Experimental Analysis of BRDF Models - Supplemental

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This document is supplemental to the paper titled *Experimental Analysis of BRDF Models*, published at Eurographics Symposium on Rendering 2005. In the following section we list the equations for the BRDF models employed in our isotropic analysis. The rest of the document contains complete fitting results for the 100 isotropic materials, together with BRDF plots in the incidence plane, and sample images. For further information, please refer to the original paper, and our project webpage at `http://groups.csail.mit.edu/graphics/brdf`.

BRDF Models

The formulations of the 7 models we use in our analysis are listed below. We enforce the diffuse contribution to Lambertian and omit the diffuse terms in the listing. The correspondences between the model parameters and our indexed parameters are shown next to the name of each model.

Common Notations	
N	normal
V, (v_x, v_y, v_z)	outgoing vector (view)
L, (l_x, l_y, l_z)	incoming vector (light)
R	mirror reflection of L
H	half vector
δ	angle between N and H

- Ward $[p_0 = \alpha]$

$$K = \rho_s \cdot \frac{1}{\sqrt{(N \cdot L)(N \cdot V)}} \cdot \frac{\exp[-\tan^2 \delta / \alpha^2]}{4\pi\alpha^2}$$

- Ward-Duer $[p_0 = \alpha]$

$$K = \rho_s \cdot \frac{1}{(N \cdot L)(N \cdot V)} \cdot \frac{\exp[-\tan^2 \delta / \alpha^2]}{4\pi\alpha^2}$$

- Blinn-Phong $[p_0 = n]$

$$K = \rho_s \cdot \frac{n+2}{2\pi} \cos^n \delta$$

- Lafortune et al. $[p_0 = C_{xy}, p_1 = C_z, p_2 = n]$

$$K = \rho_s \cdot [C_{xy}(l_x v_x + l_y v_y) + C_z l_z v_z]^n \cdot \frac{n+2}{2\pi[max(|C_z|, |C_{xy}|)]^n}$$

  where the last term is added to provide an approximate normalization.

- Cook-Torrance $[p_0 = F_0, p_1 = m]$

$$K = \frac{\rho_s}{\pi} \frac{DG}{(N \cdot L)(N \cdot V)} Fresnel(F_0, V \cdot H)$$

  where $G = min\{1, \frac{2(N \cdot H)(N \cdot V)}{(V \cdot H)}, \frac{2(N \cdot H)(N \cdot L)}{(V \cdot H)}\}$ and $D = \frac{1}{m^2 \cos^4 \delta} e^{-[(tan\delta)/m]^2}$

- Ashikhmin-Shirley $[p_0 = F_0, p_1 = n]$

$$K = \frac{n+1}{8\pi} \frac{(N \cdot H)^n}{(V \cdot H) max((N \cdot L), (N \cdot V))} Fresnel(F_0, V \cdot H)$$

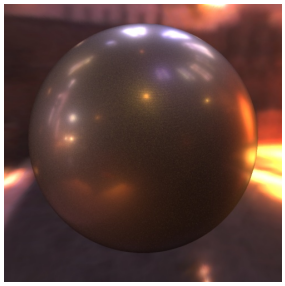
- He et al. $[p_0 = \tau, p_1 = \sigma, p_2 = n]$
  Please refer to the original paper for the full formulation. The expression τ is the auto-correlation length, σ is the standard deviation of surface height, and n is the refractive index. We employ the unpolarized version of the model, and the ideal specular term is omitted.

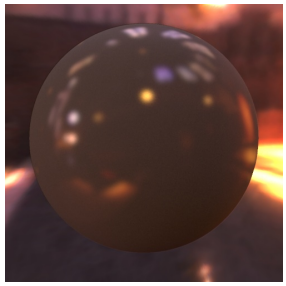
Material Name: alum-bronze

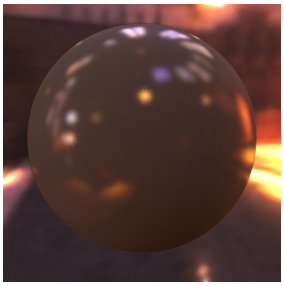
Fitted Parameters/Error

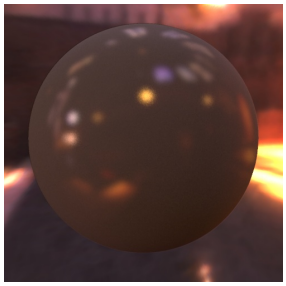
Model	d_r	d_g	d_b	s_r	s_g	s_b	p_0	p_1	p_2	Error
Ward	0.0851	0.0586	0.0294	0.0484	0.0274	0.0135	0.0357			0.0568
Ward-Duer	0.0856	0.0585	0.0291	0.032	0.0189	0.00986	0.0374			0.0454
Blinn-Phong	0.0864	0.0597	0.0302	0.015	0.00818	0.00381	1.29e+003			0.0658
Lafortune et al.	0.0923	0.0625	0.0311	0.322	0.193	0.105	-0.579	0.574	630	0.0486
Cook-Torrance	0.0886	0.0602	0.0299	0.118	0.0698	0.0371	0.203	0.0367		0.0434
He et al.	0.0866	0.0589	0.0292	0.165	0.0994	0.0532	24	0.514	2.33	0.0431
Ashikhmin-Shirley	0.0877	0.0598	0.0297	0.108	0.0637	0.0337	0.249	1.35e+003		0.0434

Rendered Images


Reference


Ward: 0.0568


Ward-Duer: 0.0454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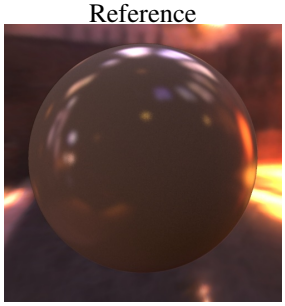
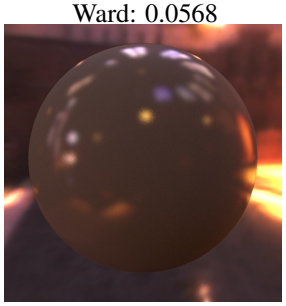
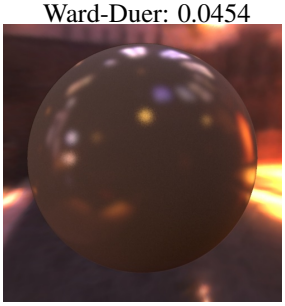
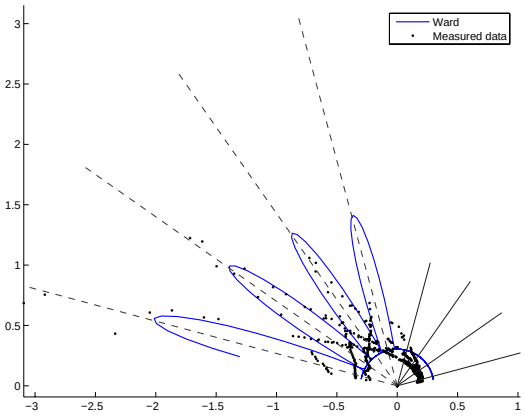

