$k = 120\ W/m.k$

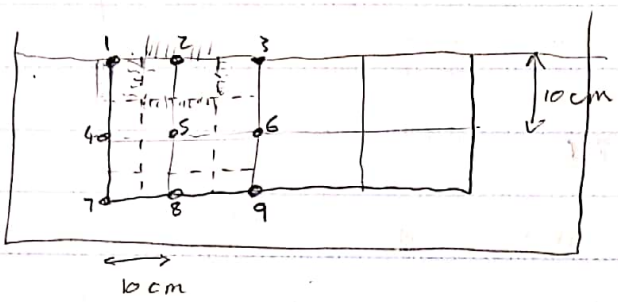
$\alpha = 3,91 \times 10^{-6}\ m^2/s$

$T_0 = 800^\circ C = (800 + 273,15)\ Kelvin$

$T_\infty = 15^\circ C = (15 + 273,15)\ Kelvin$

$h_w = 100\ W/m^2.k$

$h_{air} = 10\ W/m^2.k$



calculate to 10 mins = 600 sec

1, 2, 3, 4, 7, 8, 9 : boundary

5, 6 : internal.

time step : $\Delta t = 10\ sec.$

mesh size : $\Delta x = \Delta y = 10\ cm.$

$$\tau = \frac{\alpha \Delta t}{\Delta x^2} = 0,00391 \quad \checkmark \text{stable.}$$

→ Work Difference equations for each node:

node 1: quarter volume element:

$$h_{air} \frac{\Delta x}{2} (T_\infty - T_1^i) + h_{water} \frac{\Delta y}{2} (T_\infty - T_1^i) + k \frac{\Delta x}{2} \left(\frac{T_4^i - T_1^i}{\Delta y} \right) + k \frac{\Delta y}{2} \left(\frac{T_2^i - T_1^i}{\Delta x} \right)$$

$$= \rho \frac{\Delta x}{2} \frac{\Delta y}{2} C_p \left(\frac{T_1^{i+1} - T_1^i}{\Delta t} \right)$$

node 2: $h_{air} \Delta x (T_\infty - T_2^i) + k \frac{\Delta y}{2} \left(\frac{T_3^i - T_2^i}{\Delta x} \right) + k \frac{\Delta y}{2} \left(\frac{T_1^i - T_2^i}{\Delta x} \right) + k \Delta x \left(\frac{T_5^i - T_2^i}{\Delta y} \right)$

$$= \rho \Delta x \frac{\Delta y}{2} C_p \left(\frac{T_2^{i+1} - T_2^i}{\Delta t} \right)$$

node 3: $h_{air} \frac{\Delta x}{2} (T_\infty - T_3^i) + k \frac{\Delta y}{2} \left(\frac{T_2^i - T_3^i}{\Delta x} \right) + k \frac{\Delta x}{2} \left(\frac{T_6^i - T_3^i}{\Delta y} \right)$

$$= \rho \frac{\Delta x}{2} \frac{\Delta y}{2} C_p \left(\frac{T_3^{i+1} - T_3^i}{\Delta t} \right)$$

node 4: $h_{water}\,\Delta y\,(T_\infty - T_4^i) + k\frac{\Delta x}{2}\left(\frac{T_7^i - T_4^i}{\Delta y}\right) + k\frac{\Delta x}{2}\left(\frac{T_1^i - T_4^i}{\Delta y}\right) + k\,\Delta y\left(\frac{T_5^i - T_4^i}{\Delta x}\right)$

$$= \rho\left(\Delta y\,\frac{\Delta x}{2}\right) \times C_p \times \left(\frac{T_4^{i+1} - T_4^i}{\Delta t}\right)$$

node 5: $k\,\Delta x\,(T_2^i - T_5^i) + k\,\Delta y\,(T_6^i - T_5^i) + k\,\Delta x\,(T_8^i - T_5^i) + k\,\Delta y\,(T_4^i - T_5^i)$

$$= \rho\,(\Delta x\ \Delta y)\,C_p \times \left(\frac{T_5^{i+1} - T_5^i}{\Delta t}\right)$$

node 6: $k\frac{\Delta x}{2}\left(\frac{T_1^i - T_6^i}{\Delta y}\right) + k\frac{\Delta x}{2}\left(\frac{T_3^i - T_6^i}{\Delta y}\right) + k\Delta y\,(T_5^i - T_6^i)$

$$= \rho\left(\Delta y\,\frac{\Delta x}{2}\right) C_p\,\left(\frac{T_6^{i+1} - T_6^i}{\Delta t}\right)$$

node 7: $h_{water}\frac{\Delta y}{2}(T_\infty - T_7^i) + h_{water}\frac{\Delta x}{2}(T_\infty - T_7^i) + k\frac{\Delta x}{2}\left(\frac{T_{4i} - T_7^i}{\Delta y}\right)$

