Inverse Kinematics

r = wheel radius

b = track width

Given: Twist 2D

Return: wheel velocities

$$\text{Wheel Velocities} = \begin{bmatrix} \dot{\phi}_l \\ \dot{\phi}_r \end{bmatrix} = \frac{1}{r} \begin{bmatrix} -b & 1 & 0 \\ b & 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ V_x \\ V_y \end{bmatrix}$$

Twist

$$= \frac{1}{r} \begin{bmatrix} -b\dot{\theta} + V_x \\ b\dot{\theta} + V_y \end{bmatrix}$$

Forward Kinematics

r = Wheel radius

D = track width

Given: New wheel positions
Current wheel positions

Return: New robot configuration

1. Wheel velocities = New wheel position − old wheel position

$$\begin{bmatrix} \dot{\phi}_l \\ \dot{\phi}_r \end{bmatrix} = \begin{bmatrix} \phi_{l,2} \\ \phi_{r,2} \end{bmatrix} - \begin{bmatrix} \phi_{l,1} \\ \phi_{r,1} \end{bmatrix}$$

2. from inverse kinematics:

$$\dot{\phi}_l + \dot{\phi}_r = \frac{-D\dot{\theta} + V_x}{r} + \frac{D\dot{\theta} + V_x}{r} = \frac{2\dot{\theta}}{r}$$

$$\hookrightarrow V_x = \frac{1}{2} r (\dot{\phi}_l + \dot{\phi}_r)$$

$$\dot{\phi}_r - \dot{\phi}_l = \frac{D\dot{\theta} + V_x}{r} + \frac{D\dot{\theta} - V_x}{r} = \frac{2D\dot{\theta}}{r}$$

$$\dot{\theta} = \frac{r}{2D} (\dot{\phi}_r - \dot{\phi}_l)$$

$$V_b = \begin{bmatrix} \dot{\theta} \\ \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} \frac{r}{2D} (\dot{\phi}_r - \dot{\phi}_l) \\ \frac{r}{2} (\dot{\phi}_l + \dot{\phi}_r) \\ 0 \end{bmatrix}$$

3. $\int V_b \, dt$ = Transform to new positions

4. $T_{wb'} = T_{wb} T$