Lafortune: 0.0486


CT: 0.0434

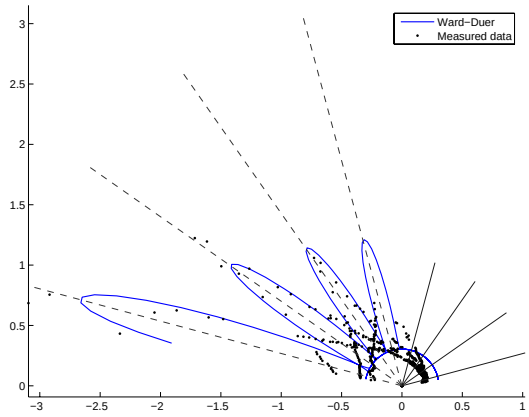

He: 0.0431


Ash: 0.0434

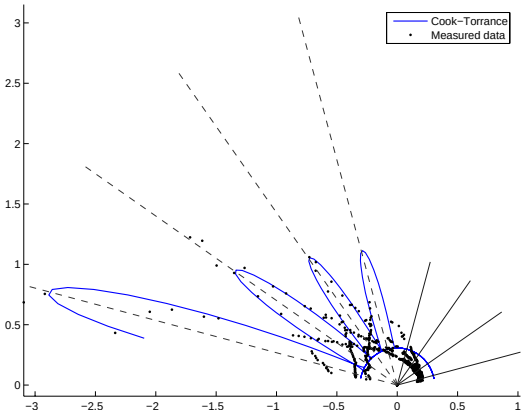
BRDF Plots in the incidence plane (Cubic root applied)



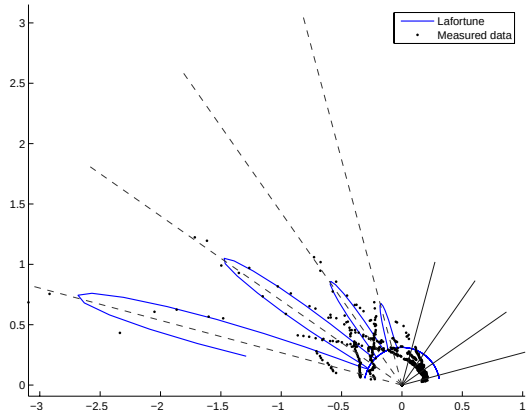
Ward



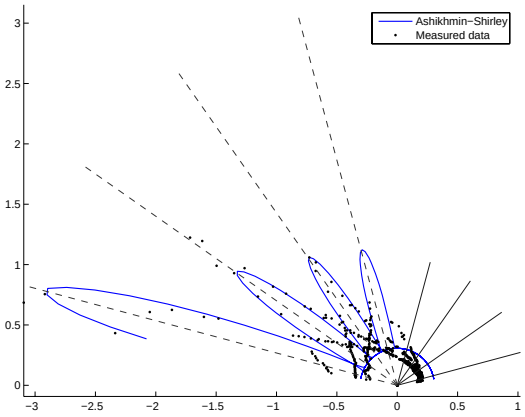
Ward-Duer



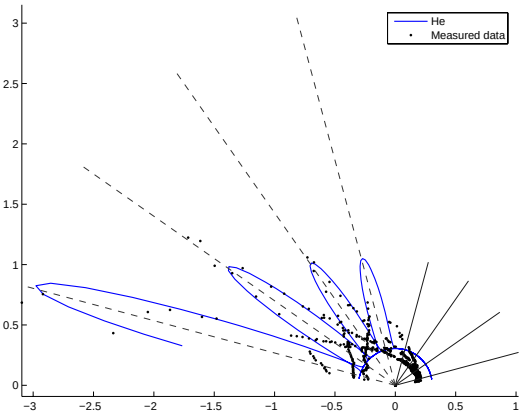
Cook-Torrance



Lafortune



Ashikhmin-Shirley



He

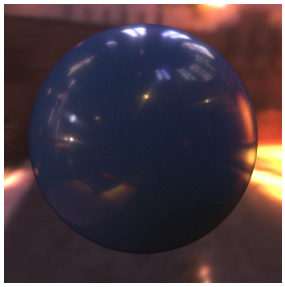
Complete Fitting Results

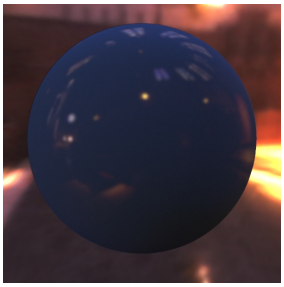
Material Name: acrylic-blue

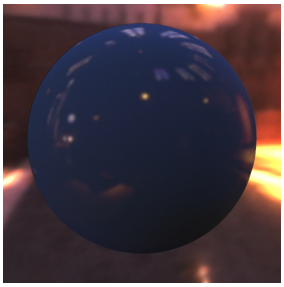
Fitted Parameters/Error

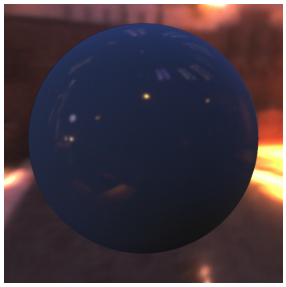
Model	d_r	d_g	d_b	s_r	s_g	s_b	p_0	p_1	p_2	Error
Ward	0.0138	0.0326	0.0636	0.00774	0.00547	0.00339	0.0162			0.0194
Ward-Duer	0.0137	0.0326	0.0637	0.00534	0.00364	0.00224	0.0162			0.0165
Blinn-Phong	0.0147	0.0332	0.064	0.0016	0.00115	0.000709	1.37e+004			0.0222
Lafortune et al.	0.0145	0.0332	0.064	0.0238	0.0164	0.01	-0.577	0.577	4.06e+003	0.0167
Cook-Torrance	0.0143	0.033	0.0639	0.0291	0.0193	0.0118	0.117	0.0137		0.0155
He et al.	0.0147	0.0334	0.0641	2.93	1.92	1.17	28.7	0.063	1.08	0.0141
Ashikhmin-Shirley	0.0143	0.0331	0.0639	0.0366	0.0241	0.0147	0.0949	1.16e+004		0.0153

