$. Then, for any initial states $\left\{\hat{q}_{B/D}(0), \hat{\omega}^B_{B/D}(0)\right\}$, the closed-loop system is globally asymptotically stable resulting in $\lim_{t\to\infty}\left\{\hat{q}_{B/D}(t), \hat{\omega}^B_{B/D}\right\} = \left\{\hat{1}, \hat{0}\right\}$.

Proof. The proof of Theorem 1 consists of two steps: first, we recast the system dynamics into the filtered system dynamics from which we may take the advantage of the unknown parameter affine representation. Then, we introduce the non-certainty equivalent adaptive controller and prove the stability of the system based on the Lyapunov-like function.

Step 1: from the definition of the error filter $\hat{s}_f$ in (12b), we have

$$\ddot{\hat{s}}_f = -\alpha \dot{\hat{s}}_f + \dot{\hat{s}}$$

where $\alpha = (k_d + k_r)$ is introduced for the notational simplicity. By substituting (9), we obtain

$$\ddot{\hat{s}}_f = -\alpha\dot{\hat{s}}_f + \left\{\left[M^{B^{-1}}\left(\hat{f}^B_c + (W_1\eta)^p + \hat{w}_{1ext} + k_p\left(M^B\left(\dot{\hat{q}}_e\right)^v\right)^p\right)^v\right]^p\right\}^\dagger \tag{13}$$

where $W_1 \in \mathbb{R}^{8\times 7}$ and $\hat{w}_{1ext} \in \mathbb{H}_d$ satisfy

$$(W_1\eta)^p + \hat{w}_{1ext} = -\hat{\omega}_e \times \left(M^B(\hat{\omega}_e)^{\dagger v}\right)^p - \left[M^B\left(\hat{q}^*_{B/D}\dot{\hat{\omega}}^D_{D/I}\hat{q}_{B/D}\right)^{\dagger v}\right.$$
$$\left. + M^B\left(\hat{\omega}^B_{D/I} \times \hat{\omega}^B_{B/D}\right)^{\dagger v}\right]^p + \hat{f}^B_g + \hat{f}^B_{gg} + \hat{f}^B_J \tag{14}$$

We add and subtract $k_d\hat{s} + \alpha k_r \hat{q}_e^\dagger + k_r \dot{\hat{q}}_e^\dagger$ at the right hand side of Eq. (13) and rewrite the inertia related terms using regressor matrices to obtain the following expression:

$$\ddot{\hat{s}}_f = -\alpha\dot{\hat{s}}_f + \left[M^{B^{-1}}\left(\hat{f}^B_c + (W\eta)^p + \hat{w}_{ext}\right)^v\right]^p - k_d\hat{s} - \alpha k_r \hat{q}_e^\dagger - k_r\dot{\hat{q}}_e^\dagger \tag{15}$$

where $W = W_1 + W_2 + W_3$ and $\hat{w}_{ext} = \hat{w}_{1ext} + \hat{w}_{2ext} + \hat{w}_{3ext}$ with

$$(W_2\eta)^p + \hat{w}_{2ext} = \left[M^B\left(k_p\dot{\hat{q}}_e\right)^v\right]^p \tag{16a}$$

$$(W_3\eta)^p + \hat{w}_{3ext} = \left[M^B\left(k_d\hat{s}^\dagger + \alpha k_r\hat{q}_e + k_r\dot{\hat{q}}_e\right)^v\right]^p \tag{16b}$$

For the unknown parameter affine system equations, we inject the control signal $-\hat{w}_{ext}$ and adopt the control filter $\hat{f}^B_f$ as follows:

$$\hat{f}^B_c = \hat{f}^B_d - \hat{w}_{ext} \tag{17a}$$

$$\dot{\hat{f}}^B_f = -\alpha\hat{f}^B_f + \hat{f}^B_d \tag{17b}$$

By plugging (17) and (12a) into Eq. (15) and rearranging terms, we have

$$\frac{d}{dt}\left\{\dot{\hat{s}}_f + k_d\hat{s}_f + k_r\hat{q}_e^\dagger - \left[M^{B^{-1}}\left(\hat{f}^B_f + (W_f\eta)^p\right)^v\right]^{p\dagger}\right\}$$
$$= -\alpha\left\{\dot{\hat{s}}_f + k_d\hat{s}_f + k_r\hat{q}_e^\dagger - \left[M^{B^{-1}}\left(\hat{f}^B_f + (W_f\eta)^p\right)^v\right]^{p\dagger}\right\} \tag{18}$$