$$+ k\frac{\Delta y}{2}\frac{(T_3^i - T_7^i)}{\Delta x} = \rho\left(\frac{\Delta y}{2}\frac{\Delta x}{2}\right) C_p\,\frac{(T_7^{i+1} - T_7^i)}{\Delta t}$$

node 8: $h_{water}\,\Delta x\,(T_\infty - T_8^i) + k\frac{\Delta y}{2}\frac{(T_7^i - T_8^i)}{\Delta x} + k\,\Delta x\,\frac{(T_5^i - T_8^i)}{\Delta y}$

$$+ k\frac{\Delta y}{2}\left(\frac{T_4^i - T_8^i}{\Delta x}\right) = \rho\left(\frac{\Delta y}{2}\,\Delta x\right) C_p\,\frac{(T_8^{i+1} - T_8^i)}{\Delta t}$$

node 9: $h_{water}\frac{\Delta x}{2}(T_\infty - T_9^i) + k\frac{\Delta y}{2}\frac{(T_8^i - T_9^i)}{\Delta x} + k\frac{\Delta x}{2}\frac{(T_6^i - T_9^i)}{\Delta y}$

$$= \rho\left(\frac{\Delta y}{2}\frac{\Delta x}{2}\right) C_p\,\frac{(T_9^{i+1} - T_9^i)}{\Delta t}$$

Node 1:

$$h_{air}\frac{\Delta x}{2}(T_\infty - T_1^i) + h_{wat}\frac{\Delta y}{2}(T_\infty - T_1^i) + k\frac{\Delta x}{2}\left(\frac{T_4^i - T_1^i}{\Delta y}\right) + k\frac{\Delta y}{2}\left(\frac{T_2^i - T_1^i}{\Delta x}\right) = \rho\frac{\Delta x}{2}\frac{\Delta y}{2}C_p\left(\frac{T_1^{i+1} - T_1^i}{\Delta t}\right)$$

$$\Rightarrow (10)\left(\frac{0.1}{2}\right)(15 - T_1^i) + (100)\left(\frac{0.1}{2}\right)(15 - T_1^i) + 120\left(\frac{0.1}{2}\right)\left(\frac{T_4^i - T_1^i}{0.1}\right) + (120)\left(\frac{0.1}{2}\right)\left(\frac{T_2^i - T_1^i}{0.1}\right) = \frac{\rho C_p \ell^2}{4\Delta t}\frac{k}{4\ell}T_1^{i+1} - T_1^i$$

$$\Rightarrow 7.5 - 0.5T_1^i + 75 - 5T_1^i + 60T_4^i - 60T_1^i + 60T_2^i - 60T_1^i = \frac{k}{4\ell}T_1^{i+1} - \frac{k}{4\ell}T_1^i$$

$$7672.63\left(T_1^{i+1} - T_1^i\right)$$

$$\Rightarrow 7672.63T_1^{i+1} - 7672.63T_1^i$$

But $\rho_{cp} = \frac{k}{\alpha} \Rightarrow \rho_{cp} = 30670537.08$

$$\Rightarrow 7672.63T_1^{i+1} = -125.5T_1^i + 60T_2^i + 60T_4^i + 82.5 + 7672.63T_1^i$$

$$T_1^{i+1} = \frac{7547.13T_1^i + 60T_2^i + 60T_4^i + 82.5}{7672.63}$$

$$T_1^{i+1} = 0.984T_1^i + 0.0073T_2^i + 0.0073T_4^i + 0.0108$$

Node 2:

$$(10)(0.1)(15 - T_1^{\,i}) + 120\left(\frac{0.1}{2}\right)\left(\frac{T_3^{\,i} - T_2^{\,i}}{0.1}\right) + 120\left(\frac{0.1}{2}\right)\left(\frac{T_1^{\,i} - T_2^{\,i}}{0.1}\right)$$

$$+ 120(0.1)\left(\frac{15^{\,i} - T_2^{\,i}}{0.1}\right) = \rho \Delta x \frac{\Delta y}{2} C_p \left(\frac{T_2^{\,i+1} - T_2^{\,i}}{10}\right)$$

But $\rho C_p = \frac{k}{\alpha} = 30690537.08$

$$\Rightarrow \quad 15 - T_1^{\,i} + 60T_3^{\,i} - 60T_2^{\,i} + 60T_1^{\,i} - 60T_2^{\,i} + 120T_5^{\,i} - 120T_2^{\,i} = 15345.269\,T_2^{\,i+1} - 15345.269\,T_2^{\,i}$$

$$15345.269\,T_2^{\,i+1} = 15 - T_1^{\,i} + 60T_3^{\,i} - 60T_2^{\,i} + 60T_1^{\,i} - 60T_2^{\,i} + 120T_5^{\,i} - 120T_2^{\,i} + 15345.269\,T_2^{\,i}$$

$$T_2^{\,i+1} = 0.00033\,T_1^{\,i} + 0.9844\,T_2^{\,i} + 0.00033\,T_3^{\,i} + 0.00073\,T_5^{\,i} + 0.00078$$