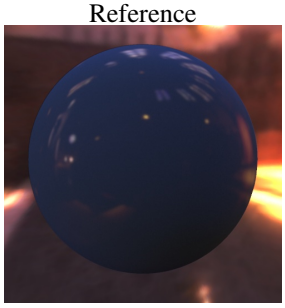
Rendered Images

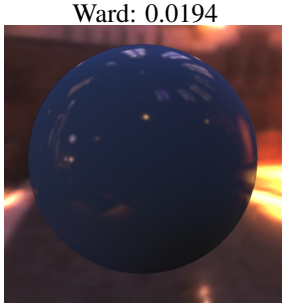

Reference

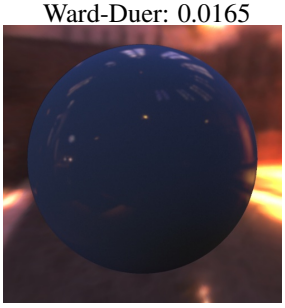

Ward: 0.0194

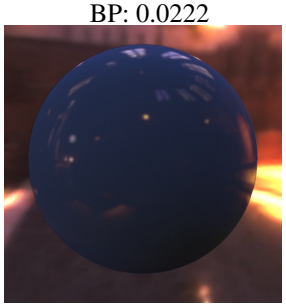

Ward-Duer: 0.0165


BP: 0.0222

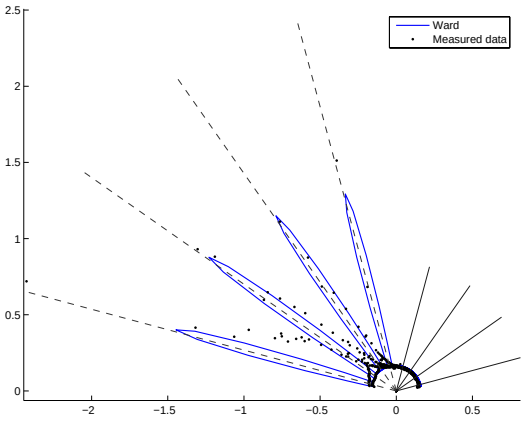

Lafortune: 0.0167


CT: 0.0155

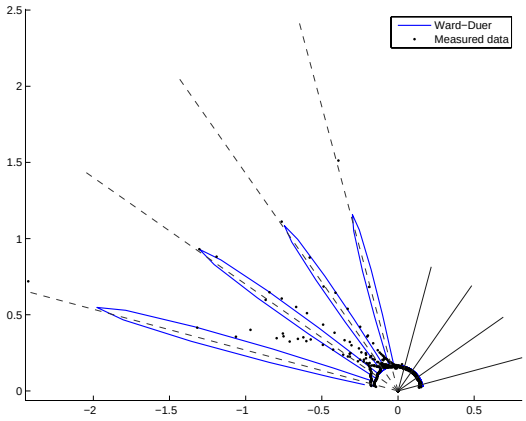

He: 0.0141


Ash: 0.0153

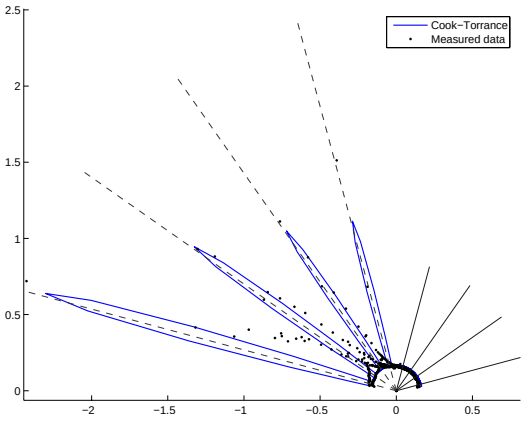
BRDF Plots in the incidence plane (Cubic root applied)