which results in the following filtered error dynamics:

$$\dot{\hat{s}}_f = -k_d\hat{s}_f - k_r\hat{q}_e^\dagger + \left[M^{B^{-1}}\left(\hat{f}^B_f + (W_f\eta)^p\right)^v\right]^{p\dagger} + \hat{\psi} \tag{19}$$

where $\hat{\psi}(t) = \dot{\hat{s}}_f + k_d\hat{s}_f + k_r\hat{q}_e^\dagger - [M^{B^{-1}}(\hat{f}^B_f + (W_f\eta)^p)^v]^{p\dagger}$ is an exponentially decaying term and its decay rate is regulated by α. Now, we define $\hat{f}^B_f$ as

$$\hat{f}^B_f = -\left[W_f(\theta+\beta)\right]^p.$$

Then, we finally get the unknown parameter affine filtered error dynamics with stabilizing terms as

$$\dot{\hat{s}}_f = -k_d\hat{s}_f - k_r\hat{q}_e^{\dagger} - \left[M^{B^{-1}}W_f z\right]^{p\dagger} + \hat{\psi} \tag{20}$$

where $z = \theta + \beta - \eta$ represents the estimation error of the unknown parameter vector. The closed-loop estimation dynamics are expressed as

$$\dot{z} = \dot{\theta} + \dot{\beta} = -\gamma W_f^T M^{B^{-1}} W_f z + \gamma W_f^T \hat{\psi}^{\dagger v} \tag{21}$$

Step 2: in this step, we prove the asymptotic stability of the closed-loop system using (20) and (21). Consider the lower bounded function given by

$$V = \left(\hat{q}_{B/D} - \hat{1}\right)^{vT}\left(\hat{q}_{B/D} - \hat{1}\right)^{v} + \frac{1}{2}\hat{s}_f^{\dagger vT}\hat{s}_f^{\dagger v} + \frac{\zeta}{2\lambda_{\min}} z^T z + \frac{\xi}{2}\hat{\psi}^{\dagger T}\hat{\psi}^{\dagger} \tag{22}$$

where the superscript T denotes the regular transpose operator; $\lambda_{\min}$ is the minimum eigenvalue of M^B; $\lambda_{\max}$ is the maximum eigenvalue of M^B; $\zeta \geq \max\{3/\gamma(k_p+k_r), 3/\gamma k_d\}$; and $\xi \geq \max\{3/\alpha(k_p+k_r), 3/\alpha k_d, 4\zeta\gamma\lambda_{\max}/\ \alpha\lambda_{\min}\}$. We recognize here that, due to the introduced isomorphism in (1), the calculation of V is simply done through a vector scalar product although all states are from dual quaternions except the estimation error z. The time derivative of V is now computed by the conventional vector algebra as follows:

$$\begin{aligned} \dot{V} &= 2\left(\hat{q}_{B/D} - \hat{1}\right)^{vT}\left(\dot{\hat{q}}_{B/D}\right)^{v} + \hat{s}_f^{\dagger vT}\left(\dot{\hat{s}}_f\right)^{\dagger v} \\ &\quad + \frac{\zeta}{\lambda_{\min}} z^T\left(\dot{\theta} + \dot{\beta}\right) + \xi\hat{\psi}^{\dagger vT}\left(\dot{\hat{\psi}}\right)^{\dagger v} \\ &= -k_p\hat{q}_e^{vT}\hat{q}_e^{v} + \left(\dot{\hat{s}}_f + \alpha\hat{s}_f\right)^{\dagger vT}\hat{q}_e^{v} + \hat{s}_f^{\dagger vT}\dot{\hat{s}}_f^{\dagger v} \\ &\quad + \frac{\zeta}{\lambda_{\min}} z^T\left(\dot{\theta} + \dot{\beta}\right) - \xi\alpha\hat{\psi}^{\dagger vT}\hat{\psi}^{\dagger v} \end{aligned} \tag{23}$$

