SpaceOps 2010 Conference
 <i>Delivering on the Dream</i>
 <i>Hosted by NASA Mars
25 - 30 April 2010, Huntsville, Alabama

AIAA 2010-1982

Attitude Control of Miniature Satellites Using Movable Masses

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The present paper presents an attitude control scheme for a miniature satellite using movable masses. The system comprises of a rigid satellite and a movable mass actuator. The nonlinear equations of motion of the system are derived using the Lagrangian approach. The control laws for motion of the movable masses are designed based on the linear-quadratic regulator technique to achieve desired satellite attitude. In order to test the performance of the proposed actuator, the governing nonlinear system is numerically simulated in conjunction with the control laws for pico-class satellites (less than 1 kg). Results of the numerical simulation state that it is feasible to stabilize the attitude of a miniature satellite using movable masses. The proposed control actuator can be useful for future space missions involving miniature satellites.

I. Introduction

The attitude control of a rigid spacecraft has been relatively well investigated (see, e.g., Refs. [1]-[12] and references therein). Several methods of attitude control have been proposed; these include momentum wheels/reaction wheels, control moment gyros, magnetic torquers, solar radiation pressure, aerodynamics forces, and movable masses. In this paper, we focus on the application of a movable mass actuator. Childs[10] proposed a control scheme for an asymmetric spacecraft using a movable mass. Kunciw and Kaplan[11] investigated the problem of the use of a movable mass control system to stabilize a tumbling asymmetric spacecraft about the maximum inertia axis, and the gradient optimization technique was applied to minimize angular velocity components along the intermediate and minimum inertia axes. In Ref. [12], a movable mass control system based on Lyapunov stability theory has been developed to convert the tumbling motions of a disabled vehicle into simple spin.

This paper investigates the problem of attitude stabilization of a miniature satellite using movable masses. The considered system comprises of a rigid satellite and a movable mass actuator. The nonlinear equations of motion of the system are derived using the Lagrangian approach. The linear-quadratic regulator technique is applied to develop the control laws for motion of the movable masses. In order to test the performance of the proposed actuator, the governing nonlinear system is numerically simulated in conjunction with the control laws for pico-class satellites (less than 1 kg).

This paper is organized as follows. The system equations of motion for a satellite with movable masses are described in Section II. The controller design using linear-quadratic regulator (LQR) approach is discussed in Section III. Simulation results are presented in Section IV followed by conclusions in Section V.

II. Equations of motion

The system consists of a rigid satellite and a control mass. The equations of motion for this system are given by [12]

$$J_1\dot{\omega}_1 = (J_2 - J_3)\omega_2\omega_3 + zf_y - yf_z \tag{1}$$
$$J_2\dot{\omega}_2 = (J_3 - J_1)\omega_3\omega_1 + xf_z - zf_x \tag{2}$$
$$J_3\dot{\omega}_3 = (J_1 - J_2)\omega_1\omega_2 + yf_x - xf_y \tag{3}$$

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where $\omega_1, \omega_2, \omega_3$ are the angular velocity components of the satellite with respect to an inertial frame I and expressed in the body frame B_l; J_1, J_2, J_3 are the principal moments of inertia of the satellite; f_x, f_y, f_z represent the force acting on the control mass expressed in component form as

$$f_x = \mu\left(\ddot{x} - 2\dot{y}\omega_3 + 2\dot{z}\omega_2 - y\dot{\omega}_3 + z\dot{\omega}_2 + y\omega_1\omega_2 + z\omega_1\omega_3 - x(\omega_2^2 + \omega_3^2)\right) \tag{4}$$

$$f_y = \mu\left(\ddot{y} - 2\dot{z}\omega_1 + 2\dot{x}\omega_3 - z\dot{\omega}_1 + x\dot{\omega}_3 + z\omega_2\omega_3 + x\omega_1\omega_2 - y(\omega_1^2 + \omega_3^2)\right) \tag{5}$$

$$f_z = \mu\left(\ddot{z} - 2\dot{x}\omega_2 + 2\dot{y}\omega_1 - x\dot{\omega}_2 + y\dot{\omega}_1 + x\omega_1\omega_3 + y\omega_2\omega_3 - z(\omega_1^2 + \omega_2^2)\right) \tag{6}$$

where μ is the reduced mass defined by

$$\mu = \frac{mM}{m+M} \tag{7}$$

with m and M the control and satellite masses, respectively.

