Warm-Up Exercises

Part III Astrostatistics

1 Product of Gaussian probability densities

Prove that the product of m multivariate Gaussian probability densities of a random d-dimensional vector $\boldsymbol{x}$:

$$\prod_{i=1}^{m} N(\boldsymbol{x}|\,\boldsymbol{\mu}_i, \boldsymbol{C}_i) \tag{1}$$

is proportional to a single Gaussian probability density in $\boldsymbol{x}$. Here the $\{\boldsymbol{\mu}_i, \boldsymbol{C}_i\}$ are m pairs of constant mean d-vectors and $d \times d$ covariance matrices. Find the mean and covariance matrix of the single resulting Gaussian. Simplify for the case of $d = 1$ and $m = 2$.

2 Sum of Gaussian random variables

Suppose x, y, are independent univariate Gaussian random variables. Their marginal distributions are given by:

$$x \sim N(\mu_x, \sigma_x^2) \tag{2}$$

$$y \sim N(\mu_y, \sigma_y^2) \tag{3}$$

Derive the probability density of $z = x + y$.