where the last equality is obtained by using the quaternion identity equation (2) along with Eqs. (4), (9) and (12b). Then, $\dot{V}$ of the closed-loop system can be upper-bounded by plugging Eqs. (20) and (21) into Eq. (23) as follows:

$$\begin{aligned} \dot{V} &= -(k_p+k_r)\hat{q}_e^{vT}\hat{q}_e^{v} - \left(M^{B^{-1}}W_f z\right)^T\hat{q}_e^{v} + \hat{\psi}^{\dagger v}\hat{q}_e^{v} - k_d\hat{s}_f^{\dagger vT}\hat{s}_f^{\dagger v} \\ &\quad - \hat{s}_f^{\dagger vT}\left(M^{B^{-1}}W_f z\right) + \hat{s}_f^{\dagger vT}\hat{\psi}^{\dagger v} - \frac{\zeta\gamma}{\lambda_{\min}} z^T W_f^T M^{B^{-1}} W_f z \\ &\quad + \frac{\zeta\gamma}{\lambda_{\min}} z^T W_f^T\hat{\psi}^{\dagger v} - \xi\alpha\hat{\psi}^{\dagger vT}\hat{\psi}^{\dagger v} \end{aligned}$$

We now complete the square expression of the right hand side as follows:

$$\begin{aligned} \dot{V} &= -\left[\frac{(k_p+k_r)}{3}\hat{q}_e^{vT}\hat{q}_e^{v} + \hat{q}_e^{vT}\left(M^{B^{-1}}W_f z\right) + \frac{\zeta\gamma}{4\lambda_{\min}} z^T W_f^T M^{B^{-1}} W_f z\right] \\ &\quad - \left[\frac{(k_p+k_r)}{3}\hat{q}_e^{vT}\hat{q}_e^{v} - \hat{q}_e^{vT}\hat{\psi}^{\dagger v} + \frac{\xi\alpha}{4}\hat{\psi}^{\dagger vT}\hat{\psi}^{\dagger v}\right] \\ &\quad - \left[\frac{k_d}{3}\hat{s}_f^{\dagger vT}\hat{s}_f^{\dagger v} + \hat{s}_f^{\dagger vT}\left(M^{B^{-1}}W_f z\right) + \frac{\zeta\gamma}{4\lambda_{\min}} z^T W_f^T M^{B^{-1}} W_f z\right] \\ &\quad - \left[\frac{k_d}{3}\hat{s}_f^{\dagger vT}\hat{s}_f^{\dagger v} - \hat{s}_f^{\dagger vT}\hat{\psi}^{\dagger v} + \frac{\xi\alpha}{4}\hat{\psi}^{\dagger vT}\hat{\psi}^{\dagger v}\right] \\ &\quad - \left[\frac{\zeta\gamma}{4\lambda_{\min}} z^T W_f^T M^{B^{-1}} W_f z - \frac{\zeta\gamma}{\lambda_{\min}}(W_f z)^T\hat{\psi}^{\dagger v} + \frac{\xi\alpha}{4}\hat{\psi}^{\dagger vT}\hat{\psi}^{\dagger v}\right] \\ &\quad - \frac{(k_p+k_r)}{3}\hat{q}_e^{vT}\hat{q}_e^{v} - \frac{k_d}{3}\hat{s}_f^{\dagger vT}\hat{s}_f^{\dagger v} \\ &\quad - \frac{\zeta\gamma}{4\lambda_{\min}} z^T W_f^T M^{B^{-1}} W_f z - \frac{\xi\alpha}{4}\hat{\psi}^{\dagger vT}\hat{\psi}^{\dagger v} \end{aligned}$$