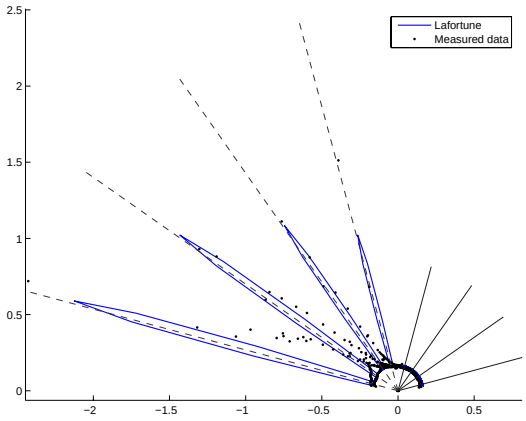
Ward



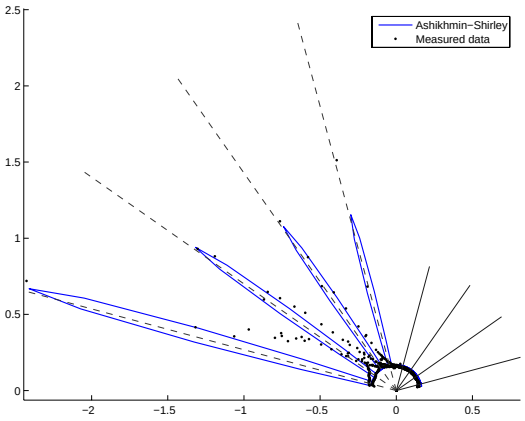
Ward-Duer



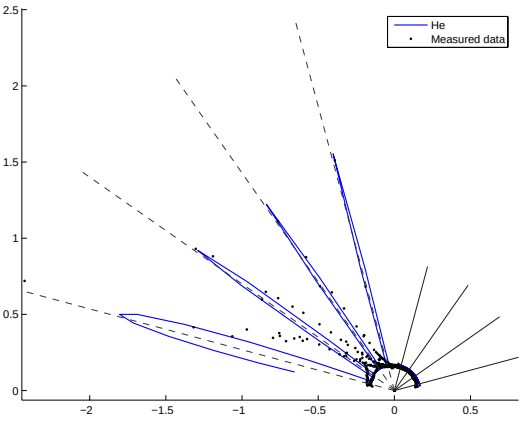
Cook-Torrance



Lafortune



Ashikhmin-Shirley



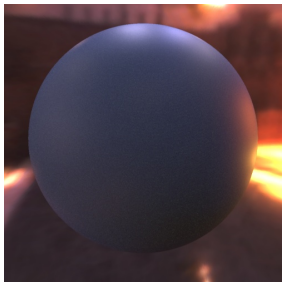
He

Material Name: blue-rubber

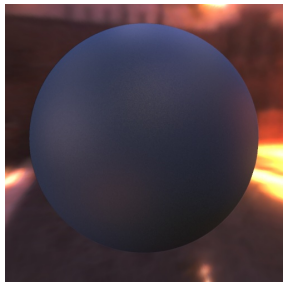
Fitted Parameters/Error

Model	d_r	d_g	d_b	s_r	s_g	s_b	p_0	p_1	p_2	Error
Ward	0.0309	0.0601	0.0891	0.0399	0.0343	0.0237	0.289			0.00956
Ward-Duer	0.029	0.0589	0.0885	0.0286	0.0241	0.0163	0.267			0.00645
Blinn-Phong	0.0425	0.0698	0.0957	0.00533	0.00471	0.00333	43.6			0.011
Lafortune et al.	0.0464	0.0736	0.0986	0.291	0.239	0.159	-0.635	0.44	32.6	0.00513
Cook-Torrance	0.0382	0.0669	0.094	0.247	0.204	0.136	0.0366	0.276		0.00139
He et al.	0.0378	0.0665	0.0938	0.549	0.454	0.302	23.8	3.48	1.31	0.00134
Ashikhmin-Shirley	0.0358	0.0649	0.0927	0.341	0.281	0.187	0.0532	20		0.00221

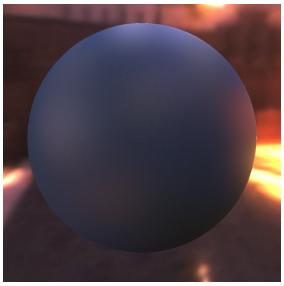
Rendered Images



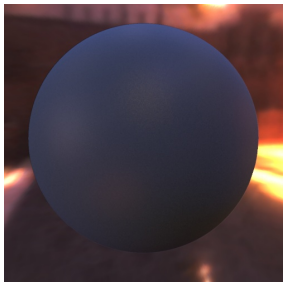
Reference



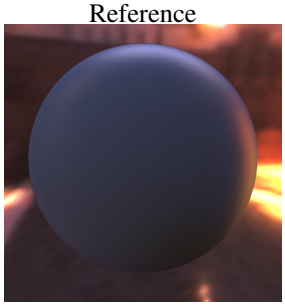
Ward: 0.00956



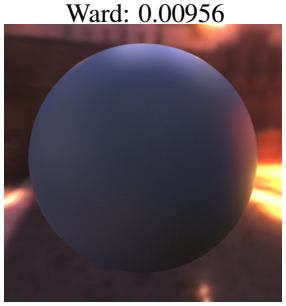
Ward-Duer: 0.00645



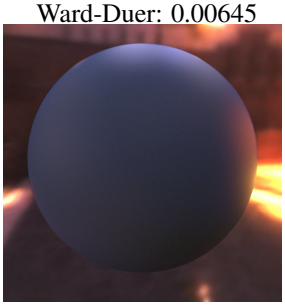
BP: 0.0110



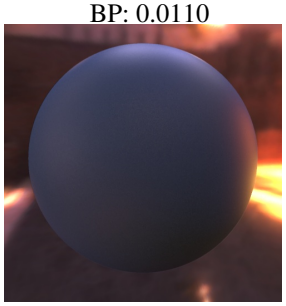
Lafortune: 0.00513



CT: 0.00139

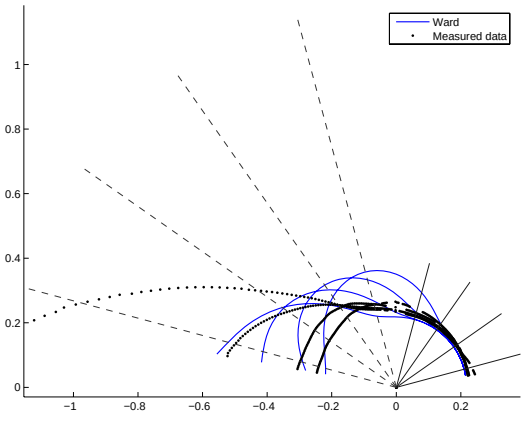


He: 0.00134

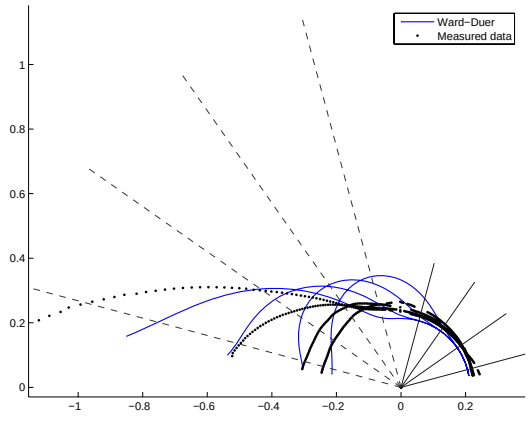


Ash: 0.00221

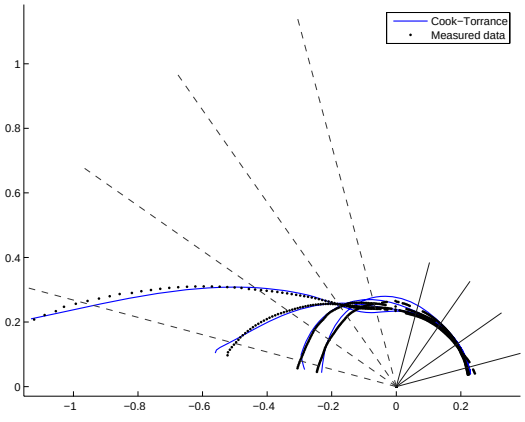
BRDF Plots in the incidence plane (Cubic root applied)