The attitude kinematics is described as

$$\begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -\sin\phi \\ 0 & \cos\gamma & \sin\gamma\cos\phi \\ 0 & -\sin\gamma & \cos\gamma\cos\phi \end{pmatrix} \begin{pmatrix} \dot{\gamma} \\ \dot{\phi} \\ \dot{\alpha} \end{pmatrix} + \dot{\theta} \begin{pmatrix} -\sin\phi \\ \sin\gamma\cos\phi \\ \cos\gamma\cos\phi \end{pmatrix} \tag{8}$$

where α (pitch), ϕ (roll), and γ (yaw) describe the attitude orientation of the satellite body-fixed reference frame relative to the local vertical local horizontal (LVLH) orbital reference frame.

Here, we assume the mass is restricted to move along a track parallel to the z axis ($x = a$ and $y = b$). Thus, applying Eqs. (4) and (5) into Eqs. (1)-(3) results in

$$\begin{aligned}(J_1 + \mu z^2)\dot{\omega}_1 - \mu a z \dot{\omega}_3 &= \mu z\left(-2\dot{z}\omega_1 + z\omega_2\omega_3 + a\omega_1\omega_2 - b(\omega_1^2 + \omega_3^2)\right) \\ &\quad + (J_2 - J_3)\omega_2\omega_3 - b f_z \end{aligned} \tag{9}$$

$$\begin{aligned}(J_2 + \mu z^2)\dot{\omega}_2 - \mu b z \dot{\omega}_3 &= -\mu z\left(2\dot{z}\omega_2 + b\omega_1\omega_2 + z\omega_1\omega_3 - a(\omega_2^2 + \omega_3^2)\right) \\ &\quad + (J_3 - J_1)\omega_3\omega_1 + a f_z \end{aligned} \tag{10}$$

$$\begin{aligned}-\mu a z\dot{\omega}_1 - \mu b z\dot{\omega}_2 + (J_3 + \mu(a^2 + b^2))\dot{\omega}_3 &= \mu b\left(2\dot{z}\omega_2 + b\omega_1\omega_2 + z\omega_1\omega_3 - a(\omega_2^2 + \omega_3^2)\right) \\ &\quad - \mu a\left(-2\dot{z}\omega_1 + z\omega_2\omega_3 + a\omega_1\omega_2 - b(\omega_1^2 + \omega_3^2)\right) \\ &\quad + (J_1 - J_2)\omega_1\omega_2 \end{aligned} \tag{11}$$

III. Controller Design using Linear-Quadratic Regulator (LQR)

For a continuous-time linear system described by

$$\dot{X} = AX + Bu \tag{12}$$

where $X \in R^n$ is the state vector of order n, and $u \in R^m$ is the control input vector of order m. The objective of the LQR problem is to find an optimal control law for $u(t)$ such that the state $X(t)$ is driven into a (small) neighborhood of the origin while minimizing a quadratic performance (L_2 performance) index on u and X. According to the LQR approach, we can design a control law by minimizing the cost functional defined as

$$J = \int_0^\infty \left(X^T Q X + u^T R u\right) dt \tag{13}$$

where $Q \in R^{n\times n}$ is a symmetric positive semi-definite matrix, and $R \in R^{m\times m}$ is a symmetric positive definite matrix. The feedback control law that minimizes the value of the cost is given by

$$u = -KX \tag{14}$$

where K is given by

$$K = R^{-1}B^T P \tag{15}$$

and P is found by solving the continuous time algebraic Riccati equation

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \tag{16}$$

For the use of the LQR to design the controller, we need to linearize the system. Consider the equilibrium point as: $\gamma = \dot{\gamma} = \phi = \dot{\phi} = \omega_1 = \omega_2 = z = \dot{z} = 0, \omega_3 = \Omega$. Then, Eqs (6), (8), (9) and (10) can be linearized about the equilibrium point as:

$$\delta\ddot{z} = a\delta\dot{\omega}_2 - b\delta\dot{\omega}_1 - a\Omega\delta\omega_1 - b\Omega\delta\omega_2 + \frac{f_z}{\mu} \tag{17}$$

$$\delta\omega_1 = \delta\dot{\gamma} - \delta\phi\dot{\theta} \tag{18}$$

$$\delta\omega_2 = \delta\dot{\phi} + \delta\gamma\dot{\theta} \tag{19}$$

$$\delta\dot{\omega}_1 = -\frac{\mu b\Omega^2}{J_1}\delta z + \frac{J_2 - J_3}{J_1}\Omega\delta\omega_2 - \frac{b}{J_1}f_z \tag{20}$$

$$\delta\dot{\omega}_2 = \frac{\mu a\Omega^2}{J_2}\delta z + \frac{J_3 - J_1}{J_2}\Omega\delta\omega_1 + \frac{a}{J_2}f_z \tag{21}$$