By using the definitions of ζ and ξ, we can find the upper bound of $\dot{V}$ as

$$\begin{aligned} \dot{V} &\leq -\frac{(k_p+k_r)}{3}\|\hat{q}_e^{v}\|^2 - \frac{k_d}{3}\|\hat{s}_f^{\dagger v}\|^2 - \frac{\zeta\gamma}{4}\|W_f z\|^2 - \frac{\xi\alpha}{4}\|\hat{\psi}^{\dagger v}\|^2 \\ &\leq -\frac{(k_p+k_r)}{3}\|\hat{q}_e^{v}\|^2 - \frac{k_d}{3}\|\hat{s}_f^{\dagger v}\|^2 - \frac{\zeta\gamma}{4}\|W_f z\|^2 \end{aligned} \tag{24}$$

Based on the last inequality of Eq. (24), we do the standard signal chasing process and apply Barbalat's lemma. First, by noticing that V is lower bounded and $\dot{V}$ is negative semi-definite, we see that the finite $V_\infty = \lim_{t\to\infty} V(t)$ exists. This makes all the internal vectorized dual quaternion signals $\mathcal{L}_\infty$ based on the definition of filter states/dynamics in (12). Further, $\hat{q}_e^{v}, \hat{s}_f^{\dagger v}, W_f z \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ since $\lim_{t\to\infty}\int_0^t V(\sigma)\,d\sigma = V_\infty - V(0)$ is finite. By applying Barbalat's lemma with the observation of $\dot{\hat{q}}_e^{v}, \dot{\hat{s}}_f^{\dagger v}, (d/dt)W_f z \in \mathcal{L}_\infty$, we conclude that $\lim_{t\to\infty}\left\{\hat{q}_e, \hat{s}_f^{\dagger}, W_f z\right\} = \left\{\hat{0}, \hat{0}, 0_{8\times 1}\right\}$. Since $\hat{s}_f^{\dagger v} \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ and $\ddot{\hat{s}}_f^{\dagger v} \in \mathcal{L}_\infty$ from the time derivative of Eq. (12b), we see that $\lim_{t\to\infty}\dot{\hat{s}}_f^{\dagger v} = 0_{8\times 1}$ by applying Barbalat's lemma again. Thus, Eq. (12b) shows $\lim_{t\to\infty}\hat{s} = \hat{0}$. Finally, we guarantee that $\lim_{t\to\infty}\{\hat{q}_e, \hat{s}\} = \left\{\hat{0}, \hat{0}\right\}$, which is equivalent to $\lim_{t\to\infty}\left\{\hat{q}_{B/D}, \hat{\omega}_{B/D}^{B}\right\} = \left\{\pm\hat{1}, \hat{0}\right\}$ [4]. Note that $\hat{q}_{B/D} = \pm\hat{1}$ means the same attitude physically. This concludes the proof.□

Remark 1. From Eq. (24), we notice that $W_f z$ goes to zero instead of the estimation error z. This means that the following attractive manifold Ξ for the parameter estimation $\theta + \beta$ is defined as

$$\Xi := \left\{z \in \mathbb{R}^7 \mid W_f z = \overline{0}\right\}$$

The attractivity of Ξ is easily shown by Eq. (21). Although the structure of the attracting manifold is similar to [11], the state space where the attracting manifold immersed consists of dual quaternions, which results in the new base space for the regressor dynamics different from [11].

Remark 2. Immediate result of Ξ is the condition for the convergence of the estimator $\theta + \beta$ to its true value η. If we simplify the estimation error dynamics of Eq. (21) by ignoring the exponentially decaying term $\hat{\psi}^{\dagger v}$ (we may do this without considering ψ in V explicitly since it decays with the exponential rate, $k_d + k_r$, regardless of system states):

$$\dot{z} = -\gamma W_f^T M^{B^{-1}} W_f z,$$

we see that z goes to zero if

$$\int_t^{t+T_{ex}} W_f^T(\sigma) W_f(\sigma)\,d\sigma > 0$$

for all $t \geq T$ for some $T \geq 0$ and $T_{ex} > 0$. This condition is also valid for W because W_f is a stable linear filter and is well known as the persistence excitation condition [11]. In