Node 3:

$$(10)\left(\frac{0.1}{2}\right)(15 - T_3^i) + (120)\left(\frac{0.1}{2}\right)\left(\frac{T_2^i - T_3^i}{0.1}\right) + (120)\left(\frac{0.1}{2}\right)\left(\frac{T_6^i - T_3^i}{0.1}\right) = \rho \frac{\Delta x}{2} \frac{\Delta y}{2} C_p \left(\frac{T_3^{i+1} - T_3^i}{10}\right)$$

But $\rho C_p = \frac{k}{\alpha} = 30690537.08$

$$\Rightarrow 0.5(15 - T_3^i) + 60(T_2^i - T_3^i) + 60(T_6^i - T_3^i) = 7672.63\, T_3^{i+1} - 7672.63\, T_3^i$$

$$7.5 - 0.5T_3^i + 60T_2^i - 60T_3^i + 60T_6^i - 60T_3^i = 7672.63T_3^{i+1} - 7672.63T_3^i$$

$$-120.5T_3^i + 60T_2^i + 60T_6^i + 7.5 = 7672.63T_3^{i+1} - 7672.63T_3^i$$

$$7672.63T_3^{i+1} = 7552.13T_3^i + 60T_2^i + 60T_6^i + 7.5$$

$$T_3^{i+1} = \frac{60T_2^i + 7552.13T_3^i + 60T_6^i + 7.5}{7672.63}$$

$$T_3^{i+1} = 0.00782\,T_2^i + 0.98429\,T_3^i + 0.00782\,T_6^i + 0.000989$$

Node 4

$$(100)(0.1)(15 - T_4^i) + (120)\left(\frac{0.1}{2}\right)\left(\frac{T_7^i - T_4^i}{0.1}\right) + (120)\left(\frac{0.1}{2}\right)\left(\frac{T_1^i - T_4^i}{0.1}\right)$$

$$+(120)(0.1)\left(\frac{T_5^i - T_4^i}{0.1}\right) = \rho(\Delta y)\left(\frac{\Delta x}{2}\right)(C_p)\left(\frac{T_4^{i+1} - T_4^i}{10}\right)$$

But $\rho C_p = \frac{k}{\alpha} = 30690537.08$

$$150 - 10T_4^i + 60T_7^i - 60T_4^i + 60T_1^i - 60T_4^i + 120T_5^i - 120T_4^i$$
$$= 15345.27\,T_4^{i+1} - 15345.27\,T_4^i$$

$$150 + 15095.27\,T_4^i + 120T_5^i + 60T_1^i + 60T_7^i = 15345.27\,T_4^{i+1}$$

$$T_4^{i+1} = \frac{60T_1^i + 15095.27\,T_4^i + 120T_5^i + 60T_7^i + 150}{15345.27}$$

$$T_4^{i+1} = 0.00391\,T_1^i + 0.98371\,T_4^i + 0.00782\,T_5^i + 0.00391\,T_7^i + 0.00977$$

Node 5

$$(120)(0.1)(T_2^i - T_5^i) + (120)(0.1)(T_6^i - T_5^i) + (120)(0.1)(T_8^i - T_5^i) + (120)(0.1)(T_4^i - T_5^i) = \rho \Delta x \Delta y C_p \left(\frac{T_5^{i+1} - T_5^i}{10}\right)$$

But $\rho C_p = \frac{k}{\alpha} = 30690537.08$

$$12T_2^i - 12T_5^i + 12T_6^i - 12T_5^i + 12T_8^i - 12T_5^i + 12T_4^i - 12T_5^i = 30690.54\,T_6^{i+1} - 30690.54\,T_5^i$$

$$12T_2^i + 12T_4^i + 30642.54\,T_5^i + 12T_6^i + 12T_8^i = 30690.54\,T_6^{i+1}$$

$$T_6^{i+1} = \frac{12T_2^i + 12T_4^i + 30642.54\,T_5^i + 12T_6^i + 12T_8^i}{30690.54}$$

$$= 0.00039\,T_2^i + 0.00039\,T_4^i + 0.99844\,T_5^i + 0.00039\,T_6^i + 0.00039\,T_8^i$$