Applying Eqs. (18) and (19) into Eqs. (20) and (21) yields

$$\delta\ddot{\gamma} = -\frac{\mu b\Omega^2}{J_1}\delta z + \delta\dot{\phi}\dot{\theta} + \frac{J_2 - J_3}{J_1}\Omega\left(\delta\dot{\phi} + \delta\gamma\dot{\theta}\right) - \frac{b}{J_1}f_z \tag{22}$$

$$\delta\ddot{\phi} = \frac{\mu a\Omega^2}{J_2}\delta z - \delta\dot{\gamma}\dot{\theta} + \frac{J_3 - J_1}{J_2}\Omega\left(\delta\dot{\gamma} - \delta\phi\dot{\theta}\right) + \frac{a}{J_2}f_z \tag{23}$$

Applying Eqs. (22) and (23) into Eq. (17) results in

$$\begin{aligned}\delta\ddot{z} = {} & \frac{J_1 a^2\Omega^2 + J_2 b^2\Omega^2}{J_1 J_2}\delta z - \frac{J_2 + J_3 - J_1}{J_2}a\Omega\left(\delta\dot{\gamma} - \delta\phi\dot{\theta}\right) - \frac{J_1 + J_2 - J_3}{J_1}b\Omega\left(\delta\dot{\phi} + \delta\gamma\dot{\theta}\right) \\ & + \left(\frac{1}{\mu} - \frac{a}{J_2} + \frac{b}{J_1}\right)f_z\end{aligned} \tag{24}$$

The constant Ω can be computed by the following equality

$$J_3^2\Omega^2 = J_1^2\omega_1^2(0) + J_2^2\omega_2^2(0) + J_3^2\omega_3^2(0) \tag{25}$$

where $\omega_i(0)$ represents the initial angular velocity ω_i.

Define $X = (\delta\gamma, \delta\phi, \delta z, \delta\dot{\gamma}, \delta\dot{\phi}, \delta\dot{z})^T$, then we have

$$\dot{X} = AX + Bu \tag{26}$$

where

$$\begin{aligned} A &= \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{J_2-J_3}{J_1}\Omega\dot{\theta} & 0 & -\frac{\mu b\Omega^2}{J_1} & 0 & \frac{J_2-J_3}{J_1}\Omega + \dot{\theta} & 0 \\ 0 & -\frac{J_3-J_1}{J_2}\Omega\dot{\theta} & \frac{\mu a\Omega^2}{J_2} & \frac{J_3-J_1}{J_2}\Omega - \dot{\theta} & 0 & 0 \\ -\frac{J_1+J_2-J_3}{J_1}b\Omega\dot{\theta} & \frac{J_2+J_3-J_1}{J_2}a\Omega\dot{\theta} & \frac{J_1a^2\Omega^2+J_2b^2\Omega^2}{J_1J_2} & -\frac{J_2+J_3-J_1}{J_2}a\Omega & -\frac{J_1+J_2-J_3}{J_1}b\Omega & 0 \end{pmatrix} \\ B &= \begin{pmatrix} 0 \\ 0 \\ 0 \\ -\frac{b}{J_1} \\ \frac{a}{J_2} \\ \frac{1}{\mu} + \frac{a^2}{J_2} + \frac{b^2}{J_1} \end{pmatrix} \\ u &= f_z \end{aligned} \tag{27}$$

IV. Results and Discussions

In order to verify the performance of the proposed control laws, numerical simulations of the governing equations of motion of the system Eqs. (8)-(11) in conjunction with the control laws Eqs. (14), are performed. The initial attitude angels are considered as: $\alpha(0) = \gamma(0) = \phi(0) = 20$ deg. The initial angular velocities are taken as: $\omega_1(0) = -0.000286$ rad/s, $\omega_2 = -0.199$ rad/s, $\omega_3 = 0.103$ rad/s. The positive definite matrix Q is set equal to $Q = \mathrm{diag}(2.5, 2.5, 2.5, 2.5, 2.5, 2.5)$, and R is considered to be $R = 100$.