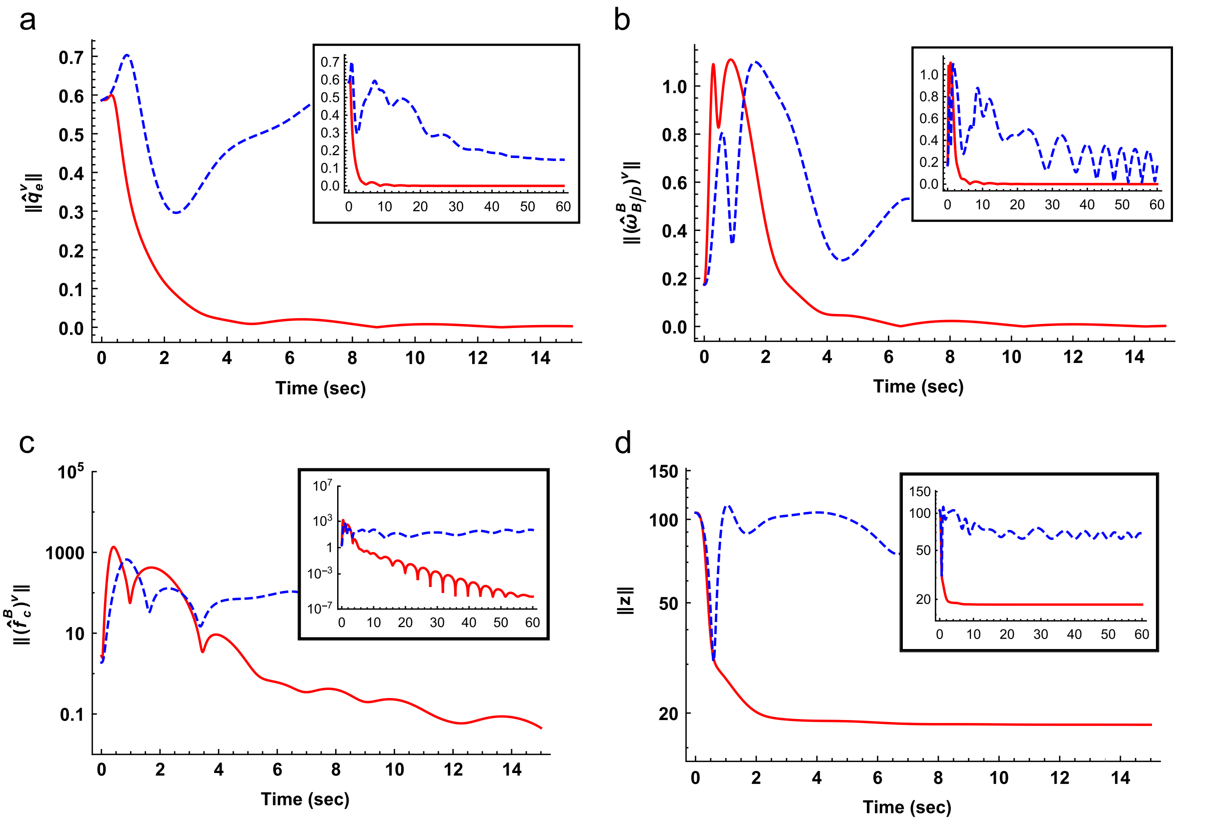

Fig. 1. Performance comparison between the proposed and the CE-based adaptive controller. The dashed line represents the system response with the CE-based adaptive controller and the solid line represents the one with the proposed controller. Each graph inside the box shows the long-term response (up to 60 s) of the same quantity. (a) Relative dual quaternion error norm. (b) Relative dual velocity error norm. (c) Control norm. (d) Estimation error norm.

addition, if any element of z starts at zero, then there will be no update for the corresponding estimator. This is the benefit of the proposed controller which cannot be shown from CE-based ones [4].

Remark 3. The implementation of the proposed non-certainty equivalent adaptive controller is possible because of the split control signals $\hat{f}_c^B = \hat{f}_d^B - \hat{w}_{ext}$. Due to $-\hat{w}_{ext}$, the system dynamics can be represented as the parameter-affine equations. This contrasts with the controller of [11]. As already mentioned, $\hat{w}_{ext}$ in the system dynamics is originated from the translational motion component in the dual quaternion expression.