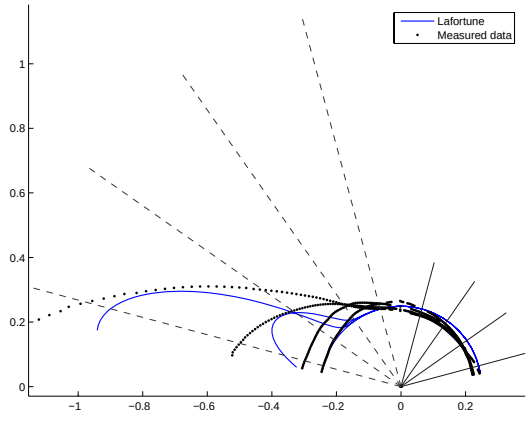
Ward



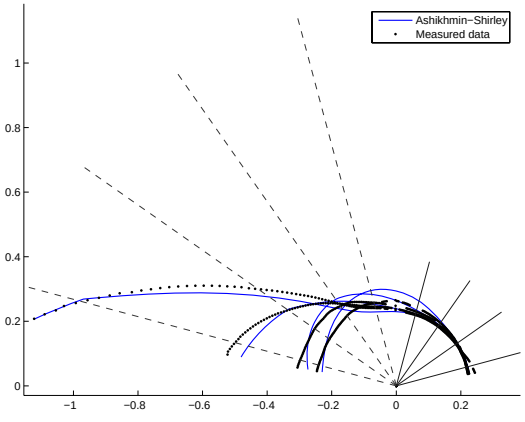
Ward-Duer



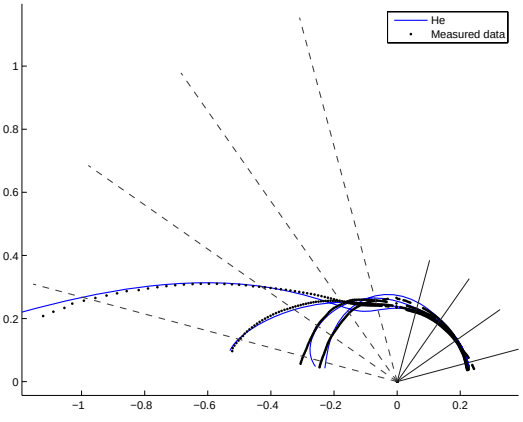
Cook-Torrance



Lafortune



Ashikhmin-Shirley



He

Material Name: chrome

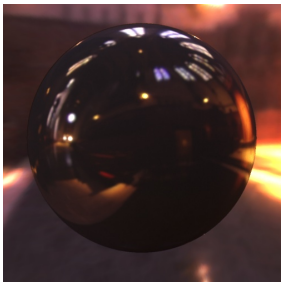
Fitted Parameters/Error

Model	d_r	d_g	d_b	s_r	s_g	s_b	p_0	p_1	p_2	Error
Ward	0.00651	0.00546	0.00498	0.0649	0.0458	0.0226	0.0101			0.562
Ward-Duer	0.0116	0.00999	0.00798	0.0423	0.0295	0.0141	0.00934			0.557
Blinn-Phong	0.00817	0.0063	0.00474	0.0213	0.0151	0.00766	1.79e+004			0.579
Lafortune et al.	0.0106	0.00755	0.00681	0.104	0.0749	0.0355	-0.577	0.577	6.63e+003	0.554
Cook-Torrance	0.0116	0.00995	0.00796	0.0428	0.0298	0.0143	0.991	0.00935		0.557
He et al.	0.0042	0.00343	0.00455	0.0841	0.0596	0.0286	37.6	0.205	8.95	0.555
Ashikhmin-Shirley	0.0124	0.0105	0.00829	0.0815	0.0568	0.0272	0.509	2.34e+004		0.557

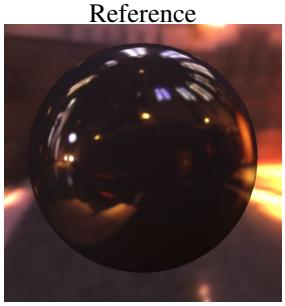
Rendered Images

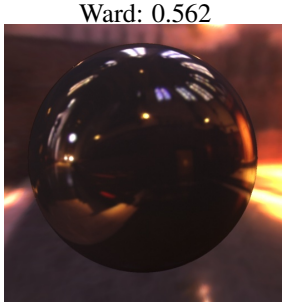

Reference

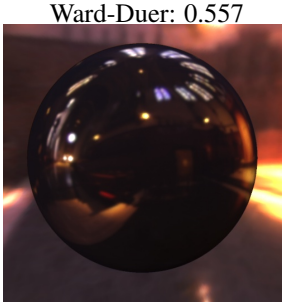

Ward: 0.562

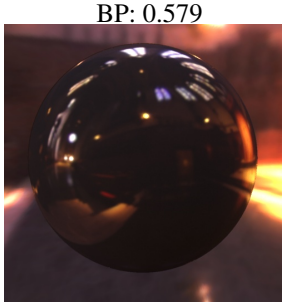

Ward-Duer: 0.557


BP: 0.579

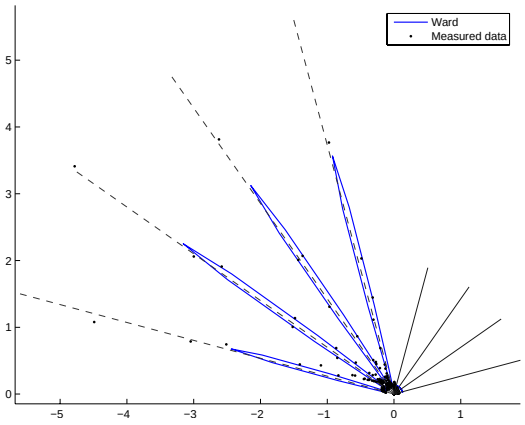

Lafortune: 0.554


CT: 0.557

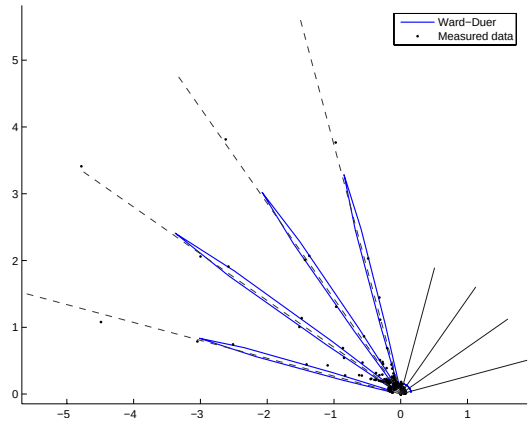

He: 0.555


Ash: 0.557

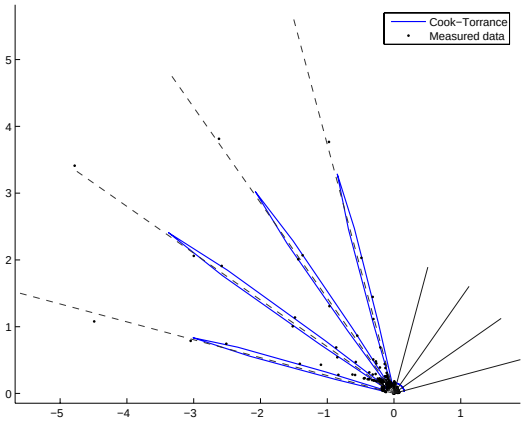
BRDF Plots in the incidence plane (Cubic root applied)