We consider the satellite to be a pico-class satellite. The numerical parameters of the pico-satellite are given as: $M = 1$kg, $J_1 = 0.0015$kg m^2, $J_2 = 0.0017$kg m^2, $J_3 = 0.0030$kg m^2. For this satellite, the control mass track offset distances are taken as $a = 0.1$m and $b = 0.05$m, respectively. The control mass is considered to be $m = 0.01$ kg for this satellite, which corresponded to 1% of the satellite mass, and the initial value of z is set to $z(0) = 0$ m. The control saturation limit is considered to be $|f_z| \leq 0.001$ N. The results using the proposed controller are shown in Figs. 1-4. Figure 1 shows the attitude angles γ and ϕ. It is observed that the proposed control law using a moving mass can stabilize the attitude angles γ and ϕ. Figure 2 shows the response of the position z, and it can be observed that the maximum mass amplitude is about 0.18 m. Figure 3 provides the response of the angular velocities, and it is found the control system using a moving mass effectively force the angular velocities ω_1 and ω_2 to zero, as the angular velocity ω_3 increases to its steady spin value, consistent with constant total angular momentum. The bounded control force is shown in Fig. 4. In summary, for a pico-satellite, a moving mass system is capable of stabilizing the satellite attitude about the maximum inertia axis, which can be accomplished with 1% of the satellite mass and a maximum displacement amplitude of approximately 0.18 m.

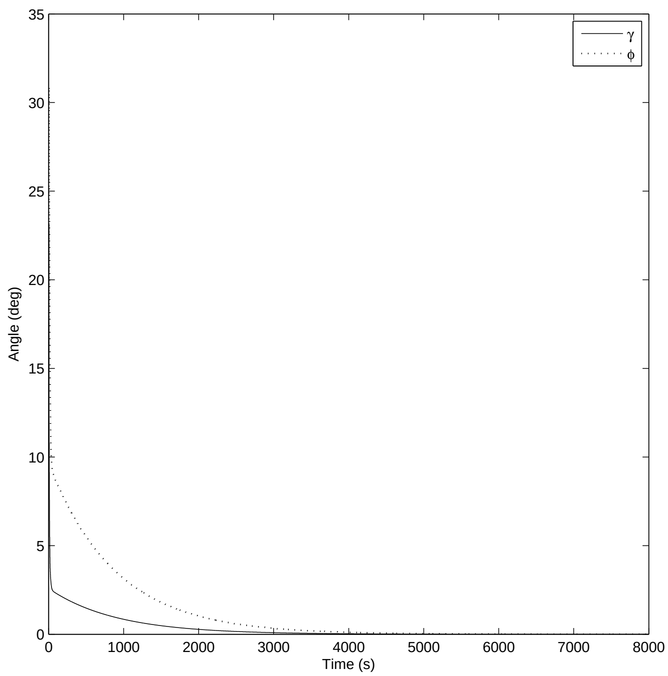

Figure 1. Response of angles γ and ϕ.

V. Conclusions

In the present paper, a attitude-stabilization control law using movable masses is proposed for a miniature satellite. The system is considered to be a rigid satellite with attached control mass. The nonlinear equations of motion of the system are derived by use of the Lagrangian approach. The moving mass control laws are developed based on the LQR technique to achieve satellite attitude stabilization. In order to test the