4. Simulations

This section demonstrates the effectiveness of the proposed controller through two numerical simulations: the one is for the small initial attitude error and the other for the relative large initial attitude error. The proposed controller is compared with the conventional adaptive controller designed with the certainty equivalence principle [4] given by

$$\hat{f}_{ce}^B = -k_d \hat{s}^{\dagger} - [(W_1 + W_2)\theta_{ce}]^p - (\hat{w}_{1ext} + \hat{w}_{2ext}) - \text{vec}[\hat{q}_e]$$
$$\dot{\theta}_{ce} = \gamma_{ce}(W_1 + W_2)^T \hat{s}^{\dagger v}$$

The mass m of the satellite is 100 kg, and the moment-of-inertia matrix $\bar{I}^B$ is

$$\bar{I}^B = \begin{bmatrix} 22 & 0.2 & 0.5 \\ 0.2 & 20 & 0.4 \\ 0.5 & 0.4 & 17 \end{bmatrix} \text{ kg m}^2.$$

The reference/desired frame is arbitrarily chosen and set to be a LVLH frame in an equatorial circular orbit whose altitude is 500 km only because of the simplicity of the reference signal. Any reference motion can be given as long as it is continuous and its time derivative is bounded. In the first simulation, the initial relative pose error of the body frame with respect to the reference frame is given by $q_{B/D} = [\sin(\pi/12)/\sqrt{3}, \sin(\pi/12)/\sqrt{3}, \sin(\pi/12)/\sqrt{3}, \cos(\pi/12)]^T$ and $\bar{r}_{B/D}^B = [0.5, 0.8, 0.1]$ km and the initial relative velocity error by $\varpi = [0.1, 0.1, 0.1]^T$ rad/s and $\bar{v}_{B/D}^B = [0.01, 0.01, 0.01]^T$ km/s. Both adaptive controllers have the same initial parameter estimation value zero. Control gains are set as follows: $k_p = 1, k_d = 2$, and the learning rate $\gamma = \gamma_{ce} = 100$ for both controllers. For the proposed controller, $k_r = 2$, $W_f(0) = 0_{8\times 7}$, and $\hat{s}_f(0) = \hat{q}_e(0) + (1/k_r)\hat{s}(0)$, wherein $\hat{s}_f(0)$ is chosen to set $\psi(t) = 0$ in order to eliminate the effect of the exponentially decaying term in the filtered dynamics of Eq. (20). The