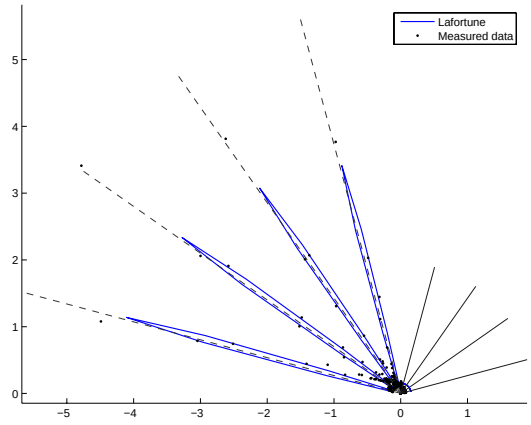
Ward



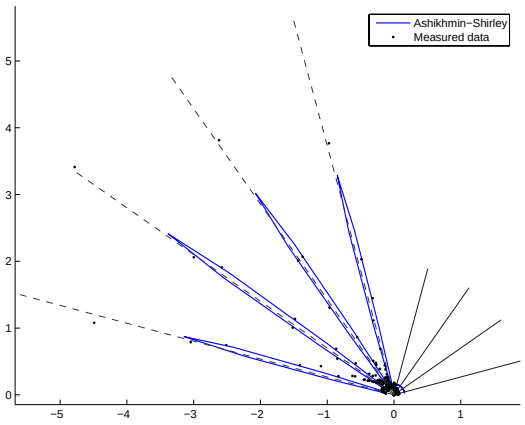
Ward-Duer



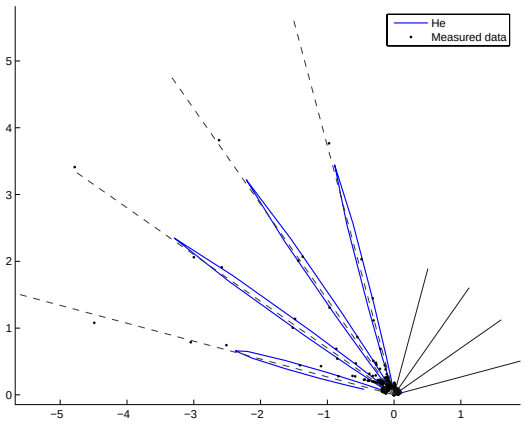
Cook-Torrance



Lafortune



Ashikhmin-Shirley



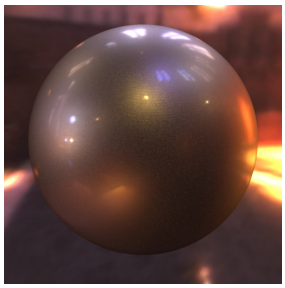
He

Material Name: ch-ball-gold-metallic2

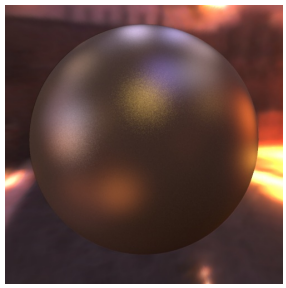
Fitted Parameters/Error

Model	d_r	d_g	d_b	s_r	s_g	s_b	p_0	p_1	p_2	Error
Ward	0.0651	0.0467	0.0232	0.149	0.111	0.0629	0.153			0.0277
Ward-Duer	0.0806	0.0581	0.0294	0.0864	0.0644	0.037	0.147			0.0292
Blinn-Phong	0.0633	0.0452	0.0226	0.0459	0.0342	0.0193	79.2			0.0305
Lafortune et al.	0.066	0.0474	0.0238	0.233	0.173	0.0978	-0.583	0.566	27.4	0.029
Cook-Torrance	0.0777	0.056	0.0282	0.142	0.106	0.0607	0.629	0.144		0.0286
He et al.	0.0719	0.0517	0.0258	0.229	0.17	0.0975	28	2.11	5.18	0.028
Ashikhmin-Shirley	0.072	0.0517	0.0257	0.107	0.0797	0.0458	0.999	80.1		0.0273