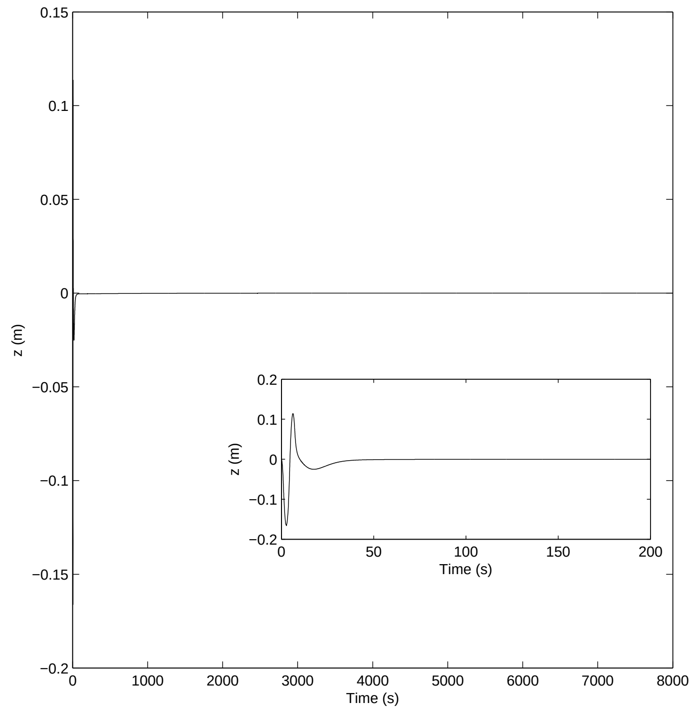

Figure 2. Response of the control mass position z.

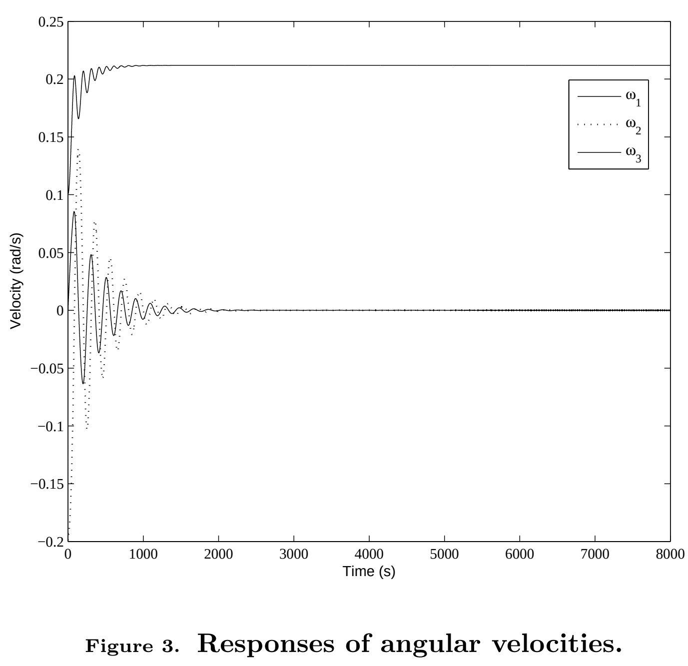

Figure 3. Responses of angular velocities.

performance of the proposed actuator, the governing nonlinear system is numerically simulated in conjunction with the control laws for pico-class satellites (less than 1 kg). Numerical simulations have been demonstrated the performance of the proposed control algorithm. The future work is to investigate the problem of attitude stabilization of a miniature satellite using two and three moving masses with consideration of external

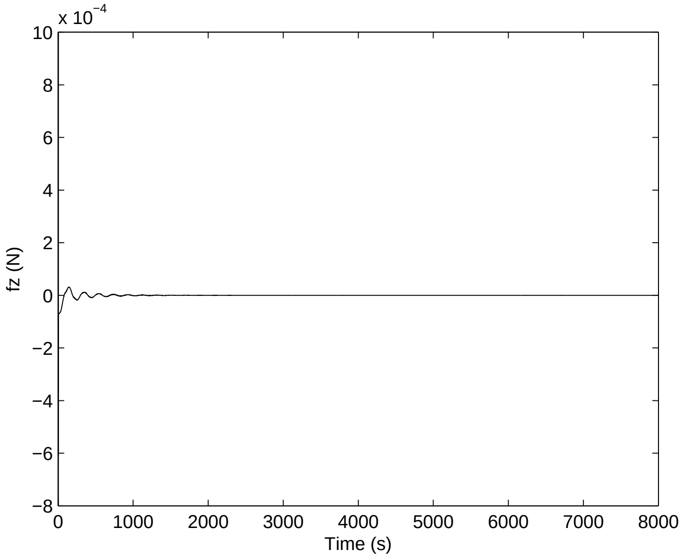

Figure 4. The applied control force f_z.

disturbances, variations in system parameters by using the nonlinear sliding mode control approach.

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