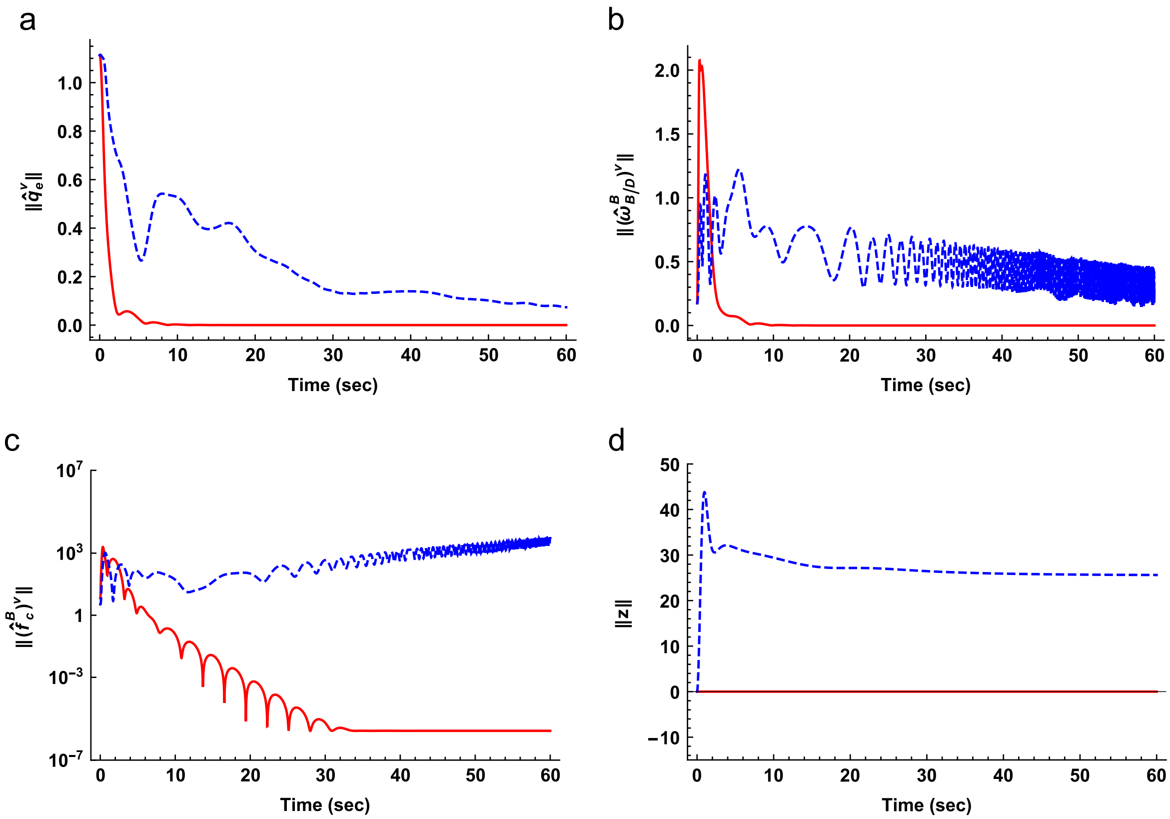

Fig. 2. Performance comparison with a large initial attitude error and adaptation algorithm comparison. The dashed line represents the CE-based adaptive controller and the solid line represents the proposed controller. (a) Relative dual quaternion error norm. (b) Relative dual velocity error norm. (c) Control norm. (d) Adaptation scheme.

comparison results are summarized in Fig. 1 and clearly show the superior transient performance of the proposed controller. In Fig. 1a, the pose error is measured by $\hat{q}_e$. As pointed out in the proof of Theorem 1, $\hat{q}_e = \hat{0}$ (i.e., $\|\hat{q}_e^v\| = 0$) means $\hat{q}_{B/D} = \hat{1}$. The convergence to zero already occurs after about 15 s for the proposed controller while $\hat{f}_{ce}^B$ is still trying to reduce the error. The performance improvement is also shown in Fig. 1b, where the convergence of the dual relative error (translational and rotational velocity error) is compared. More importantly, Fig. 1c shows the better performance, which does not mean the increased control efforts. It shows a reduced control force in total except for the aggressive transient region due to the fast convergence of the relative errors. This is because of the fast adaptation algorithm as shown in Fig. 1d. In Fig. 1d, we verify that the convergence of the unknown parameter estimator occurs before 10 s for the proposed controller. Further performance comparison is given in Fig. 2 when the initial attitude error is relatively large. $q_{B/D} = [\sin(\pi/3)/\sqrt{3}, \sin(\pi/3)/\sqrt{3}, \sin(\pi/3)/\sqrt{3}, \cos(\pi/3)]^T$ is set as an initial attitude error which is equivalent to 120° orientation error about the axis-of-rotation [1, 1, 1] relative to the reference frame. Corresponding simulation results are shown in Fig. 2a–c. The benefit of the non-certainty equivalent adaptation algorithm can be shown more clearly in Fig. 2d. In Fig. 2d, we observe the estimation error evolution for each controllers when both controllers start with the same initial parameter estimation value η (i.e., parameter estimators are set to be true value initially). The CE-based algorithm drives the estimation θ_{ce} away from the true value while the proposed method guarantees that the estimation $\theta + \beta$ stays at the true value all the time as expected.