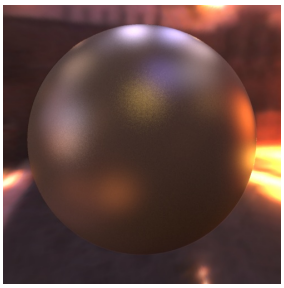
Rendered Images



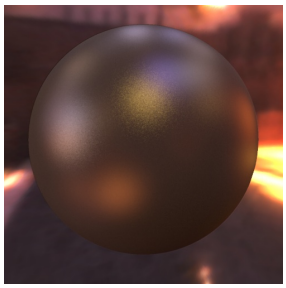
Reference



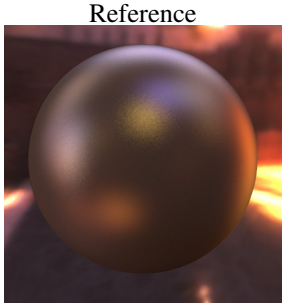
Ward: 0.0277



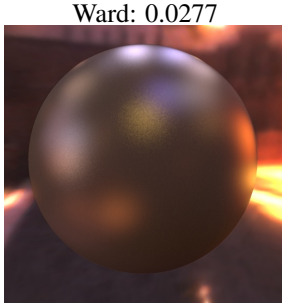
Ward-Duer: 0.0292



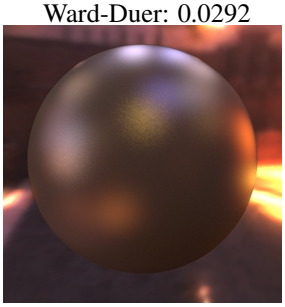
BP: 0.0305



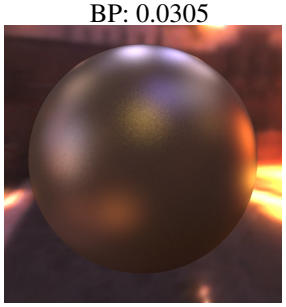
Lafortune: 0.0290



CT: 0.0286

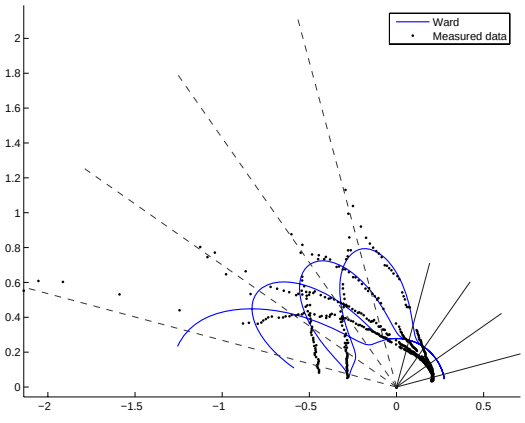


He: 0.0280

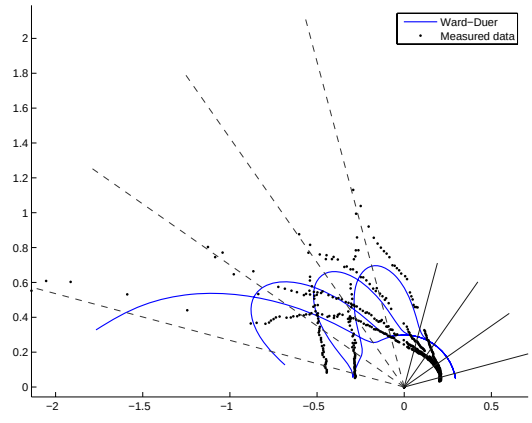


Ash: 0.0273

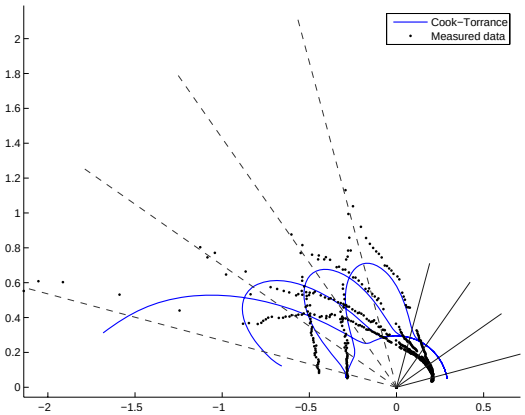
BRDF Plots in the incidence plane (Cubic root applied)



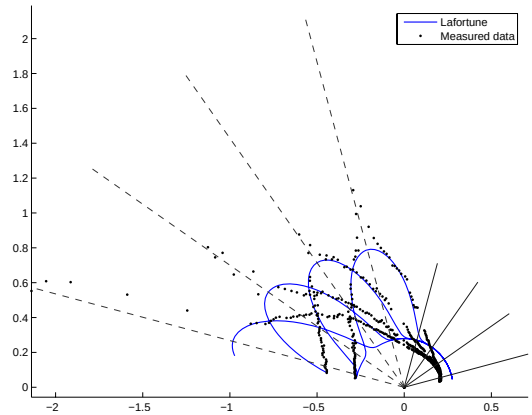
Ward



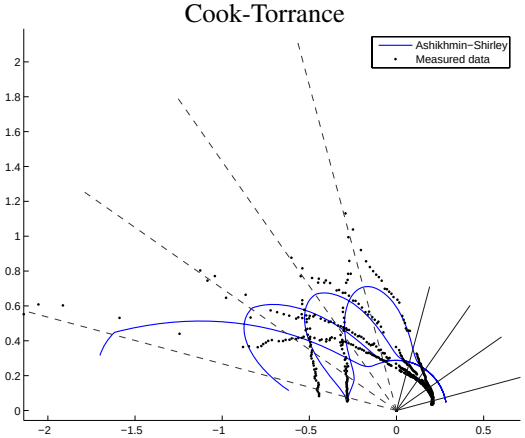
Ward-Duer



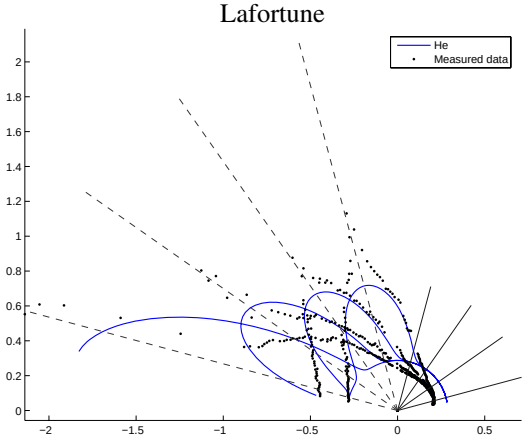
Cook-Torrance



Lafortune



Ashikhmin-Shirley



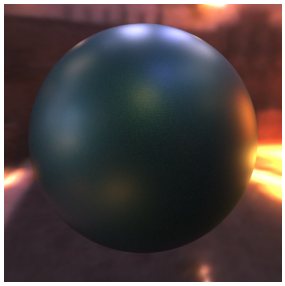
He

Material Name: green-metallic-paint

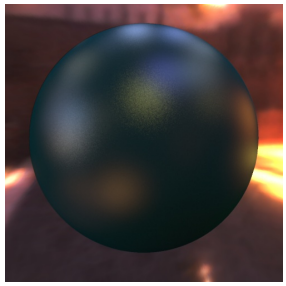
Fitted Parameters/Error

Model	d_r	d_g	d_b	s_r	s_g	s_b	p_0	p_1	p_2	Error
Ward	0	0.028	0.0242	0.0508	0.0655	0.0471	0.158			0.0197
Ward-Duer	0	0.0334	0.0283	0.0346	0.0379	0.0271	0.145			0.0115
Blinn-Phong	0.00479	0.0318	0.0267	0.0126	0.019	0.0138	72			0.0243
Lafortune et al.	0.015	0.0537	0.0428	0.22	0.217	0.153	-0.591	0.548	76	0.0176
Cook-Torrance	0.00303	0.0415	0.0341	0.164	0.165	0.117	0.137	0.143		0.00705
He et al.	0.00318	0.0416	0.0343	0.231	0.232	0.164	28	2.02	1.92	0.00709
Ashikhmin-Shirley	0.000627	0.0388	0.0322	0.207	0.21	0.149	0.146	79.6		0.00684