Node 6:

$$(120)\left(\frac{0.1}{2}\right)\left(\frac{T_9^i - T_6^i}{0.1}\right) + (120)\left(\frac{0.1}{2}\right)\left(\frac{T_3^i - T_6^i}{0.1}\right) + (120)(0.1)(T_5^i - T_6^i)$$
$$= \rho \Delta y \frac{\Delta x}{2} C_p \left(\frac{T_6^{i+1} - T_6^i}{10}\right)$$

$$\rightarrow \rho C_p = 30690537.08$$

$$60T_9^i - 60T_6^i + 60T_3^i - 60T_6^i + 12T_5^i - 12T_6^i = 15345.27\,T_6^{i+1} - 15345.27\,T_6^i$$

$$15345.27\,T_6^{i+1} = 60T_3^i + 12T_5^i - 60T_6^i - 60T_6^i - 12T_6^i + 15345.27T_6^i + 60T_9^i$$

$$T_6^{i+1} = \frac{60T_3^i + 12T_5^i + 15213.27T_6^i + 60T_9^i}{15345.27}$$

$$T_6^{i+1} = 0.00391\,T_3^i + 0.000782\,T_5^i + 0.991398\,T_6^i + 0.00391\,T_9^i$$

Node 7:

$$(100)\left(\frac{0.1}{2}\right)(15 - T_7^i) + (100)\left(\frac{0.1}{2}\right)(15 - T_7^i) + (120)\left(\frac{0.1}{2}\right)\left(\frac{T_4^i - T_7^i}{0.1}\right) + (120)\left(\frac{0.1}{2}\right)\left(\frac{T_8^i - T_7^i}{0.1}\right) = \rho \frac{\Delta y}{2} \frac{\Delta x}{2} C_p \left(\frac{T_7^{i+1} - T_7^i}{10}\right)$$

$$\rho C_p = 3069053 7.08$$

$$75 - 5T_7^i + 75 - 5T_7^i + 60T_4^i - 60T_7^i + 60T_8^i - 60T_7^i = 7672.63\,T_7^{i+1} - 7672.63\,T_7^i$$

$$7672.63\,T_7^{i+1} = 60T_4^i - 5T_7^i - 5T_7^i - 60T_7^i - 60T_7^i + 7672.63\,T_7^i + 60T_8^i + 75 + 75$$

$$T_7^{i+1} = \frac{60T_4^i + 7342.63T_7^i + 60T_8^i + 150}{7672.63}$$

$$T_7^{i+1} = 0.00782\,T_4^i + 0.98306\,T_7^i + 0.00782\,T_8^i + 0.01955$$

Node 8:

$$(100)(0.1)(15 - T_8^i) + (120)\left(\frac{0.1}{2}\right)\left(\frac{T_7^i - T_8^i}{0.1}\right) + (120)(0.1)\left(\frac{T_5^i - T_8^i}{0.1}\right)$$

$$+ (120)\left(\frac{0.1}{2}\right)\left(\frac{T_7^i - T_8^i}{0.1}\right) = \rho \frac{\Delta y}{2} \frac{\Delta x}{I} C_p \left(\frac{T_8^{i+1} - T_8^i}{10}\right)$$

$$150 - 10T_8^i + 60T_7^i - 60T_8^i + 120T_5^i - 120T_8^i + 60T_7^i - 60T_8^i$$

$$= 15345.27\, T_8^{i+1} - 15345.27\, T_8^i$$

$$15345.27\, T_8^{i+1} = 120T_5^i + 60T_7^i - 10T_8^i - 60T_8^i - 120T_8^i - 60T_8^i$$

$$+ 15345.27\, T_8^i + 60T_7^i + 150$$

$$T_8^{i+1} = \frac{120T_5^i + 60T_7^i + 15075.27\, T_8^i + 60T_7^i + 150}{15345.27}$$

$$T_8^{i+1} : 0.007820\, T_5^i + 0.00391\, T_7^i + 0.98371\, T_8^i + 0.00391\, T_7^i$$

$$+ 150$$

Node 9:

$$(100)\left(\frac{0.1}{2}\right)(15 - T_9^i) + (120)\left(\frac{0.1}{2}\right)\left(\frac{T_8^i - T_9^i}{0.1}\right) + (120)\left(\frac{0.1}{2}\right)\left(\frac{T_6^i - T_9^i}{0.1}\right) = \rho \frac{\Delta y}{2} \frac{\Delta x}{2} C_p \left(\frac{T_9^{i+1} - T_9^i}{10}\right)$$

$$75 - 15T_9^i + 60T_8^i - 60T_9^i + 60T_6^i - 60T_9^i = 7672.63\,T_9^{i+1} - 7672.63\,T_9^i$$

$$7672.63\,T_9^{i+1} = 60T_6^i + 60T_8^i - 15T_9^i - 60T_9^i - 60T_9^i + 7672.63\,T_9^i + 75$$

$$T_9^{i+1} = \frac{60T_6^i + 60T_8^i + 7567.63T_9^i + 75}{7672.63}$$

$$T_9^{i+1} = 0.00782\,T_6^i + 0.00782\,T_8^i + 0.98631\,T_9^i + 0.00978$$