5. Conclusion

A new non-certainty equivalent adaptive controller is proposed for the satellite pose tracking problem in Earth orbit. Main gravitational disturbances such as the gravity-gradient torque and the non-sphere earth effect are explicitly considered to increase the accuracy of the considered model. The non-certainty equivalence formulation is possible because of the newly introduced dual filter and vectorizer, which enable us to rewrite the system equation in the parameter-affine dual quaternion form along with the injected control signals to cancel the translational components. It is shown that the proposed controller guarantees the global asymptotic convergence of 6-DOF tracking errors for arbitrary uncertainties in the dual inertia matrix. The proposed adaptive controller demonstrates the significantly improved transient and steady-state performance compared to the CE-based adaptive controller without increased control efforts. The price for the better performance is only the increased system dimensions due to the additional filter states. One of the potential research topics can be to find the optimal number of filter states instead of filtering an entire sparse regressor matrix.

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References

[1] Nicholas A. Aspragathos, John K. Dimitros, A comparative study of three methods for robot kinematics, IEEE Trans. Syst. Man Cybern. Part B: Cybern. 28 (2) (1998) 135–145.
[2] A. Astolfi, R. Ortega, Immersion and invariance: a new tool for stabilization and adaptive control of nonlinear systems, IEEE Trans. Autom. Control 48 (2003) 590–606.
[3] Nuno Filipe, Panagiotis Tsiotras, Rigid body motion tracking without linear and angular velocity feedback using dual quaternions, in: European Control Conference (ECC), Zurich, 2013, IEEE, pp. 329–334.
[4] Nuno Filipe, Panagiotis Tsiotras, Adaptive position and attitude-tracking controller for satellite proximity operations using dual quaternions, J. Guid. Control Dyn. (2014) 1–12.
[5] Janez Funda, Richard P. Paul, A computational analysis of screw transformations in robotics, IEEE Trans. Robot. Autom. 6 (3) (1990) 348–356.
[6] Janez Funda, Russell H. Taylor, Richard P. Paul, On homogeneous transforms, quaternions, and computational efficiency, IEEE Trans. Robot. Autom. 6 (3) (1990) 382–388.
[7] Dapeng Han, Qing Wei, Zexiang Li, Weimeng Sun, Control of oriented mechanical systems: a method based on dual quaternion, in: 17th IFAC World Congress, Seoul, 2008, pp. 3836–3841.
[8] R. Ortega, L. Hsu, A. Astolfi, Immersion and invariance adaptive control of linear multivariable systems, Syst. Control Lett. 49 (1) (2003) 37–47.
[9] Hoang-Lan Pham, Véronique Perdereau, Bruno Vilhena Adorno, Philippe Fraisse, et al., Position and orientation control of robot manipulators using dual quaternion feedback, in: IROS'10: International Conference on Intelligent Robots and Systems, 2010.
[10] H. Schaub, J.L. Junkins, Analytical mechanics of space systems, in: AIAA Education Series, 2003 (Chapter 3).
[11] D. Seo, M.R. Akella, High-performance spacecraft adaptive attitude-tracking control through attracting-manifold design, J. Guid. Control Dyn. 31 (4) (2008) 884–891.
[12] Jianying Wang, Zhaowei Sun, 6-dof robust adaptive terminal sliding mode control for spacecraft formation flying, Acta Astronaut. 73 (0) (2012) 76–87.
[13] Xiangke WangDapeng HanChangbin Yu, Zhiqiang Zheng, The geometric structure of unit dual quaternion with application in kinematic control, J. Math. Anal. Appl. 389 (2) (2012) 1352–1364.
[14] Xiangke Wang, C. Yu, Unit-dual-quaternion-based pid control scheme for rigid-body transformation, in: Proceedings of 18th World Congress International Federation of Automatic Control, 2011, pp. 9296–9301.
[15] Xiangke Wang, Changbin Yu, Unit dual quaternion-based feedback linearization tracking problem for attitude and position dynamics, Syst. Control Lett. 62 (3) (2013) 225–233.
[16] Yuanxin Wu, Xiaoping Hu, Dewen Hu, Tao Li, Junxiang Lian, Strapdown inertial navigation system algorithms based on dual quaternions, IEEE Trans. Aerospace Electron. Syst. 41 (1) (January 2005) 110–132.