1. Let A = {x ∈ R: |x−1| < 1} and let B = {x ∈ R: |x−2| < 2}. Prove that A⊆B and find A′, B′ and B\A.
   Proof. Suppose a ∈ {x ∈ R:|x−1| < 1}.
   This means that a ∈ R and |a-1| < 1 so -1 < a ≤ 2.
   Consequently, a must also be 0 < a ≤ 4.
   Therefore, a ∈ {x ∈ R: |x−2| < 2}, so it follows that A⊆B.
   A' = {x ∈ R: |x−1| ≥ 1}, B' = {x ∈ R: |x−2| ≥ 2}
   B\A is equal to {x ∈ R: |x−2| < 2} since this set contains elements not within A.

2. a. A ⊂ B ⇔ A ∪ B = B
   $\Longrightarrow (A \cup B)' = B' \Longrightarrow$ (A ∪ B) ′ = B′
   Using Demorgan's Law
   $\Longrightarrow A' \cap B' = B' \Longrightarrow$ A′ ∩ B′ = B′
   $\Leftrightarrow B' \subset A' \Leftrightarrow$ B′ ⊂ A′
   b. By definition of ∩, A ∩ B = {x: x ∈ A ∧ x ∈ B}. Therefore, for A ∩ A' must be in both A and A', but A' is the set of all numbers not in A. Therefore, there will be nothing in the set resulting from A ∩ A', hence A ∩ A' = ∅.
   c. A ∪ A' = the universal set. It follows that if x exists in an arbitrary set X, then it must also exist in the universal set.

3. Let $x \in A\backslash(B \cap C)$.
   Then $x \in A$ but $x \notin B \cap C$.
   From this, $x \notin B$ or $x \notin$ C (since it does not belong to both B and C).
   Thus, either $x \in A \setminus B$ or $x \in A \setminus$ C. That is, $x \in (A \setminus B) \cup (A \setminus C)$.
   $x \in A \setminus (B \cap C) \Longrightarrow x \in (A\backslash B) \cup (A\backslash C)$.
   Therefore, $(A\backslash B) \cup (A\backslash C) \subseteq A \setminus (B \cap C)$.
   From this we can deduce that A \ (B ∩ C) = (A \ B) ∪ (A \ C).

4. a. X = {1,2} or {1,2,3}
   b. X = {3}
   c. X = {1, 2, 4}

5. Suppose $(x, y) \in$ (A ∪ B) × (C ∪ D).
   From this, $x \in$ A ∪ B, so $x \in$ A or $x \in$ B, and $y \in$ C ∪ D, so $y \in$ C or y ∈ D.
   => ($x \in$ A and $y \in$ C) or ($x \in$ B and $y \in$ C) or ($x \in$ A and $y \in$ D) or ($x \in$ B and $y \in$ D)
   => $(x, y) \in (A \times$ C) or $(x, y) \in$ (B × C) or $(x, y) \in (A \times$ D) or $(x, y) \in$ (B × D)
   => $(x, y) \in$ (A × C) ∪ (B × C) ∪ (A × D) ∪ (B × D)
   These steps can be performed in reverse with the exact same result, meaning that if $(x, y) \in$ (A ∪ B) × (C ∪ D), then $(x, y) \in$ (A × C) ∪ (B × C) ∪ (A × D) ∪ (B × D), and if $(x, y) \in$ (A × C) ∪ (B × C) ∪ (A × D) ∪ (B × D), then $(x, y) \in$ (A ∪ B) × (C ∪ D).
   Hence it is proved that (A ∪ B) × (C ∪ D) = (A × C) ∪ (B × C) ∪ (A × D) ∪ (B × D).