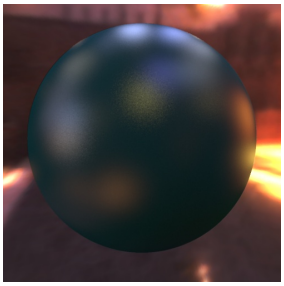
Rendered Images



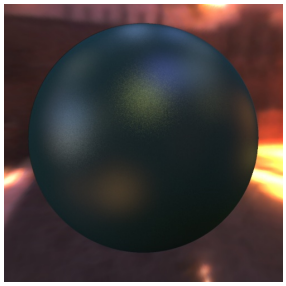
Reference



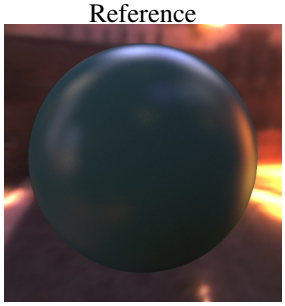
Ward: 0.0197



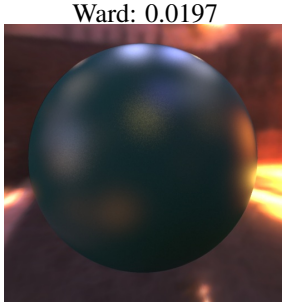
Ward-Duer: 0.0115



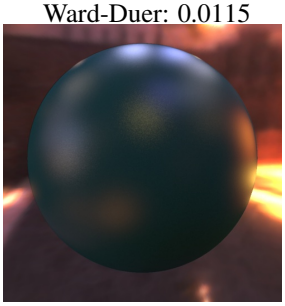
BP: 0.0243



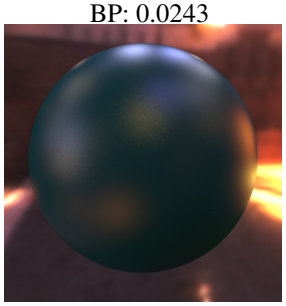
Lafortune: 0.0176



CT: 0.00705

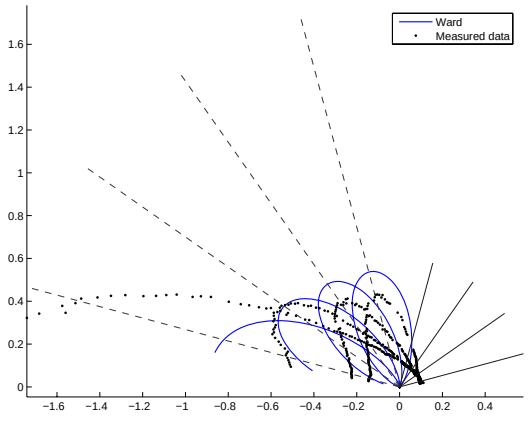


He: 0.00709

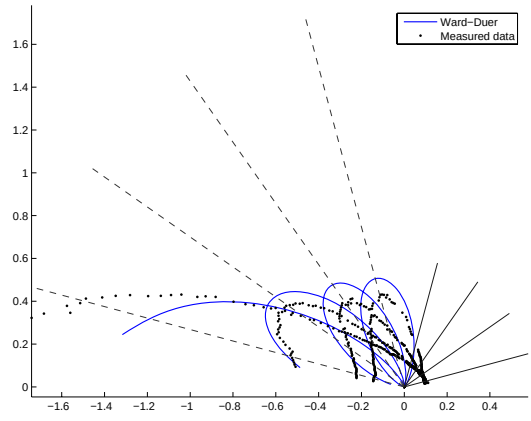


Ash: 0.00684

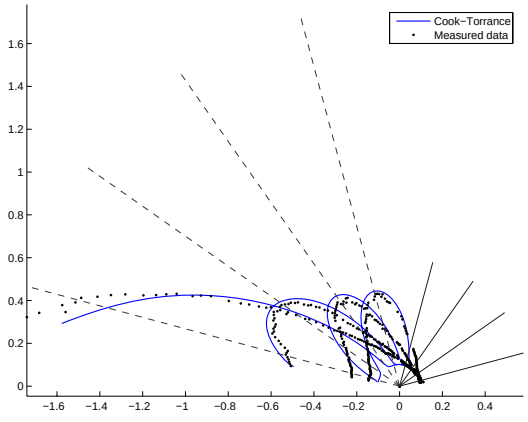
BRDF Plots in the incidence plane (Cubic root applied)



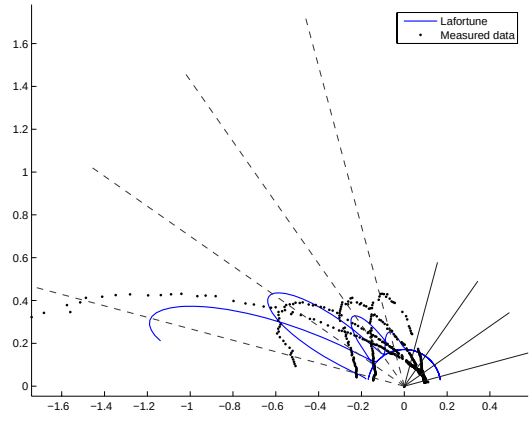
Ward



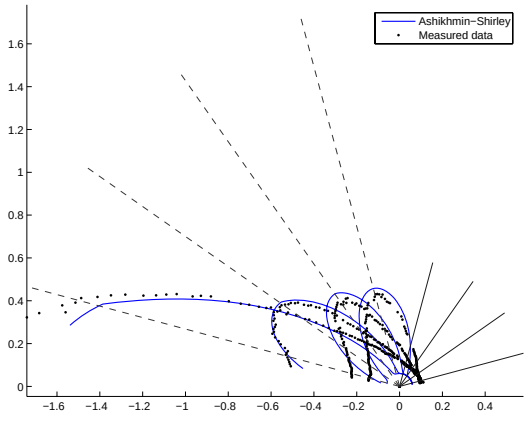
Ward-Duer



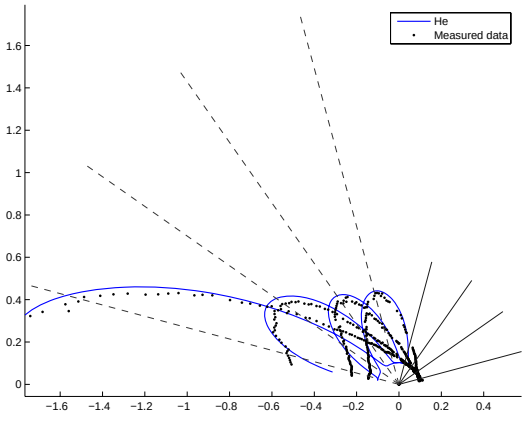
Cook-Torrance



Lafortune



Ashikhmin-Shirley



He